King Fahd University of Petroleum and Minerals
Prep-Year Math Program
Math 002 - Term 151
Recitation (7.5)

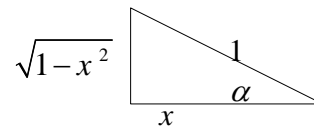
Question 1:   Find the exact value of $\tan\left[\sin^{-1}\left(-\dfrac{4}{5}\right) - \cos^{-1}\left(\dfrac{12}{13}\right)\right]$.   **Answer:** $-\dfrac{63}{16}$

Question 2 Find the exact value of $\sec^{-1}(-2) + \tan^{-1}\left(\tan\dfrac{3\pi}{5}\right)$ **Answer:** $\dfrac{4\pi}{15}$

Question 3: Verify the identity $\tan(2\cos^{-1}x) = \dfrac{2x\sqrt{1-x^2}}{2x^2-1}$.

Solution:

Let $\alpha = \cos^{-1}x$ $\Rightarrow$ $\cos\alpha = x$ $\Rightarrow$ $\cos\alpha = \dfrac{x}{1}$

$\tan\left(2\cos^{-1}x\right) = \tan 2\alpha$

$$= \frac{\sin 2\alpha}{\cos 2\alpha} = \frac{2\sin\alpha\cos\alpha}{2\cos^2\alpha - 1} = \frac{2\left(\sqrt{1-x^2}\right)x}{2x^2-1} = \frac{2x\sqrt{1-x^2}}{2x^2-1}$$

Another Method:

$$\tan\left(2\cos^{-1}x\right) = \tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{2\dfrac{\sqrt{1-x^2}}{x}}{1-\dfrac{1-x^2}{x^2}} = \frac{2x\sqrt{1-x^2}}{x^2-(1-x^2)} = \frac{2x\sqrt{1-x^2}}{2x^2-1}$$

Question 4: $6\cos\left[\arctan\sqrt{3} + \arcsin\dfrac{1}{3}\right] = ?$

Answer: $2\sqrt{2} - \sqrt{3}$

Question 5 $\csc^{-1}\left(\dfrac{-2\sqrt{3}}{3}\right) + \cos^{-1}\left(\sin\dfrac{\pi}{5}\right) =$

A) $\dfrac{\pi}{20}$ D) $\dfrac{2\pi}{15}$

B) $-\dfrac{2\pi}{15}$ E) $\dfrac{3\pi}{20}$

C) $-\dfrac{\pi}{30}$

Answer: (c) $-\dfrac{\pi}{30}$