Serial #: ________ ID ______________ NAME ____

Show all necessary steps for full marks.

Question 1: (7 points) (1.7 Textbook Review Exercise 68, Page 171):

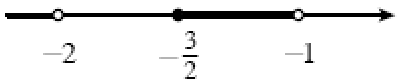
Find the solution set of the following inequality in interval notation. $\frac{1}{x+1}+\frac{1}{x+2}\le 0$.

Solution:

68. $\frac{1}{x+1}+\frac{1}{x+2}\le 0 \Leftrightarrow \frac{x+2}{(x+1)(x+2)}+\frac{x+1}{(x+1)(x+2)}\le 0 \Leftrightarrow \frac{x+2+x+1}{(x+1)(x+2)}\le 0 \Leftrightarrow \frac{2x+3}{(x+1)(x+2)}\le 0$. The expression on the left of the inequality changes sign when $x=-\frac{3}{2}$, $x=-1$, and $x=-2$. Thus we must check the intervals in the following table.

Interval	$(-\infty,-2)$	$\left(-2,-\frac{3}{2}\right)$	$\left(-\frac{3}{2},-1\right)$	$(-1,\infty)$
Sign of $2x+3$	$-$	$-$	$+$	$+$
Sign of $x+1$	$-$	$-$	$-$	$+$
Sign of $x+2$	$-$	$+$	$+$	$+$
Sign of $\frac{2x+3}{(x+1)(x+2)}$	$-$	$+$	$-$	$+$

From the table, the solution set is $\left\{x \mid x<-2 \text{ or } -\frac{3}{2}\le x<-1\right\}$. The points $x=-2$ and $x=-1$ are excluded from the solution because the expression is undefined at those values. Interval: $(-\infty,-2)\cup\left[-\frac{3}{2},-1\right)$.

Graph: 

Question 2: (7 points): If A is the solution set of $|2x-3|\le 5$ and B is the solution set of $|x-2|>1$, then $A\cap B=?$

Solution:

(a) $[-1,1)\cup(3,4]$

(b) $(3,4]$

(c) $(3,\infty)$

(d) $[-4,1)\cup(3,\infty)$

(e) ϕ (the empty set)

$-5 \le 2x-3 \le 5$

$-2 \le 2x \le 8$

$-1 \le x \le 4$

$\therefore A=[-1,4]$

$x-2<-1$ or $x-2>1$

$x<1$ $\quad$ $x>3$

$\therefore B=(-\infty,1)\cup(3,\infty)$

$A\cap B=[-1,1)\cup(3,4]$

A : [−1//////////////////////////////// 4]

B : /////////////////////1) (3/////////////////

Answer: $A\cap B=[-1,1)\cup(3,4]$

Question 3: (6 points): Given $f(x)=\dfrac{3x}{x+1}$. Find the difference quotient $\dfrac{f(a+h)-f(a)}{h}$, where $h \neq 0$

Solution:

$$\begin{aligned}
\frac{f(a+h)-f(a)}{h} &= \frac{1}{h}\left[f(a+h)-f(a)\right] \\
&= \frac{1}{h}\left[\frac{3(a+h)}{a+h+1}-\frac{3a}{a+1}\right] \\
&= \frac{1}{h}\cdot\frac{(3a+3h)(a+1)-3a(a+h+1)}{(a+h+1)(a+1)} \\
&= \frac{3a^2+3a+3ah+3h-3a^2-3ah-3a}{h(a+h+1)(a+1)} \\
&= \frac{3h}{h(a+h+1)(a+1)} \\
&= \frac{3}{(a+h-1)(a+1)}
\end{aligned}$$