

Problem S4.1

Obtain the inverse Laplace transform of $F(s) = \frac{2}{s(s+1)}$

Solution:

$F(s)$ has two simple poles (at $s = 0$, $s = -1$)

$$F(s) = \frac{2}{s(s+1)} = \frac{a_1}{s} + \frac{a_2}{s+1}$$

$$a_1 = s \left. \frac{2}{s(s+1)} \right|_{s=0} = \left. \frac{2}{(s+1)} \right|_{s=0} = 2$$

$$a_2 = (s+1) \left. \frac{2}{s(s+1)} \right|_{s=-1} = \left. \frac{2}{s} \right|_{s=-1} = -2$$

$$F(s) = \frac{2}{s(s+1)} = \frac{2}{s} + \frac{-2}{s+1}$$

$$f(t) = 2e^{0t} - 2e^{-t} = 2 - 2e^{-t}$$