King Fahd University of Petroleum and Minerals

Prep-Year Math Program

Math 002 Class Test 2B
Textbook Sections: 6.3 to 8.3
Term 153
Time Allowed: 70 Minutes

Student's Name: …………… ………………………………………………
ID #:……………………….   **Section:** 13…….   **Serial Number:** …….

Provide neat and complete solutions.
Show all necessary steps for full credit and write the answer in simplest form.

No Calculators, Cameras, or Mobiles are allowed during this exam.

Question	**Points**	Student's Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	5	
7	5	
8	5	
9	6	
10	4	
Total	**50**	____ 50
		____ 100

Q1. (5 points) (7.1 Recitation Q#1): If $\sec\theta = \dfrac{x+4}{x}$, then find $\csc\theta = ?$

Solution:

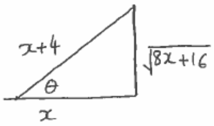
Question5

If $sec\theta = \frac{x+4}{x}$, then $csc\theta =$

a) $\pm\frac{(x+4)\sqrt{2x+4}}{4(x+2)}$ b) $\pm\frac{(x+4)\sqrt{x+2}}{8(x+2)}$ c) $\pm\frac{\sqrt{2x+4}}{(x+4)}$

d) $\pm\frac{(x+4)\sqrt{x+2}}{2x+4}$ e) $\pm\frac{2\sqrt{x+2}}{x+4}$

$$\csc\theta = \pm\frac{(x+4)}{\sqrt{8x+16}} = \pm\frac{(x+4)}{\sqrt{8}\,\sqrt{x+2}}\cdot\left(\frac{\sqrt{8}\,\sqrt{x+2}}{\sqrt{8}\,\sqrt{x+2}}\right)$$



$$= \pm\frac{\sqrt{8}\,(x+4)\sqrt{x+2}}{8\,(x+2)} = \pm\frac{2\sqrt{2}\,(x+4)\sqrt{x+2}}{8\,(x+2)}$$

$$= \pm\frac{(x+4)\sqrt{2x+4}}{4\,(x+2)}$$

Q2. (5 points) (7.2 Recitation Q#4): If $\sin^4 x - \cos^4 x = A\cos Bx$, then $A = ?$ and $B = ?$

Solution:

$$\sin^4 x - \cos^4 x = (\sin^2 x - \cos^2 x)\underbrace{(\sin^2 x + \cos^2 x)}_{=1}$$

$$= \sin^2 x - \cos^2 x$$

$$= \sin^2 x - (1 - \sin^2 x)$$

$$= \sin^2 x - 1 + \sin^2 x$$

$$= 2\sin^2 x - 1$$

$$= -(1 - 2\sin^2 x)$$

$$= -\cos 2x$$

Then $\boxed{A = -1}$ and $\boxed{B = 2}$

Q3. (5 points)(7.5 Textbook Example 6): Evaluate $\tan\left(2\arcsin\dfrac{2}{5}\right) = ?$

Solution:

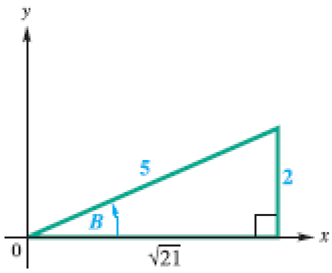


Figure 24

(b) Let $\arcsin\frac{2}{5} = B$. Then, from the double-angle tangent identity,

$$\tan\left(2\arcsin\frac{2}{5}\right) = \tan 2B = \frac{2\tan B}{1 - \tan^2 B}. \quad \text{(Section 7.4)}$$

Since $\arcsin\frac{2}{5} = B$, $\sin B = \frac{2}{5}$. Sketch a triangle in quadrant I, find the length of the third side, and then find $\tan B$. From the triangle in Figure 24, $\tan B = \frac{2}{\sqrt{21}}$, and

$$\tan\left(2\arcsin\frac{2}{5}\right) = \frac{2\left(\frac{2}{\sqrt{21}}\right)}{1 - \left(\frac{2}{\sqrt{21}}\right)^2} = \frac{\frac{4}{\sqrt{21}}}{1 - \frac{4}{21}} = \frac{\frac{4}{\sqrt{21}}\cdot\frac{\sqrt{21}}{\sqrt{21}}}{\frac{17}{21}} = \frac{\frac{4\sqrt{21}}{21}}{\frac{17}{21}} = \frac{4\sqrt{21}}{17}.$$

Be careful simplifying the complex fraction.

