King Fahd University of Petroleum and Minerals

Prep-Year Math Program

Math 001 Class Test I
Textbook Sections: P.2 to P.8
Term 171
Time Allowed: 90 Minutes

Time: 7:20 pm – 8:50 pm

Student's Name: …………… ………………………………………………
ID #:………………………. **Section:** ……. **Serial Number:** …….

Provide neat and complete solutions.
Show all necessary steps for full credit and write the answer in simplest form.

No Calculators, Cameras, or Mobiles are allowed during this exam.

Question	**Points**	Student's Score
1	4	
2	5	
3	4	
4	4	
5	4	
6	8	
7	4	
8	5	
9	4	
10	4	
11	4	
Total	**50**	____ / 50
		____ / 100

Q1. (4 points): If $x=\frac{1}{8}$, $y=-4$ and $w=-\frac{5}{7}$, then $\frac{(y+x)(1-w)}{2w}=?$

Solution:

$$\frac{(y+x)(1-w)}{2w}=\frac{\left(-4+\frac{1}{8}\right)\left(1+\frac{5}{7}\right)}{2\left(-\frac{5}{7}\right)}=\frac{\frac{-32+1}{8}\cdot\frac{7+5}{7}}{-\frac{10}{7}}=\frac{\frac{-31}{8}\cdot\frac{12}{7}}{-\frac{10}{7}}=\frac{31}{8}\cdot\frac{12}{7}\cdot\frac{7}{10}=\frac{31}{2}\cdot\frac{3}{10}=\frac{93}{20}$$

Q2. (5 points): Write TRUE or FALSE

(a) Any irrational number has a multiplicative inverse. ☐

(b) Every natural number is either prime or composite number. ☐

(c) Any irrational number has a terminating or repeating decimal expansion. ☐

(d) If x is a negative number then $|-x|=-x$. ☐

(e) $\sqrt{(3-\pi)^2}=3-\pi$. ☐

Solution:

(a): True — If x is any irrational number then $\frac{1}{x}$ is also an irrational number.

(b): False — 1 is a natural number but 1 is neither prime nor composite.

(c): False — Any irrational number has no terminating and has no repeating decimal expansion.

(d): True — $|-x|=|x|=-x$ because x is a negative number

(e): False — $\sqrt{(3-\pi)^2}=|3-\pi|=-(3-\pi)=-3+\pi=\pi-3$

Q3. (4 points): If $X=(a-2b)^2$ and $Y=(2a+b)^3$, then find $X-Y$.

Solution:

$$X-Y=(a-2b)^2-(2a+b)^3$$
$$=a^2-2a2b+(2b)^2-\left[(2a)^3+3(2a)^2b+3(2a)b^2+b^3\right]$$
$$=a^2-4ab+4b^2-8a^3-12a^2b-6ab^2-b^3$$

Q4. (4 points): If $A=\{x\,|x\le -3\}\cup\{x\,|x>1\}$, $B=\{x\,|-6\le x<8\}$ and $C=\{x\,|1\ge x>-1\}$ then find $(A\cap B)\cup C$.

Solution:

A: ←—— $-3]$ $(1$ ——→

B: $[-6$ ———— $8)$

$A\cap B$: $[-6$ —— $-3]$ $(1$ —— $8)$

C: $(-1$ —— $1]$

$$A\cap B\cup C=[-6,-3]\cup(1,8)\cup(-1,1]=[-6,-3]\cup(-1,8)$$

Answer: $[-6,-3]\cup(-1,8)$

Q5. (4 points): Simplify the expression $-7xy\sqrt[4]{32x^5y^6}+2x^2\sqrt[4]{2^9xy^{10}}$

Solution: Since the index is even, then $x>0$ and $y>0$.

$$\begin{aligned}-7xy\sqrt[4]{32x^5y^6}+2x^2\sqrt[4]{2^9xy^{10}} &= -7xy\sqrt[4]{2^4 2x^4xy^4y^2}+2x^2\sqrt[4]{2^8 2xy^8y^2}\\ &= -7xy\,2xy\sqrt[4]{2xy^2}+2x^2 2^2y^2\sqrt[4]{2xy^2}\\ &= -14x^2y^2\sqrt[4]{2xy^2}+8x^2y^2\sqrt[4]{2xy^2}\\ &= -6x^2y^2\sqrt[4]{2xy^2}\end{aligned}$$

Q6. (8 points): Factor **completely** each of the following expressions

(a) y^3-1-y^2+y

(b) $2(a+b)^2-5(a+b)-3$

(c) $8r^3-64t^6$

(d) $\frac{1}{2}x^{-1/2}(3x+4)^{1/2}+\frac{3}{2}x^{1/2}(3x+4)^{-1/2}$

Solution:

(a): $$\begin{aligned}y^3-1-y^2+y &= y^3-1^3-y(y-1)\\ &=(y-1)(y^2+y+1)-y(y-1)\\ &=(y-1)(y^2+y+1-y)=(y-1)(y^2+1)\end{aligned}$$

(b): $$\begin{aligned}2(a+b)^2-5(a+b)-3 &= [2(a+b)+1][(a+b)-3]\\ &=(2a+2b+1)(a+b-3)\end{aligned}$$

$2(a+b) \quad 1$

$(a+b) \quad -3$

(c): $$\begin{aligned}8r^3-64t^6 &= 8(r^3-8t^6)=8\left[r^3-(2t^2)^3\right]=8(r-2t^2)(r^2+r2t^2+(2t^2)^2)\\ &=8(r-2t^2)(r^2+2rt^2+4t^4)\end{aligned}$$

