King Fahd University of Petroleum and Minerals

Prep-Year Math Program

Math 001 Class Test I
Textbook Sections: R.1 to R.7
Term 161
Time Allowed: 90 Minutes

Time: 5:30 pm – 7:00 pm

Student's Name: ....
ID #:........................... **Section:** .... **Serial Number:** ....

Provide neat and complete solutions.
Show all necessary steps for full credit and write the answer in simplest form.

No Calculators, Cameras, or Mobiles are allowed during this exam.

Question	**Points**	Student's Score
1	8	
2	4	
3	4	
4	4	
5	4	
6	4	
7	4	
8	4	
9	4	
10	4	
11	3	
12	3	
Total	**50**	____ / 50
		____ / 100

Q1. (8 points): Write TRUE or FALSE

(a) Every rational number has a multiplicative inverse. $\square$

(b) Every even integer has an additive inverse. $\square$

(c) The set $\{-1, 0, 1\}$ is closed with respect to multiplication. $\square$

(d) Any irrational number has a multiplicative inverse. $\square$

(e) If x is any integer and y is any irrational number, then $\dfrac{x}{y}$ is irrational. $\square$

(f) Any irrational number has a terminating or repeating decimal expansion. $\square$

(g) If x is a negative number then $|-x| = -x$. $\square$

(h) $\sqrt{(3-\pi)^2} = 3-\pi$. $\square$

Solution:

(a): $\boxed{\text{False}}$. Because 0 is a rational number but 0 does not have a multiplicative inverse.

(b): $\boxed{\text{True}}$. Every even integer has an additive inverse.

(c): $\boxed{\text{True}}$. The set $\{-1, 0, 1\}$ is closed with respect to multiplication.

(d): $\boxed{\text{True}}$. If x is any irrational number then $\dfrac{1}{x}$ is also an irrational number.

(e): $\boxed{\text{False}}$. Because $\dfrac{0}{\sqrt{2}} = 0$ is a rational number.

(f): $\boxed{\text{False}}$. No any irrational number has a terminating or repeating decimal expansion.

(g): $\boxed{\text{True}}$. $|-x| = |x| = -x$ because x is a negative number

(h): $\boxed{\text{False}}$. $\sqrt{(3-\pi)^2} = |3-\pi| = -(3-\pi) = -3+\pi = \pi-3$.

Q2. (4 points): If $\dfrac{1}{3} \le x < \dfrac{2}{3}$, then write the expression $\left|x-\dfrac{2}{3}\right| - \left|\dfrac{1}{9}-x\right| + \left|\dfrac{1}{3}+x\right|$ without the absolute value symbols:

Solution: Note that $\dfrac{1}{9} < \dfrac{1}{3} \le x < \dfrac{2}{3}$

$$\begin{aligned}
\left|x-\frac{2}{3}\right| - \left|\frac{1}{9}-x\right| + \left|\frac{1}{3}+x\right| &= \left|\frac{2}{3}-x\right| - \left|x-\frac{1}{9}\right| + \left|\frac{1}{3}+x\right| \\
&= \frac{2}{3}-x-\left(x-\frac{1}{9}\right)+\frac{1}{3}+x \qquad \text{since } \frac{1}{9} < \frac{1}{3} \\
&= \frac{2}{3}-x-x+\frac{1}{9}+\frac{1}{3}+x \\
&= \frac{2}{3}+\frac{1}{9}+\frac{1}{3}-x \\
&= \frac{6}{9}+\frac{1}{9}+\frac{3}{9}-x \\
&= \frac{10}{9}-x
\end{aligned}$$

Another Method:

$$\frac{1}{3}\le x<\frac{2}{3} \Rightarrow \frac{1}{3}-\frac{2}{3}\le x-\frac{2}{3}<0$$

$$\frac{1}{3}\le x<\frac{2}{3} \Rightarrow -\frac{1}{3}\ge -x>-\frac{2}{3} \Rightarrow \frac{1}{9}-\frac{1}{3}\ge\frac{1}{9}-x>\frac{1}{9}-\frac{2}{3} \Rightarrow 0>\frac{1}{9}-x$$

$$\left|x-\frac{2}{3}\right|-\left|\frac{1}{9}-x\right|+\left|\frac{1}{3}+x\right|=-\left(x-\frac{2}{3}\right)-\left[-\left(\frac{1}{9}-x\right)\right]+\frac{1}{3}+x$$

$$=-x+\frac{2}{3}+\frac{1}{9}-x+\frac{1}{3}+x=\frac{10}{9}-x$$

Q3. (4 points): If $X=(a-2b)^3$ and $Y=(2a+b)^3$, then find $X-Y$.

Solution:

$$\begin{aligned}X-Y&=(a-2b)^3-(2a+b)^3\\&=a^3-3a^2 2b+3a(2b)^2-(2b)^3-\left[(2a)^3+3(2a)^2b+3(2a)b^2+b^3\right]\\&=a^3-6a^2b+12ab^2-8b^3-8a^3-12a^2b-6ab^2-b^3\\&=-7a^3-18a^2b+6ab^2-9b^3\end{aligned}$$

Q4. (4 points): If $\frac{2x^3-11x^2+28}{x-5}=Q(x)+\frac{R(x)}{x-5}$, find $Q(x)=?$ and $R(x)=?$

Solution:

$$\frac{2x^3-11x^2+28}{x-5}$$

$$\begin{array}{r|l} & 2x^2 \quad -x \quad -5 \\ \hline x-5 & 2x^3-11x^2+0x+28 \\ & \underline{2x^3-10x^2} \\ & -x^2+0x \\ & \underline{-x^2+5x} \\ & -5x+28 \\ & \underline{-5x+25} \\ & 3 \end{array}$$

