Serial #: ________ ID ______________ NAME ______________________________________

Show all necessary steps for full marks.

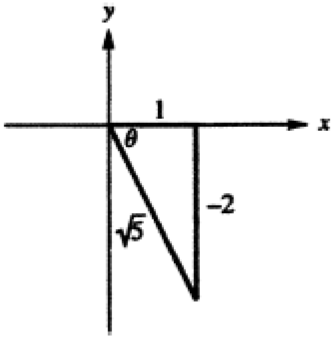
Question1: (4 points) (7.5 Textbook Exercise 88): $\cos\left(2\tan^{-1}(-2)\right)=?$

Solution:

88. $\cos\left(2\tan^{-1}(-2)\right)$

Let $\theta = \arctan(-2)$, so that $\tan\theta = -2$. Since arctan is defined only in quadrants I and IV, and -2 is negative, θ is in quadrant IV. Sketch θ and label a triangle with the hypotenuse equal to

$$\sqrt{(-2)^2+1^2}=\sqrt{4+1}=\sqrt{5}.$$



$$\cos\theta = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\cos\left(2\tan^{-1}(-2)\right) = \cos 2\theta = 2\cos^2\theta - 1$$

$$= 2\left(\frac{1}{\sqrt{5}}\right)^2 - 1 = \frac{2}{5} - 1 = -\frac{3}{5}$$

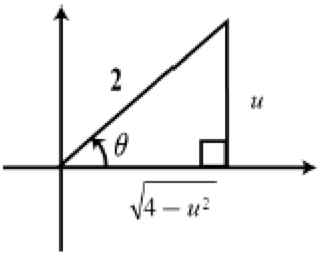
Question 2: (4 points) (7.5 Textbook Exercise 107): Write the expression $\sec\left(arc\cot\frac{\sqrt{4-u^2}}{u}\right)$ as an algebraic expression in u, for $u>0$.

Solution:

107. $\sec\left(\operatorname{arc}\cot\frac{\sqrt{4-u^2}}{u}\right)$

Let $\theta = \operatorname{arc}\cot\frac{\sqrt{4-u^2}}{u}$, so $\cot\theta = \frac{\sqrt{4-u^2}}{u}$.

Since $u>0$, $0<\theta<\frac{\pi}{2}$.



From the Pythagorean theorem,

$$r=\sqrt{\left(\sqrt{4-u^2}\right)^2+u^2}$$
$$=\sqrt{4-u^2+u^2}=\sqrt{4}=2.$$

Therefore, $\sec\theta=\dfrac{2}{\sqrt{4-u^2}}=\dfrac{2\sqrt{4-u^2}}{4-u^2}$.

Thus, $\sec\left(\operatorname{arccot}\dfrac{\sqrt{4-u^2}}{u}\right)=\dfrac{2\sqrt{4-u^2}}{4-u^2}$.

Question 3: (6 points) (7.6 Textbook Exercise 90): Given the equation $\cos\theta-1=\cos 2\theta$.
(a): Solve the equation over the interval $[0°,360°)$. **(b):** Find all solutions.

Solution (a):

90. $\cos\theta-1=\cos 2\theta$

$\cos\theta-1=\cos 2\theta\Rightarrow\cos\theta-1=2\cos^2\theta-1\Rightarrow$
$2\cos^2\theta-\cos\theta=0\Rightarrow\cos\theta(2\cos\theta-1)=0$

Over the interval $[0°,360°)$, we have

$\cos\theta=0\Rightarrow\theta=90°$ or $270°$

Since both 90° and 270° are solutions, we can write the general solution as $90°+180°n$.

$2\cos\theta-1=0\Rightarrow 2\cos\theta=1\Rightarrow\cos\theta=\dfrac{1}{2}\Rightarrow$

$\theta=60°$ or $300°$

$SS=\{60°,\ 90°,\ 270°,\ 300°\}$

(b):

Solution set: $\{60°+360°n,\ 90°+180°n,\ 300°+360°n$, where n is any integer$\}$

Question 4: (6 points) (7.7 Textbook 7.7 Exercises 36 and 37): Solve the following equations:

(a): $\arccos x+2\arcsin\dfrac{\sqrt{3}}{2}=\pi$

(b): $\sin^{-1}x+\tan^{-1}\sqrt{3}=\dfrac{2\pi}{3}$

Solution:

$\arccos x+2\arcsin\dfrac{\sqrt{3}}{2}=\pi\Rightarrow$

$\arccos x=\pi-2\arcsin\dfrac{\sqrt{3}}{2}\Rightarrow$

$\arccos x=\pi-2\left(\dfrac{\pi}{3}\right)$

$\arccos x=\pi-\dfrac{2\pi}{3}\Rightarrow\arccos x=\dfrac{\pi}{3}$

$x=\cos\dfrac{\pi}{3}\Rightarrow x=\dfrac{1}{2}$

Solution set: $\left\{\dfrac{1}{2}\right\}$

36. $\sin^{-1}x+\tan^{-1}\sqrt{3}=\dfrac{2\pi}{3}$

$\sin^{-1}x+\tan^{-1}\sqrt{3}=\dfrac{2\pi}{3}\Rightarrow$

$\sin^{-1}x+\dfrac{\pi}{3}=\dfrac{2\pi}{3}\Rightarrow\sin^{-1}x=\dfrac{\pi}{3}$

$x=\sin\dfrac{\pi}{3}\Rightarrow x=\dfrac{\sqrt{3}}{2}$

Solution set: $\left\{\dfrac{\sqrt{3}}{2}\right\}$

Question 5: **(2 points) (7.6 Recitation Q#1):** Solve the following equations

$\sin\frac{x}{2}+\cos x=1$, for $0\le x\le\pi$.

Solution: Given $\sin\frac{x}{2}+\cos x=1$, $0\le x<\pi$

$\sin\frac{x}{2}=1-\cos x$, for $0\le\frac{x}{2}<\frac{\pi}{2}$. $\Rightarrow$ $\frac{x}{2}$ is in Quadrant I

$+\sqrt{\frac{1-\cos x}{2}}=1-\cos x$ (because $\frac{x}{2}$ is in quadrant I)

$\frac{1-\cos x}{2}=1-2\cos x+\cos^2 x$ (Squaring both sides)

$0=2\cos^2 x-3\cos x+1$

$2\cos x \quad -1$

$\cos x \quad -1$

$0=(2\cos x-1)(\cos x-1)$

$2\cos x-1=0$, $\cos x-1=0$

$\cos x=\frac{1}{2}$, $\cos x=1$

$x=\frac{\pi}{3}$, $\frac{5\pi}{3}$, $x=0$, 2π

$x=\frac{\pi}{3}$, $x=0$ are the possible solutions in the given interval.

Because squaring both sides, we need to check the above possible solutions:

$x=\frac{\pi}{3}\Rightarrow \sin\frac{\frac{\pi}{3}}{2}+\cos\frac{\pi}{3}=1$ OK

$x=0 \Rightarrow \sin\frac{0}{2}+\cos 0=1$ OK

Answer: $SS=\left\{0,\frac{\pi}{3}\right\}$