

Problem S4.8

Obtain the inverse Laplace transform of $\frac{s+1}{s(s+3)^2}$

Solution:

$$\frac{s+1}{s(s+3)^2} = \frac{A}{s} + \frac{B}{(s+3)^2} + \frac{C}{s+3}$$

$$A = s \frac{s+1}{s(s+3)^2} \Big|_{s=0} = \frac{1}{9}$$

$$B = (s+3)^2 \frac{s+1}{s(s+3)^2} \Big|_{s=-3} = \frac{s+1}{s} \Big|_{s=-3} = \frac{2}{3}$$

$$C = \frac{d}{ds} \left((s+3)^2 \frac{s+1}{s(s+3)^2} \right) \Big|_{s=-3} = \frac{d}{ds} \left(\frac{s+1}{s} \right) \Big|_{s=-3} = -\frac{1}{9}$$

$$\frac{s+1}{s(s+3)^2} = \frac{1}{9} + \frac{2}{(s+3)^2} + \frac{-1}{s+3}$$

$$f(t) = \frac{1}{9} + \frac{2}{3}te^{-3t} - \frac{1}{9}e^{-3t}$$