$Q(x)=2x^2-x-5$, $R(x)=3$

$$\frac{2x^3-11x^2+28}{x-5}=2x^2-x-5+\frac{3}{x-5}$$

Q5. (4 points)(Textbook Review exercise 67): Factor $48a^8-12a^7b-90a^6b^2$

Solution:

67. $48a^8-12a^7b-90a^6b^2$

$$=6a^6(8a^2-2ab-15b^2)$$

$$=6a^6(4a+5b)(2a-3b)$$

Q6. (4 points)(Textbook Review exercise 80): Simplify $\dfrac{27m^3-n^3}{3m-n}\div\dfrac{9m^2+3mn+n^2}{9m^2-n^2}=?$

Solution:

80. $$\frac{27m^3-n^3}{3m-n}\div\frac{9m^2+3mn+n^2}{9m^2-n^2}$$
$$=\frac{27m^3-n^3}{3m-n}\cdot\frac{9m^2-n^2}{9m^2+3mn+n^2}$$
$$=\frac{(3m)^3-n^3}{3m-n}\cdot\frac{(3m)^2-n^2}{9m^2+3mn+n^2}$$
$$=\frac{(3m-n)[(3m)^2+3mn+n^2]}{3m-n}\cdot\frac{(3m+n)(3m-n)}{9m^2+3mn+n^2}$$
$$=\frac{(3m-n)(9m^2+3mn+n^2)(3m+n)(3m-n)}{(3m-n)(9m^2+3mn+n^2)}$$
$$=(3m+n)(3m-n)$$

Q7. (4 points) (R.6 Recitation Q1a): If Simplify $\dfrac{2}{4+x}+\dfrac{16}{x^2-16}+\dfrac{6}{4-x}$

Solution:

$$\frac{2}{4+x}+\frac{16}{x^2-16}+\frac{6}{4-x}=\frac{2}{4+x}+\frac{6}{4-x}+\frac{16}{(x-4)(x+4)}$$
$$=\frac{2}{x+4}-\frac{6}{x-4}+\frac{16}{(x-4)(x+4)}$$
$$=\frac{2x-8-6x-24}{(x-4)(x+4)}+\frac{16}{(x-4)(x+4)}$$
$$=\frac{-4x-32}{(x-4)(x+4)}+\frac{16}{(x-4)(x+4)}$$
$$=\frac{-4x-16}{(x-4)(x+4)}=\frac{-4(x+4)}{(x-4)(x+4)}=\frac{-4}{x-4}$$

Q8. (4 points) (R.4 Recitation Q4): Factor of $4x^2-8xy-5y^2-4x+10y$

Solution:

$2x \quad -5y$

$2x \quad +y$

$$4x^2-8xy-5y^2-4x+10y=(2x-5y)(2x+y)-2(2x-5y)$$
$$=(2x-5y)[(2x+y)-2]$$
$$=(2x-5y)(2x+y-2)$$

Q9. (4 points) (R.6 Textbook exercise 96): Factor completely $7(5t+3)^{-5/3}+(5t+3)^{-2/3}-21(5t+3)^{1/3}$

Solution:

$$7(5t+3)^{-5/3}+(5t+3)^{-2/3}-21(5t+3)^{1/3}=(5t+3)^{-5/3}\left[7+(5t+3)^{3/3}-21(5t+3)^{6/3}\right]$$
$$=(5t+3)^{-5/3}\left[7+(5t+3)-21(5t+3)^2\right]$$

Q10. (4 points) (R.7 Recitation Q4): Find the value of $\dfrac{2}{\sqrt[3]{81}}+\dfrac{4}{\sqrt[3]{24}}-\dfrac{1}{\sqrt[3]{3}}$

Solution:

$$\frac{2}{\sqrt[3]{81}}+\frac{4}{\sqrt[3]{24}}-\frac{1}{\sqrt[3]{3}}=\frac{2}{\sqrt[3]{3^3(3)}}+\frac{4}{\sqrt[3]{2^3(3)}}-\frac{1}{\sqrt[3]{3}}$$
$$=\frac{2}{3\sqrt[3]{3}}+\frac{4}{2\sqrt[3]{3}}-\frac{1}{\sqrt[3]{3}}$$
$$=\frac{1}{\sqrt[3]{3}}\left(\frac{2}{3}+2-1\right)$$
$$=\frac{\sqrt[3]{9}}{\sqrt[3]{3}\sqrt[3]{9}}\left(\frac{2}{3}+1\right)$$
$$=\frac{\sqrt[3]{9}}{3}\left(\frac{5}{3}\right)=\frac{5\sqrt[3]{9}}{9}$$

Q11. (3 points)(R.7 Textbook Exercise 86): Simplify $\dfrac{\sqrt[3]{8m^2n^3}\cdot\sqrt[3]{2m^2}}{\sqrt[3]{32m\,4n^3}}$

Solution:

$$\frac{\sqrt[3]{8m^2n^3}\cdot\sqrt[3]{2m^2}}{\sqrt[3]{32m\,4n^3}}=\frac{\sqrt[3]{2^3m^2n^32m^2}}{\sqrt[3]{2^5m\,2^2n^3}}=\sqrt[3]{\frac{2^4m^4n^3}{2^7m\,n^3}}=\sqrt[3]{\frac{m^3}{2^3}}=\frac{m}{2}$$

Q12. (3 points) Find the value of the expression $-17+3\left[8x-4(3x-2)\right]$ when $x=-\dfrac{3}{4}$ is

Solution:

$$-17+3\left[8x-4(3x-2)\right]=-17+3[8x-12x+8]$$
$$=-17+3[-4x+8]$$
$$=-17+3\left[(-4)\left(-\frac{3}{4}\right)+8\right]$$
$$=-17+3[3+8]$$
$$=-17+33$$
$$=16$$