Code A	Serial:	ID:	**Name:**

Q1. (7 points): If $f(x)=\left(\frac{2}{3}\right)^{2-3x}$ is written as $f(x)=k\,a^x$, then $8a-27k=?$

(A) 15 (B) 39 (C) 19 (D) −15 (E) −19

Solution:

$$f(x)=\left(\frac{2}{3}\right)^{2-3x}$$

$$=\left(\frac{2}{3}\right)^{2}\cdot\left(\frac{2}{3}\right)^{-3x}$$

$$=\frac{4}{9}\cdot\left[\left(\frac{2}{3}\right)^{-3}\right]^{x}$$

$$=\frac{4}{9}\cdot\left(\frac{27}{8}\right)^{x}=k\,a^x \Rightarrow \boxed{k=\frac{4}{9}} \quad \boxed{a=\frac{27}{8}}$$

$$8a-27k=8\left(\frac{27}{8}\right)-27\left(\frac{4}{9}\right)=27-3(4)=15$$

Q2. (7 points): Given $f(x)=\frac{1}{5}x^2-\frac{4}{25}x+k$ where $x\ge 2$. If $f^{-1}(3)=5$, then $k=?$

Solution:

$$f^{-1}(3)=5$$

$$f\left(f^{-1}(3)\right)=f(5)$$

$$3=f(5)$$

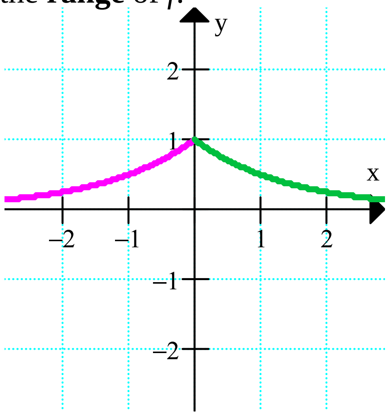
$$3=\frac{1}{5}\cdot 25-\frac{4}{25}\cdot 5+k$$

$$3=5-\frac{4}{5}+k$$

$$k=-2+\frac{4}{5} \Rightarrow \boxed{k=-\frac{6}{5}}$$

Q3. (8 points): **Graph** the function $f(x)=2^{-|x|}-1$ and find the **range** of f.

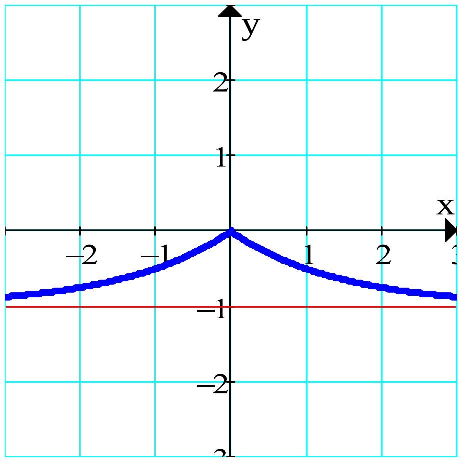
Solution: $f(x)=2^{-|x|}-1=\begin{cases}2^{-x}-1 & \text{if } x\ge 0\\ 2^{x}-1 & \text{if } x<0\end{cases}$



Let $f_1(x)=2^{-|x|}$ then $f_1(x)=\begin{cases}2^{-x} & \text{if } x\ge 0\\ 2^{x} & \text{if } x<0\end{cases}$

Shifting the above graph one unit downward vertically, gives the graph of $f(x)=2^{-|x|}-1$:

The range of f is $(-1,0]$



The range of f also can be obtained algebraically:

$0<\dfrac{1}{2^{|x|}}\le 1$ is true for any $x\in(-\infty,\infty)$

$0-1<\dfrac{1}{2^{|x|}}-1\le 1-1$

$-1<2^{-|x|}-1\le 0$

$-1<y\le 0$

$R_f=(-1,0]$

Q4. (7 points): Given $f(x)=\log_4\left(\dfrac{3-x}{x^2+x-2}\right)$. Find domain of $f(x)$.

Solution: $\dfrac{3-x}{x^2+x-2}>0$

$\dfrac{3-x}{(x+2)(x-1)}>0 \Rightarrow$ The critical values are -2, 1 and 3

	$-\infty$ to -2	-2 to 1	1 to 3	3 to $+\infty$
$3-x$	+	+	+	−
$x+2$	−	+	+	+
$x-1$	−	−	+	+
$\dfrac{3-x}{(x+2)(x-1)}$	+	−	+	−

$SS=(-\infty,2)\cup(1,3)$

$D_f=(-\infty,-2)\cup(1,3)$

Q5. (7 points): If a point on the edge of a circle with radius 30 cm is rotating with angular speed of $\frac{\pi}{10}$ radian per second, then **find the distance** traveled by the point in 45 seconds.

(A) 135π cm (B) 150π cm (C) 180π cm (D) 90π cm (E) 145π cm

Solution: Given: $r = 30$ cm $\omega = \frac{\pi}{10}\frac{\text{rad}}{\text{sec}}$ $t = 45$ sec

$$v = \frac{s}{t}$$
$$s = v\,t$$
$$= r\,w\,t$$
$$= (30cm)\left(\frac{\pi}{10}\frac{rad}{\sec}\right)(45\sec)$$
$$= 135\pi \text{ cm}$$

Answer: $\boxed{135\pi \text{ cm}}$

Q6. (7 points): If α is the reference angle of $675°$ and β is the least positive coterminal angle of $-240°$, then find $\alpha + \beta$.

Solution:

$675° - 360° = 315°$ is in Quadrant IV $\Rightarrow$ $\alpha = 360° - 315° = 45°$

$-240° + 360° = 120° = \beta$ is the smallest positive coterminal angle

Answer: $\alpha + \beta = 45° + 120° = 165°$

Q7. (7 points): If $\tan\theta = 4$ and $P(-3, n)$ is a point on the terminal side of θ where θ is in standard position, then find $\sec\theta + \sin\theta = ?$.

Solution:

$\tan\theta = 4$

The point $P(-3, n)$ is on the terminal side of θ.

$$\tan\theta = \frac{y}{x} = 4$$

$$\frac{n}{-3} = 4 \Rightarrow \boxed{n = -12} \Rightarrow r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + (-12)^2} = \sqrt{9+144} = \sqrt{9+9(16)} = \sqrt{9(1+16)} = 3\sqrt{17}$$

$$\sec\theta = \frac{r}{x} = \frac{3\sqrt{17}}{-3} = -\sqrt{17} \Rightarrow \boxed{\sec\theta = -\sqrt{17}}$$

$$\sin\theta = \frac{n}{r} = \frac{-12}{3\sqrt{17}} = -\frac{4}{\sqrt{17}} \Rightarrow \boxed{\sin\theta = -\frac{4\sqrt{17}}{17}}$$

$$\sec\theta + \sin\theta = -\sqrt{17} - \frac{4\sqrt{17}}{17} \Rightarrow \boxed{\sec\theta + \sin\theta = \frac{-21\sqrt{17}}{17}}$$