Q4. (5 points)(7.6 Textbook Exercise 21): Solve the equation $-2\sin^2 x = 3\sin x + 1$ over the interval $[0, 2\pi)$

Solution:

21. $-2\sin^2 x = 3\sin x + 1$

$$-2\sin^2 x = 3\sin x + 1$$
$$2\sin^2 x + 3\sin x + 1 = 0$$
$$(2\sin x + 1)(\sin x + 1) = 0$$
$$2\sin x + 1 = 0 \Rightarrow \sin x = -\frac{1}{2} \text{ or}$$
$$\sin x + 1 = 0 \Rightarrow \sin x = -1$$

Over the interval $[0, 2\pi)$, the equation $\sin x = -\frac{1}{2}$ has two solutions. The angles in quadrants **III** and IV that have a reference angle of $\frac{\pi}{6}$ are $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$.

In the same interval, $\sin x = -1$ when the angle is $\frac{3\pi}{2}$. Solution set: $\left\{\frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\right\}$

Q5. (5 points): Find the exact value of $\dfrac{\tan 69^\circ + \cot 24^\circ}{1 - \tan 69^\circ \tan 66^\circ} = ?$

Solution:

$$\frac{\tan 69^\circ + \cot 24^\circ}{1 - \tan 69^\circ \tan 66^\circ} = \frac{\tan 69^\circ + \tan(90^\circ - 24^\circ)}{1 - \tan 69^\circ \tan 66^\circ}$$
$$= \frac{\tan 69^\circ + \tan 66^\circ}{1 - \tan 69^\circ \tan 66^\circ}$$
$$= \tan(69^\circ + 66^\circ)$$
$$= \tan 135^\circ$$
$$= -\tan 45^\circ$$
$$= -1$$

Q6. (5 points): Solve the equation $\cos^{-1}\frac{x}{2} + \sin^{-1}\left(\frac{-3}{5}\right) - \frac{\pi}{3} = 0$

Solution: $\cos^{-1}\frac{x}{2} = \frac{\pi}{3} - \sin^{-1}\left(\frac{-3}{5}\right) \Rightarrow \cos\left(\cos^{-1}\frac{x}{2}\right) = \cos\left(\frac{\pi}{3} - \sin^{-1}\left(\frac{-3}{5}\right)\right) \Rightarrow$

$$\frac{x}{2} = \cos\left(\frac{\pi}{3} - \sin^{-1}\left(\frac{-3}{5}\right)\right)$$

Let $\theta = \sin^{-1}\left(\frac{-3}{5}\right)$ where $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$. $\Rightarrow$ $\sin\theta = \frac{-3}{5}$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ $\Rightarrow$

θ is in Quadrant IV $y = -3$, $r = 5$, $x = +\sqrt{r^2 - y^2} = \sqrt{25 - 9} = \sqrt{16} = 4$

$$\frac{x}{2} = \cos\left(\frac{\pi}{3} - \theta\right) = \cos\frac{\pi}{3}\cos\theta + \sin\frac{\pi}{3}\sin\theta = \frac{1}{2}\left(\frac{4}{5}\right) + \frac{\sqrt{3}}{2}\left(\frac{-3}{5}\right)$$

$$x = \frac{4 - 3\sqrt{3}}{5} \Rightarrow SS = \left\{\frac{4 - 3\sqrt{3}}{5}\right\}$$

Q7. (5 points): Given the vectors $u = \langle -2, 2\sqrt{3} \rangle$ and $\mathbf{v} = -2\sqrt{3}i + 2j$. Find the following

(a) $\|u\|$

(b) The unit vector e in the opposite direction of u.

