Serial #: _____ ID _____ NAME _____

Show all necessary steps for full marks.

Question 1: (5 points): Given the polynomial $f(x)=\left(3x^2-2\right)^2-\left(2x^2-x-3\right)\left(2x^2-x+3\right)$

(a): Write down the following :

(b): Write $f(x)$ in standard form.

The leading coefficient	The constant Term	The coefficient of x^2	Degree

Solution (a):

$$f(x)=\left(3x^2-2\right)^2-\left(2x^2-x-3\right)\left(2x^2-x+3\right)$$

$$=9x^4-12x^2+4-\left[\left(2x^2-x\right)-3\right]\left[\left(2x^2-x\right)+3\right]$$

$$=9x^4-12x^2+4-\left[\left(2x^2-x\right)^2-3^2\right]$$

$$=9x^4-12x^2+4-\left[4x^4-4x^3+x^2-9\right]$$

$$=5x^4+4x^3-13x^2+13$$

(b):

The leading coefficient	The constant Term	The coefficient of x^2	Degree
5	13	-13	4

Question 2: (5 points): If $\dfrac{4x^3-3x^2+x+1}{x+2}=4x^2+mx+23+\dfrac{n}{x+2}$, find m and n.

Solution:

$$\begin{array}{r} 4x^2-11x+23 \\ x+2\overline{)\,4x^3-3x^2+x+1} \\ \underline{4x^3+8x^2} \\ -11x^2+x \\ \underline{-11x^2-22x} \\ 23x+1 \\ \underline{23x+46} \\ -45 \end{array}$$

$$If \quad \frac{4x^3-3x^2+x+1}{x+2}=4x^2-11x+23+\frac{-45}{x+2} \quad \text{find } m \text{ and } n.$$

$$\boxed{m=-11 \quad , \quad n=-45}$$

Question 3: (4 points) (R.4 Textbook Exercise 23): Factor completely $p^2q^2 - 10 - 2q^2 + 5p^2$

Solution:

23. $p^2q^2 - 10 - 2q^2 + 5p^2$

$= p^2q^2 - 2q^2 + 5p^2 - 10$

$= q^2(p^2 - 2) + 5(p^2 - 2)$

$= (p^2 - 2)(q^2 + 5)$

Question 4: (3 points) (R.4 Textbook Exercise 40): Factor completely $18x^5 + 15x^4z - 75x^3z^2$

Solution:

40. First, factor out the greatest common factor, $3x^3$:

$18x^5 + 15x^4z - 75x^3z^2 = 3x^3\left(6x^2 + 5xz - 25z^2\right)$

Now factor the trinomial by trial and error: $6x^2 + 5xz - 25z^2 = (3x - 5z)(2x + 5z)$

Thus,

$18x^5 + 15x^4z - 75x^3z^2 = 3x^3\left(6x^2 + 5xz - 25z^2\right)$

$= 3x^3(3x - 5z)(2x + 5z)$

Question 5: (3 points) (R.4 Textbook Exercise 97): Factor $64 + (3x + 2)^3$

Solution:

97. $64 + (3x + 2)^3$

$= 4^3 + (3x + 2)^3$

$= \left[4 + (3x + 2)\right]\left[4^2 - (4)(3x + 2) + (3x + 2)^2\right]$

$= \left[4 + (3x + 2)\right]\left[\begin{array}{l} 4^2 - (4)(3x + 2) \\ \quad + 9x^2 + 12x + 4 \end{array}\right]$

$= (4 + 3x + 2)(16 - 12x - 8 + 9x^2 + 12x + 4)$

$= (3x + 6)(9x^2 + 12)$

$= 3(x + 2)(3)(3x^2 + 4) = 9(x + 2)(3x^2 + 4)$