Serial #: _____ ID _____ NAME _____

Show all necessary steps for full marks.

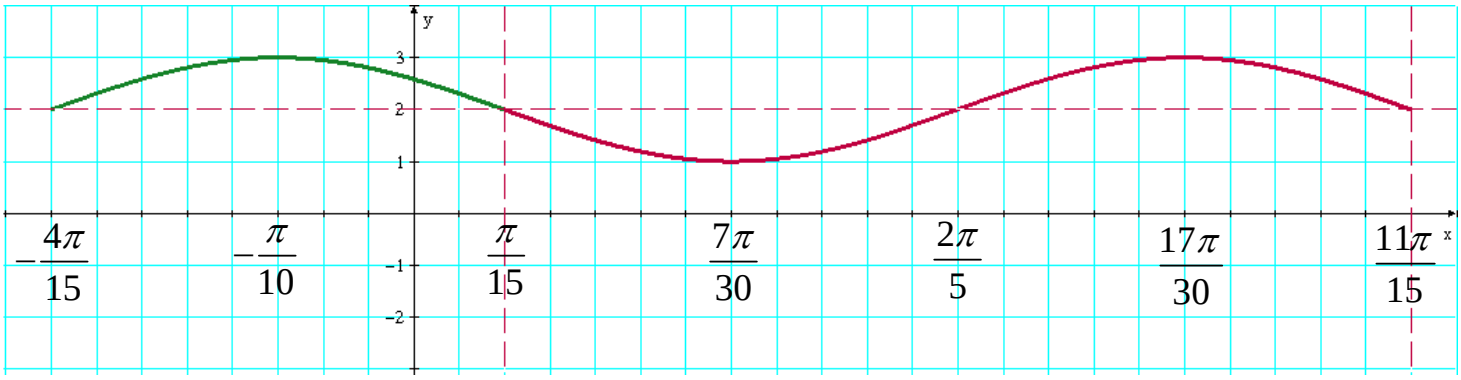
Question 1: (9 points) (6.4 Textbook Exercise 31): Given $y = 2 - \sin\left(3x - \frac{\pi}{5}\right)$ where $x \in \left[-\frac{4\pi}{15}, \frac{11\pi}{15}\right]$

(a): Graph the function over the given interval

(b): Find the intervals where the function is increasing.

(c): Find the intervals where the function is decreasing.

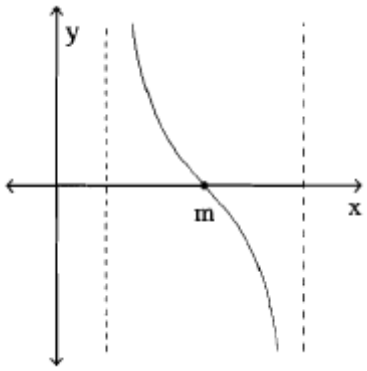
Solution: (a):



(b): Increasing on $\left[-\frac{4\pi}{15}, \frac{-\pi}{10}\right]$ and on $\left[\frac{7\pi}{30}, \frac{17\pi}{30}\right]$

(c): decreasing on $\left[-\frac{\pi}{10}, \frac{7\pi}{30}\right]$ and on $\left[\frac{17\pi}{30}, \frac{11\pi}{15}\right]$

Question 2: (5 points): If the adjacent figure represents a part of the graph of $y = \cot\left[\frac{x}{2} - \frac{\pi}{4}\right]$, then $m = ?$



Solution:

$$y = \cot\left[\frac{x}{2} - \frac{\pi}{4}\right] \Rightarrow P = \frac{\pi}{1/2} = 2\pi$$

Phase shift: $\frac{x}{2} - \frac{\pi}{4} = 0 \Rightarrow x = \frac{\pi}{2} \Rightarrow m = \frac{\pi}{2} + \frac{1}{2}p = \frac{\pi}{2} + \frac{1}{2}2\pi = \frac{3\pi}{2}$

Another Method:

$$0<\frac{x}{2}-\frac{\pi}{4}<\pi \quad \Rightarrow \quad \frac{\pi}{4}<\frac{x}{2}<\pi+\frac{\pi}{4} \quad \Rightarrow \quad \frac{\pi}{2}<x<2\pi+\frac{\pi}{2} \quad \Rightarrow \quad \frac{\pi}{2}<x<\frac{5\pi}{2}$$

Then m is the middle point $m=\frac{1}{2}\left(\frac{\pi}{2}+\frac{5\pi}{2}\right)=\frac{1}{2}\left(\frac{6\pi}{2}\right)=\frac{3\pi}{2}$

Question 3: (6 points): Find the vertical asymptotes of $y=-3\csc\left(\frac{x}{3}-\frac{\pi}{6}\right)+2$ in the interval $[-4\pi,4\pi]$.

Solution:

$$y=-3\csc\left(\frac{x}{3}-\frac{\pi}{6}\right)+2=-3\frac{1}{\sin\left(\frac{x}{3}-\frac{\pi}{6}\right)}+2$$

$\frac{x}{3}-\frac{\pi}{6}=n\pi$, where n is an integer.

$$2x-\pi=6n\pi$$
$$2x=\pi+6n\pi$$
$$2x=(1+6n)\pi$$
$$x=\frac{(1+6n)\pi}{2}$$

$n=0 \Rightarrow \boxed{x=\frac{\pi}{2}} \in[-4\pi,4\pi]$

$n=-1 \Rightarrow \boxed{x=-\frac{5\pi}{2}} \in[-4\pi,4\pi]$

$n=1 \Rightarrow \boxed{x=\frac{7\pi}{2}} \in[-4\pi,4\pi]$

$n=2 \Rightarrow x=\frac{13\pi}{2} \notin[-4\pi,4\pi]$

$n=-2 \Rightarrow x=-\frac{11\pi}{2} \notin[-4\pi,4\pi]$