OR: $$\begin{aligned}8r^3-64t^6 &= (2r)^3-(4t^2)^3=(2r-4t^2)(4r^2+2r\cdot 4t^2+16t^4)\\ &=2(r-2t^2)4(r^2+2rt^2+4t^4)=8(r-2t^2)(r^2+2rt^2+4t^4)\end{aligned}$$

(d):

$$\frac{1}{2}x^{-1/2}(3x+4)^{1/2}+\frac{3}{2}x^{1/2}(3x+4)^{-1/2}=\frac{1}{2}x^{-1/2}(3x+4)^{-1/2}[(3x+4)+3x]$$

$$=\frac{1}{2}x^{-1/2}(3x+4)^{-1/2}(6x+4)=\frac{1}{2}x^{-1/2}(3x+4)^{-1/2}2(3x+2)=\boxed{x^{-1/2}(3x+4)^{-1/2}(3x+2)}$$

Q7. (4 points) If the coefficient of x^3 in the product $x^2\left(kx-\frac{2}{k}\right)\left(5x+\frac{1}{k}\right)$ is $-\frac{7}{4}$, then find k

Solution: $x^2\left(kx-\frac{2}{k}\right)\left(5x+\frac{1}{k}\right)=\left(kx^3-\frac{2}{k}x^2\right)\left(5x+\frac{1}{k}\right)$

Multiplying terms that give x^3:

$$kx^3\left(\frac{1}{k}\right)+\left(-\frac{2}{k}x^2\right)(5x)=-\frac{7}{4}x^3$$

$$x^3-\frac{10}{k}x^3=-\frac{7}{4}x^3$$

$$\left(1-\frac{10}{k}\right)x^3=-\frac{7}{4}x^3$$

$$1-\frac{10}{k}=-\frac{7}{4}$$

$$-\frac{10}{k}=-\frac{7}{4}-1$$

$$\frac{10}{k}=\frac{11}{4}$$

$$\boxed{k=\frac{40}{11}}$$

Q8. **(5 points)** Simplify the expression $\frac{x^2-1}{x^3-1}\cdot\frac{xy-2y+3x-6}{xy+3x+y+3}=?$

Solution:

$$\frac{x^2-1}{x^3-1}\cdot\frac{xy-2y+3x-6}{xy+3x+y+3}=\frac{(x-1)(x+1)}{(x-1)(x^2+x+1)}\cdot\frac{y(x-2)+3(x-2)}{x(y+3)+(y+3)}$$

$$=\frac{x+1}{x^2+x+1}\cdot\frac{(x-2)(y+3)}{(y+3)(x+1)}$$

$$=\frac{x-2}{x^2+x+1}$$

Q9. **(4 points)** The expression $\frac{x^2y^{-2}-y^2x^{-2}}{yx^{-1}+xy^{-1}}$ simplifies to

Solution:

$$\frac{x^2y^{-2}-y^2x^{-2}}{yx^{-1}+xy^{-1}}=\frac{x^2y^2\left(x^2y^{-2}-y^2x^{-2}\right)}{x^2y^2\left(yx^{-1}+xy^{-1}\right)}$$

$$=\frac{x^4-y^4}{xy^3+x^3y}=\frac{\left(x^2-y^2\right)\left(x^2+y^2\right)}{xy\left(y^2+x^2\right)}=\frac{x^2-y^2}{xy}=\frac{(x-y)(x+y)}{xy}$$

Q10. **(4 points)** Rationalize the denominator $\frac{6}{3+2\sqrt{12}-\sqrt{3}}-\frac{2}{\sqrt[3]{2}}=?$

Solution:

$$\frac{6}{3+2\sqrt{12}-\sqrt{3}}-\frac{2}{\sqrt[3]{2}}=\frac{6}{3+2\sqrt{4}\sqrt{3}-\sqrt{3}}-\frac{2}{\sqrt[3]{2}}=\frac{6}{3+4\sqrt{3}-\sqrt{3}}-\frac{2}{\sqrt[3]{2}}=\frac{6}{3+3\sqrt{3}}-\frac{2}{\sqrt[3]{2}}$$

$$=\frac{2}{1+\sqrt{3}}\cdot\frac{1-\sqrt{3}}{1-\sqrt{3}}-\frac{2}{\sqrt[3]{2}}\cdot\frac{\sqrt[3]{4}}{\sqrt[3]{4}}=\frac{2(1-\sqrt{3})}{1-3}-\frac{2\sqrt[3]{4}}{2}=-1(1-\sqrt{3})-\sqrt[3]{4}=-1+\sqrt{3}-\sqrt[3]{4}$$

Q11. (4 points): Solve the equations for k .

(a): $-k = (5k + 3)(3x + 1)$

(b): $\frac{k+1}{b} = \frac{k-1}{b} + \frac{b+1}{k}$

Solution (a):

$-k = (5k + 3)(3x + 1)$

$-k = 15kx + 5k + 9x + 3$

$-6k - 15kx = 9x + 3$

$k(-6 - 15x) = 9x + 3$

$k = \frac{9x + 3}{-15x - 6} = \frac{3(3x + 1)}{3(-5x - 2)} = \frac{3x + 1}{-5x - 2}$

(b): $\frac{k+1}{b} = \frac{k-1}{b} + \frac{b+1}{k}$

Multiply both sides by kb :

$kb \cdot \frac{k+1}{b} = kb \cdot \frac{k-1}{b} + kb \cdot \frac{b+1}{k}$

$k^2 + k = k^2 - k + b^2 + b$

$2k = b^2 + b$

$k = \frac{b(b+1)}{2}$