(c) The direction angle of $\mathbf{v} = -2\sqrt{3}i + 2j$.

(d) The vector $2u - \sqrt{3}v$.

(e) The smallest nonnegative angle between u and v.

Solution:

(a): $\|u\| = \sqrt{(-2)^2 + \left(2\sqrt{3}\right)^2} = \sqrt{4+12} = 4$

(b): $e = -\frac{\vec{u}}{\|\vec{u}\|} = -\frac{<-2, 2\sqrt{3}>}{4} = < \frac{1}{2}, -\frac{\sqrt{3}}{2} >$

(c): Let θ be the direction angle of $\mathbf{v} = -2\sqrt{3}i + 2j$. Then

$$\tan\theta = \frac{2}{-2\sqrt{3}} = -\frac{1}{\sqrt{3}} \quad , \quad \theta \in QII \quad \Rightarrow \quad \theta = 180° - 30° = 150° = \frac{5\pi}{6}$$

(d): $2\,\mathbf{u} - \sqrt{3}\,\mathbf{v} = 2\left\langle -2, 2\sqrt{3} \right\rangle - \sqrt{3}\left\langle -2\sqrt{3}, 2 \right\rangle = \left\langle -4, 4\sqrt{3} \right\rangle + \left\langle 6, -2\sqrt{3} \right\rangle = \left\langle 2, 2\sqrt{3} \right\rangle$

(e): Let α be the smallest nonnegative angle between $\mathbf{u}$ and $\mathbf{v}$. Then

$$\cos\alpha = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{\left\langle -2, 2\sqrt{3}\right\rangle \cdot \left\langle -2\sqrt{3}, 2\right\rangle}{(4)(4)} = \frac{4\sqrt{3} + 4\sqrt{3}}{16} = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad \alpha = 30° = \frac{\pi}{6}$$

Q8. (5 points): Find the **amplitude**, **range**, **period**, **phase shif**t, and **sketch the graph** the function $f(x) = \cos\frac{x}{2} - \sqrt{3}\sin\frac{x}{2}$.

Solution: $a = -\sqrt{3},\ b = 1 \ \Rightarrow\ \alpha$ is in Quadrant II. $k = \sqrt{a^2 + b^2} = \sqrt{\left(-\sqrt{3}\right)^2 + (1)^2} = \sqrt{4} = 2$

amplitude $= 2$, period $= 4\pi$

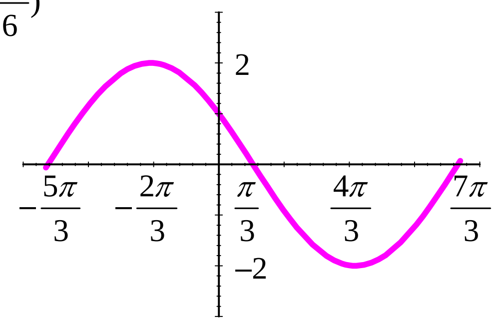
$$\left.\begin{array}{l} \sin\alpha' = |\sin\alpha| = \left|\frac{b}{k}\right| = \left|\frac{1}{2}\right| = \frac{1}{2} \\ \cos\alpha' = |\cos\alpha| = \left|\frac{a}{k}\right| = \left|\frac{-\sqrt{3}}{2}\right| = \frac{\sqrt{3}}{2} \end{array}\right\} \Rightarrow \alpha' = \frac{\pi}{6} \Rightarrow \alpha = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$f(x) = -\sqrt{3}\sin\frac{x}{2} + \cos\frac{x}{2} = k\sin\left(\frac{x}{2} + \alpha\right) = 2\sin\left(\frac{x}{2} + \frac{5\pi}{6}\right)$$

$$f(x) = 2\sin\left(\frac{x}{2} + \frac{5\pi}{6}\right)$$

phase shift $= -\frac{5\pi}{3}$

The period is $P = \frac{2\pi}{\frac{1}{2}} = 4\pi$



Q9. (6 points): Given $y = 3\sin\left(2x - \frac{\pi}{4}\right) + 4$, $x \in \left[\frac{3\pi}{8}, \frac{11\pi}{8}\right]$.

