Serial #: ________ ID ______________ NAME ____

Show all necessary steps for full marks.

Question 1: (4 points) (6.2 Textbook Exercises 12-16): Find the exact value of the following:

(a): $\cot\frac{5\pi}{6}=?$ **(b):** $\cos\left(-\frac{4\pi}{3}\right)=?$ **(c):** $\tan\left(-\frac{17\pi}{3}\right)=?$ **(d):** $\cos\frac{7\pi}{4}=?$

Solution:

12. Since $\frac{5\pi}{6}$ is in quadrant II, the reference angle is $\pi-\frac{5\pi}{6}=\frac{6\pi}{6}-\frac{5\pi}{6}=\frac{\pi}{6}$. In quadrant II, the cotangent is negative. Thus,

$$\cot\frac{5\pi}{6}=-\cot\frac{\pi}{6}=-\sqrt{3}.$$

Converting $\frac{5\pi}{6}$ to degrees, we have $\frac{5\pi}{6}=\frac{5}{6}(180°)=150°$. The reference angle is $180°-150°=30°$. Thus,

$$\cot\frac{5\pi}{6}=\cot 150°=-\cot 30°=-\sqrt{3}.$$

13. $-\frac{4\pi}{3}$ is coterminal with $-\frac{4\pi}{3}+2\pi=-\frac{4\pi}{3}+\frac{6\pi}{3}=\frac{2\pi}{3}$. Since $\frac{2\pi}{3}$ is in quadrant II, the reference angle is $\pi-\frac{2\pi}{3}=\frac{3\pi}{3}-\frac{2\pi}{3}=\frac{\pi}{3}$. In quadrant II, the cosine is negative. Thus,

$$\cos\left(-\frac{4\pi}{3}\right)=\cos\frac{2\pi}{3}=-\cos\frac{\pi}{3}=-\frac{1}{2}.$$

14. $-\frac{17\pi}{3}$ is coterminal with $-\frac{17\pi}{3}+6\pi=-\frac{17\pi}{3}+\frac{18\pi}{3}=\frac{\pi}{3}$. Since $\frac{\pi}{3}$ is in quadrant I, $\frac{\pi}{3}$ is the reference angle. In quadrant I, the tangent is positive. Thus,

$\tan\left(-\frac{17\pi}{3}\right)=\tan\frac{\pi}{3}=\sqrt{3}$. Converting $\frac{\pi}{3}$ to degrees, we have $\frac{\pi}{3}=\frac{1}{3}(180°)=60°$. Thus,

$$\tan\frac{\pi}{3}=\tan 60°=\sqrt{3}.$$

$$\tan\left(-\frac{17\pi}{3}\right)=\tan\frac{\pi}{3}=\sqrt{3}.$$

15. Since $\frac{7\pi}{4}$ is in quadrant IV, the reference angle is $2\pi-\frac{7\pi}{4}=\frac{8\pi}{4}-\frac{7\pi}{4}=\frac{\pi}{4}$. In quadrant IV, the cosine is positive. Thus,

$$\cos\frac{7\pi}{4}=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}.$$

Converting $\frac{7\pi}{4}$ to degrees, we have $\frac{7\pi}{4}=\frac{7}{4}(180°)=315°$. The reference angle is $360°-315°=45°$. Thus,

$$\cos\frac{7\pi}{4}=\cos 315°=\cos 45°=\frac{\sqrt{2}}{2}.$$

Question 2: (5 points): (6.5 Additional Exercise 10):

(a): Graph the function $y=3\tan\left(2x+\frac{\pi}{2}\right)$ Over the Interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

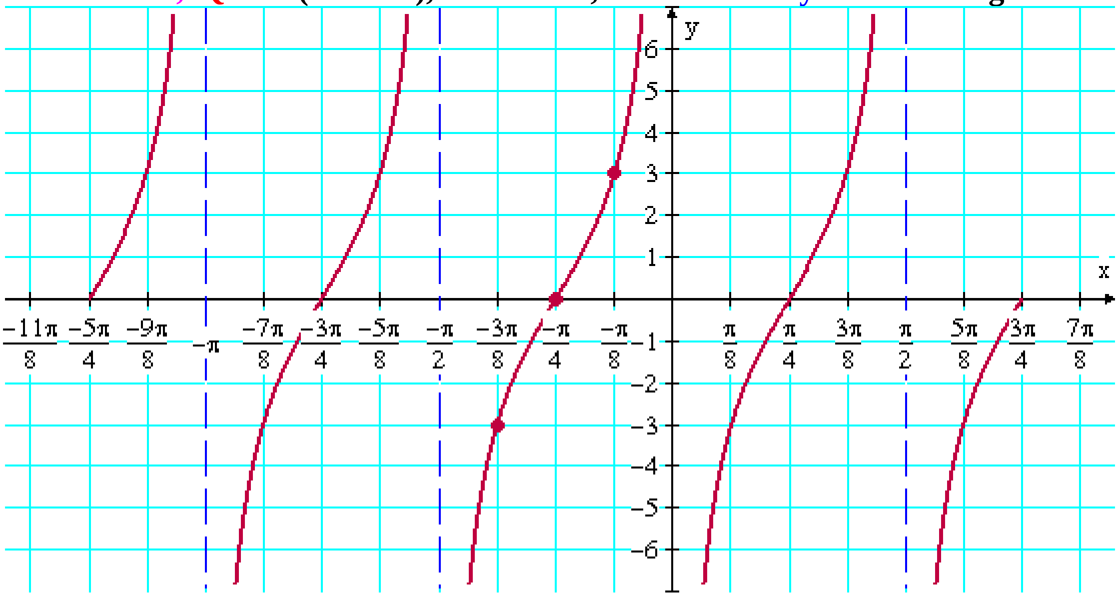
(b): Find the x-intercepts of the Graph of the function over the Interval $\left[-\frac{5\pi}{4},\frac{3\pi}{4}\right]$

Solution (a):

$$-\frac{\pi}{2}<2x+\frac{\pi}{2}<\frac{\pi}{2}$$

$$-\pi<2x<0$$

$$-\frac{\pi}{2}<x<0$$



(b): $x = -\frac{5\pi}{4}$, $x = -\frac{3\pi}{4}$, $x = -\frac{\pi}{4}$. $x = \frac{\pi}{4}$ and $x = \frac{3\pi}{4}$

Question 3: **(6 points):** **(a):** Sketch the graph of $y = 3\sin\left(2x - \frac{\pi}{4}\right) + 4$, $x \in \left[\frac{3\pi}{8}, \frac{11\pi}{8}\right]$.

(b): Determine the interval where the function is increasing.

(c): Determine the interval where the function is decreasing.

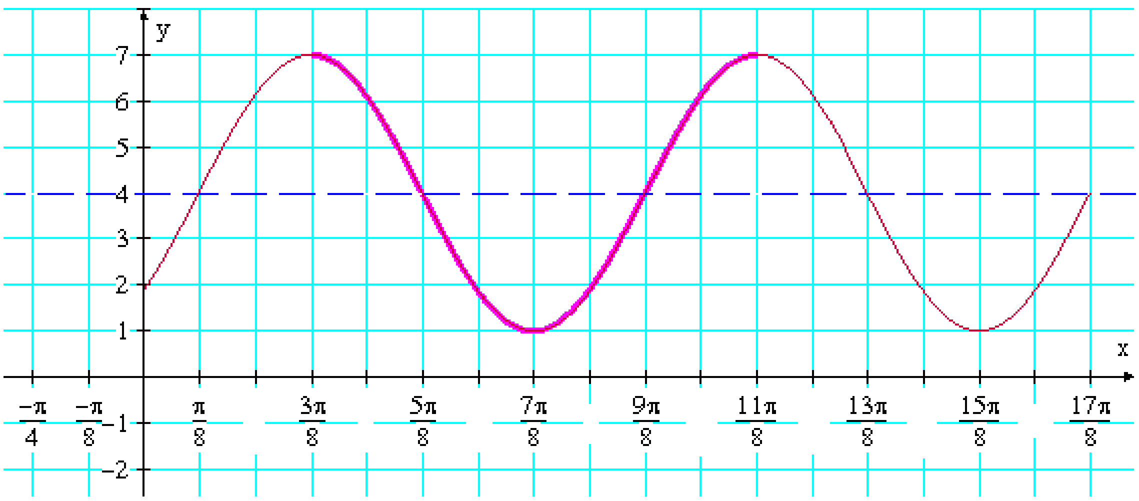
(d): Determine the **coordinates of the lowest point** of the graph.

