Serial #: _____ ID _____ NAME _____

**Show all necessary steps for full marks.**

## Q1. (7 points)(4.3 Textbook Exercise 54):

Graph the function $f(x) = \log_{1/3}(3-x)$ and give the **domain** and the **range**. Find the intervals where the graph is **above** the *x*-axis.

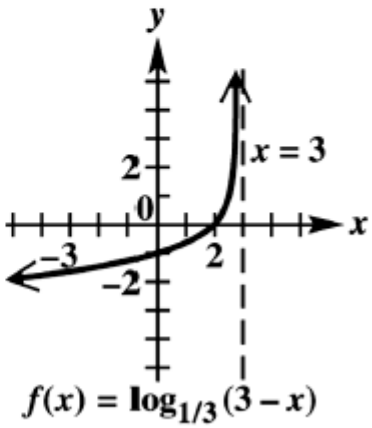
## Solution:

**54.** $f(x) = \log_{1/3}(3-x)$

Since $f(x) = y = \log_{1/3}(3-x)$, we can write the exponential form as

$3 - x = \left(\frac{1}{3}\right)^y \Rightarrow x = 3 - \left(\frac{1}{3}\right)^y$ to find ordered pairs that satisfy the equation. It is easier to choose values for $y$ and find the corresponding values of $x$. Make a table of values.

| $x$ | $y = \log_{1/3}(3-x)$ |
|---|---|
| $-6$ | $-2$ |
| $0$ | $-1$ |
| $2$ | $0$ |
| $\frac{8}{3} \approx 2.7$ | $1$ |
| $\frac{26}{9} \approx 2.9$ | $2$ |




The graph has the line $x = 3$ as a vertical asymptote.

$D_f = (-\infty, 3)$ and $R_f = (-\infty, \infty)$

The graph is above the x-axis on the interval $(2.3)$.

## Q2. (3 points) Find the domain of the function $f(x) = \log(x^2 + 7x + 10) - 10$.

## Solution:

Q3.

The domain of the function, $f(x) = \log(x^2+7x+10) - 10$ is:

✓ A) $(-\infty, -5) \cup (-2, \infty)$

B) $(-\infty, 2) \cup (5, \infty)$

C) $(-5, -2)$

D) $(-\infty, -5] \cup [-2, \infty)$

E) $(-\infty, -7) \cup (0, \infty)$

$f(x) = \log(x+2)(x+5) - 10$

$(x+2)(x+5) > 0$

$(-\infty, -5) \cup (-2, \infty)$

**Q3. (5 points)** (4.4 Additional Exercise 13): Assume all variables represent positive real numbers, write the logarithmic expression: $2-\log_3 x^2-8\log_9 y+\log_{\sqrt{3}} xy$ as a single logarithm with a base of 3

**Solution:**

$$
\begin{aligned}
2-\log_3 x^2-8\log_9 y+\log_{\sqrt{3}} xy &= \log_3 3^2-\log_3 x^2-8\frac{\log_3 y}{\log_3 9}+\frac{\log_3 xy}{\log_3 \sqrt{3}} \\
&= \log_3 9-\log_3 x^2-8\frac{\log_3 y}{2}+\frac{\log_3 xy}{\frac{1}{2}} \\
&= \log_3 9-\log_3 x^2-4\log_3 y+2\log_3 xy \\
&= \log_3 9-\log_3 x^2-\log_3 y^4+\log_3 (xy)^2 \\
&= \log_3 9+\log_3 (xy)^2-\left(\log_3 x^2+\log_3 y^4\right) \\
&= \log_3 9(xy)^2-\log_3 x^2y^4 \\
&= \log_3 \frac{9(xy)^2}{x^2y^4} \\
&= \log_3 \frac{9}{y^2}
\end{aligned}
$$

**Q4. (5 points) (4.5 Textbook Exercise 61):** Find the solution set of $\log_2(x^2-100)-\log_2(x+10)=1$

**Solution:**

**61.** $\log_2\left(x^2-100\right)-\log_2(x+10)=1$

$$
\begin{aligned}
\log_2\left(\frac{x^2-100}{x+10}\right) &= 1 \\
\frac{x^2-100}{x+10} &= 2^1=2 \\
x^2-100 &= 2x+20 \\
x^2-2x-120 &= 0 \\
(x+10)(x-12) &= 0 \\
x &= -10,\ 12
\end{aligned}
$$

If $-10$ is substituted for $x$ in $\log_2(x+10)$, the argument becomes 0. Since this is not allowed, we reject this proposed solution.
Solution set: {12}