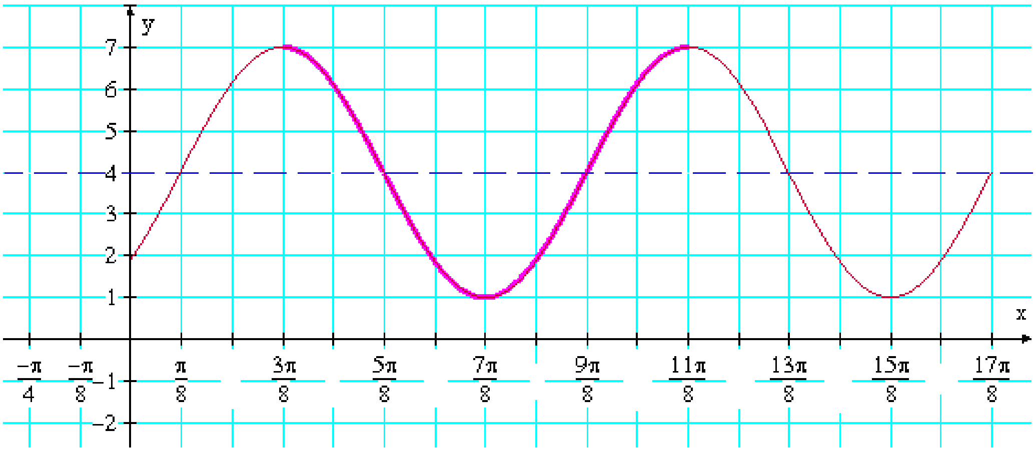
(a): Determine the interval where the function is increasing.
(b): Determine the interval where the function is decreasing.
(c): Determine the **coordinates of the lowest point** of the graph.

Solution: $P = \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

Phase Shift $= x_0 = -\frac{c}{b} = -\frac{-\pi/4}{2} = \frac{\pi}{8}$ units to the right

$\frac{1}{4}P = \frac{\pi}{4} = \frac{2\pi}{8} \quad \Rightarrow \quad x_0 + \frac{1}{4}P = \frac{\pi}{8} + \frac{\pi}{4} = \frac{3\pi}{8}$

The Key Points are: $\frac{\pi}{8}$ $\frac{3\pi}{8}$ $\frac{5\pi}{8}$ $\frac{7\pi}{8}$ $\frac{9\pi}{8}$ $\frac{11\pi}{8}$



(a): Given $x \in \left[\frac{3\pi}{8}, \frac{11\pi}{8}\right]$ then the function is **increasing on** $\left[\frac{7\pi}{8}, \frac{11\pi}{8}\right]$

(b): Given $x \in \left[\frac{3\pi}{8}, \frac{11\pi}{8}\right]$ then the function is **decreasing on** $\left[\frac{3\pi}{8}, \frac{7\pi}{8}\right]$

(c): Given $x \in \left[\frac{3\pi}{8}, \frac{11\pi}{8}\right]$ then the coordinates of the lowest point is $\left(\frac{7\pi}{8}, 1\right)$

Q10. (4 points) (6.6 Textbook Exercise 32): Graph $y = 3\tan\left(\frac{3}{4}x - \pi\right)$ over a two period.

Solution: $-\frac{\pi}{2} < \frac{3}{4}x - \pi < \frac{\pi}{2} \quad \Rightarrow \quad -2\pi < 3x - 4\pi < 2\pi \quad \Rightarrow \quad 2\pi < 3x < 6\pi$

$\Rightarrow \quad \frac{2\pi}{3} < x < 2\pi$

Graph of $y = 3\tan\left(\frac{3}{4}x - \pi\right)$: