Problem S4.2

Obtain the inverse Laplace transform of $F(s) = \dfrac{1}{s^2 + 5s + 4}$

Solution:

The first step is to factor $s^2 + 5s + 4 = (s+4)(s+1)$ which means that F(s) has two simple poles.

$$F(s) = \frac{1}{(s+4)(s+1)} = \frac{a_1}{s+4} + \frac{a_2}{s+1}$$

$$a_1 = (s+4)\ \frac{1}{(s+4)(s+1)}\Bigg|_{s=-4} = \frac{1}{(s+1)}\Bigg|_{s=-4} = \frac{1}{-3}$$

$$a_2 = (s+1)\ \frac{1}{(s+4)(s+1)}\Bigg|_{s=-1} = \frac{1}{(s+4)}\Bigg|_{s=-1} = \frac{1}{3}$$

$$F(s) = \frac{1}{(s+4)(s+1)} = -\frac{\frac{1}{3}}{s+4} + \frac{\frac{1}{3}}{s+1}$$

$$f(t) = -\frac{1}{3}e^{-4t} + \frac{1}{3}e^{-t}$$