Solution: **(a):** $P = \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

Phase Shift $= x_0 = -\frac{c}{b} = -\frac{-\pi/4}{2} = \frac{\pi}{8}$ units to the right

$\frac{1}{4}P = \frac{\pi}{4} = \frac{2\pi}{8} \quad \Rightarrow \quad x_0 + \frac{1}{4}P = \frac{\pi}{8} + \frac{\pi}{4} = \frac{3\pi}{8}$

The Key Points are: $\frac{\pi}{8}$ $\frac{3\pi}{8}$ $\frac{5\pi}{8}$ $\frac{7\pi}{8}$ $\frac{9\pi}{8}$ $\frac{11\pi}{8}$



(b): Given $x \in \left[\frac{3\pi}{8}, \frac{11\pi}{8}\right]$ then the function is **increasing on** $\left[\frac{7\pi}{8}, \frac{11\pi}{8}\right]$

(c): Given $x \in \left[\frac{3\pi}{8}, \frac{11\pi}{8}\right]$ then the function is **decreasing on** $\left[\frac{3\pi}{8}, \frac{7\pi}{8}\right]$

(d): Given $x \in \left[\frac{3\pi}{8}, \frac{11\pi}{8}\right]$ then the coordinates of the lowest point is $\left(\frac{7\pi}{8}, 1\right)$

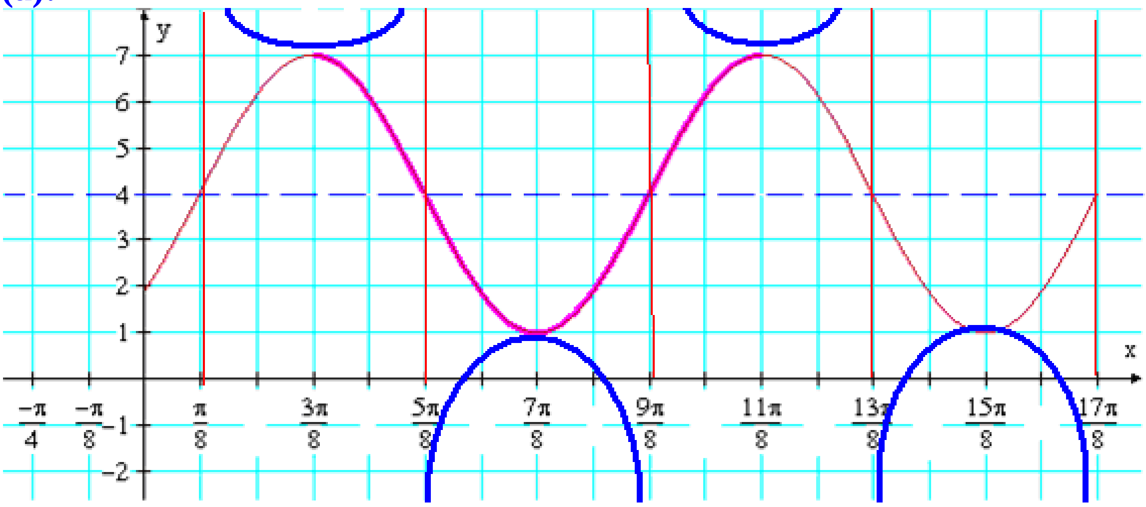
Question 4: (5 points): **(a):** Sketch the graph of $y = 3\csc\left(2x - \frac{\pi}{4}\right) + 4$ over the interval $\left[\frac{3\pi}{8}, \frac{11\pi}{8}\right]$.

(b): Determine the interval where the function is increasing.

(c): Determine the interval where the function is decreasing.

Solution:

(a):



(b): Increasing on the intervals: $\left[\frac{3\pi}{8}, \frac{5\pi}{8}\right)$ and $\left(\frac{5\pi}{8}, \frac{7\pi}{8}\right]$

(c): Increasing on the intervals: $\left[\frac{7\pi}{8}, \frac{9\pi}{8}\right)$ and $\left(\frac{9\pi}{8}, \frac{11\pi}{8}\right]$