Serial #: _________ ID ______________ NAME ____

Show all necessary steps for full marks.

Question 1: (7 points): (6.3 Textbook Exercise 41): Given $y=\frac{1}{2}-\frac{1}{2}\cos\left(2x-\frac{\pi}{3}\right)$

(a): Find the amplitude, period horizontal shift and find the range.

(b): Graph the function over one period.

(c): Find the interval where the function is increasing.

(d): Find the interval where the function is decreasing.

Solution: (a): The amplitude is $\frac{1}{2}$, The period is π

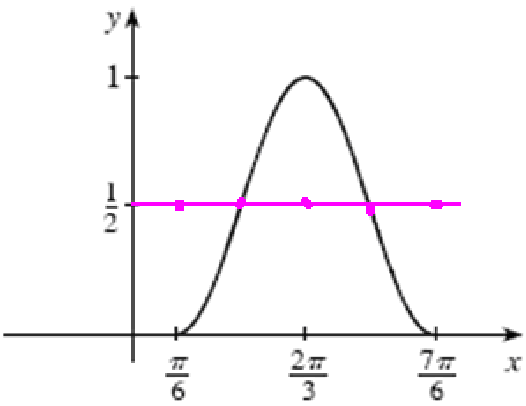
The horizontal shift is $\frac{\pi}{6}$ units to the right.

The range is $\left[-|a|+d,|a|+d\right]=\left[-\frac{1}{2}+\frac{1}{2},\frac{1}{2}+\frac{1}{2}\right]=[0,1]$

(b): $0\le 2x-\frac{\pi}{3}\le 2\pi$

$\frac{\pi}{3}\le 2x\le 2\pi+\frac{\pi}{3}$

$\frac{\pi}{6}\le 2x\le\frac{7\pi}{6}$



(c): Increasing on $\left[\frac{\pi}{6},\frac{2\pi}{3}\right]$ **(d):** Increasing on $\left[\frac{2\pi}{3},\frac{7\pi}{6}\right]$

Question 2: (6 points): Find the range of the function $y=1-\frac{1}{2}\csc\left(x-\frac{3\pi}{4}\right)$

Solution: Method I: $\left(-\infty,-|a|+d\right]\cup\left[|a|+d,\infty\right)=\left(-\infty,-\left|-\frac{1}{2}\right|+1\right]\cup\left[\left|-\frac{1}{2}\right|+1,\infty\right)$

$=\left(-\infty,-\frac{1}{2}+1\right]\cup\left[\frac{1}{2}+1,\infty\right)$

$=\left(-\infty,\frac{1}{2}\right]\cup\left[\frac{3}{2},\infty\right)$

Method II: If $y=a\csc(bx+c)+d$ Then $Range=\left(-\infty,-|a|+d\right]\cup\left[|a|+d,\infty\right)$

$$y = 1 - \frac{1}{2}\csc\left(x - \frac{3\pi}{4}\right)$$

$$\csc\left(x - \frac{3\pi}{4}\right) \le -1 \quad \text{or} \quad \csc\left(x - \frac{3\pi}{4}\right) \ge 1$$

$$-\frac{1}{2}\csc\left(x - \frac{3\pi}{4}\right) \ge \frac{1}{2} \quad \text{or} \quad -\frac{1}{2}\csc\left(x - \frac{3\pi}{4}\right) \le -\frac{1}{2}$$

$$1 - \frac{1}{2}\csc\left(x - \frac{3\pi}{4}\right) \ge 1 + \frac{1}{2} \quad \text{or} \quad 1 - \frac{1}{2}\csc\left(x - \frac{3\pi}{4}\right) \le 1 - \frac{1}{2}$$

$$y \ge \frac{3}{2} \quad \text{or} \quad y \le \frac{1}{2}$$

$$y \le \frac{1}{2} \quad \text{or} \quad y \ge \frac{3}{2} \qquad \Rightarrow \qquad Range = \left(-\infty, \frac{1}{2}\right] \cup \left[\frac{3}{2}, \infty\right)$$

Question 3: (7 points): Given the function $f(x) = 2\tan\left(\frac{x}{2} - \frac{\pi}{3}\right)$ where $x \in \left[-\frac{\pi}{3}, \frac{11\pi}{3}\right]$

(I): Sketch the graph of $f(x)$ over the given interval.
Using the above graph and the key points, find the following
(II): Find x-intercepts of f over the given interval.
(III): Find a general formula for all equations of vertical asymptotes.
(IV): Find all asymptotes over the given interval. $x \in \left[-\frac{\pi}{3}, \frac{11\pi}{3}\right]$

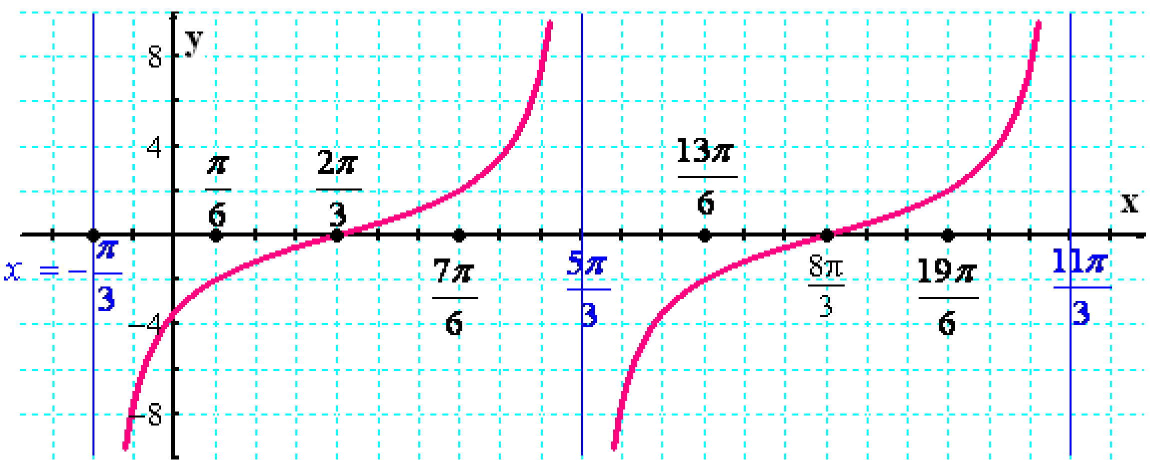
Solution: (I):

$$-\frac{\pi}{2} < \frac{x}{2} - \frac{\pi}{3} < \frac{\pi}{2}$$

$$-3\pi < 3x - 2\pi < 3\pi$$

$$-\pi < 3x < 5\pi$$

$$-\frac{\pi}{3} < x < \frac{5\pi}{3}$$



(II): Zeros: $\frac{2\pi}{3}$, $\frac{8\pi}{3}$.

(III): $x = -\frac{\pi}{3} + n(P) = -\frac{\pi}{3} + n(2\pi) = \frac{-\pi + 6n\pi}{3}$

$\boxed{x = (-1 + 6n)\frac{\pi}{3}}$ is the general form of the asymptotes

(IV): Vertical Asymptotes: $x = -\frac{\pi}{3}$, $x = \frac{5\pi}{3}$, $x = \frac{11\pi}{3}$