King Fahd University of Petroleum and Minerals

Prep-Year Math Program

Math 002 Class Test II
Textbook Sections: 6.2 to 8.3
Term 151
Time Allowed: 90 Minutes

Time: 6:00 pm – 7:30 pm

Student's Name: …………… ………………………………………………
ID #:………………………. **Section:** ……. **Serial Number:** …….

Provide neat and complete solutions.
Show all necessary steps for full credit and write the answer in simplest form.

No Calculators, Cameras, or Mobiles are allowed during this exam.

Question	**Points**	Student's Score
1	5	
2	6	
3	6	
4	3	
5	4	
6	4	
7	4	
8	4	
9	4	
10	4	
11	6	
Total	**50**	____ 50
		____ 100

Q1. (5 points) (6.2 Textbook Exercises 7-11): Find the exact value of the following:

(a): $\sin\frac{7\pi}{6}$

(b): $\cos\frac{5\pi}{3}$

(c): $\tan\frac{3\pi}{4}$

(d): $\sec\frac{2\pi}{3}$

(e): $\csc\frac{11\pi}{6}$

Solution:

$$\sin\frac{7\pi}{6} = \sin 210° = -\sin 30° = -\frac{1}{2}.$$

$$\cos\frac{5\pi}{3} = \cos 300° = \cos 60° = \frac{1}{2}.$$

$$\tan\frac{3\pi}{4} = \tan 135° = -\tan 45° = -1.$$

$$\sec\frac{2\pi}{3} = \sec 120° = -\sec 60° = -2.$$

$$\csc\frac{11\pi}{6} = \csc 330° = -\csc 30° = -2.$$

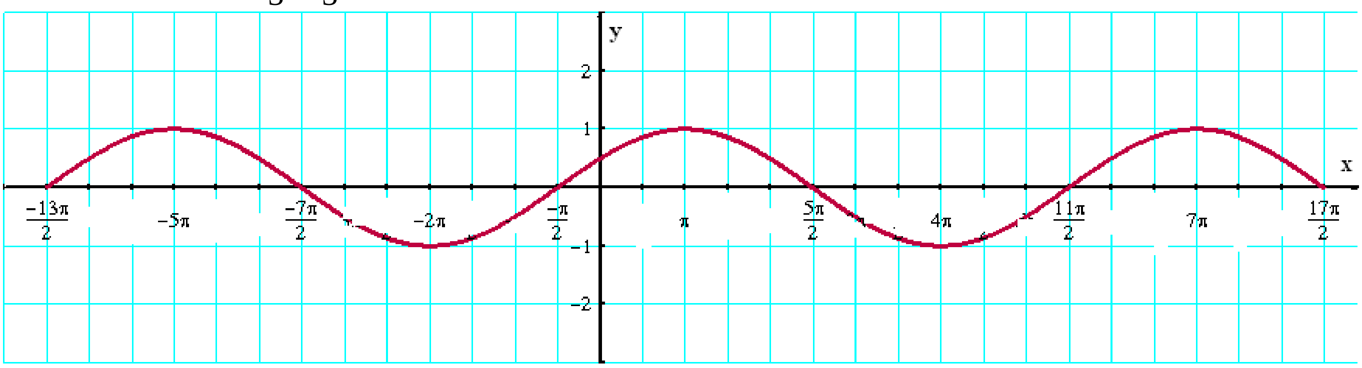
Q2. (6 points) (6.4 Additional Exercises 15): Given $y = \cos\left(\frac{x}{3} - \frac{\pi}{3}\right)$, $-\frac{13\pi}{2} \le x \le \frac{17\pi}{2}$.

(a): Sketch the graph of function over the interval $\left[-\frac{13\pi}{2}, \frac{17\pi}{2}\right]$.

(b): Find the intervals where the function is below the *x*-axis.

(c): Find the intervals where the function is decreasing over the given interval.

Solution (a): $0 \le \frac{x}{3} - \frac{\pi}{3} \le 2\pi \;\Rightarrow\; 0 \le x - \pi \le 6\pi \;\Rightarrow\; \pi \le x \le 7\pi$



(b): The graph is below the x-axis on $\left(-\frac{7\pi}{2}, -\frac{\pi}{2}\right)$ and on $\left(\frac{5\pi}{2}, \frac{11\pi}{2}\right)$

(c) The graph is decreasing on $(-5\pi, -2\pi)$, $(\pi, 4\pi)$ and $\left(7\pi, \frac{17\pi}{2}\right)$

Q3. (6 points): (Recitation 6.5):

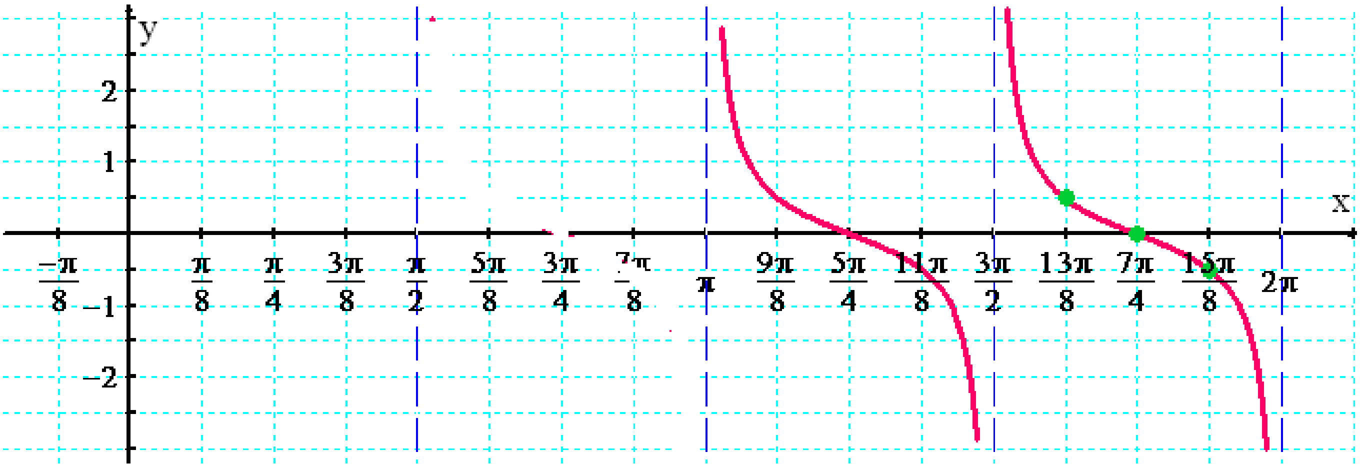
(a): Graph the function $y = \frac{1}{2}\cot(2x - 3\pi)$ in the Interval $[\pi, 2\pi]$.

(b): Determine the intervals where the graph of the function is below the x-axis.

(c): Determine the equations of vertical asymptotes over the interval $[\pi, 2\pi]$.

Solution (a): $0 < 2x - 3\pi < \pi \Rightarrow 3\pi < 2x < 4\pi \Rightarrow \frac{3\pi}{2} < x < 2\pi$.

$y = \frac{1}{2}\cot(2x - 3\pi)$:



(b): The graph is below the x-axis on $\left(\frac{5\pi}{4}, \frac{3\pi}{2}\right)$ and $\left(\frac{7\pi}{4}, 2\pi\right)$.

(c): The vertical asymptotes are: $x = \pi$, $x = \frac{3\pi}{2}$ and $x = 2\pi$

Q4. (3 points) (6.6 Additional Exercise 9): The adjacent graph represents part of the graph of

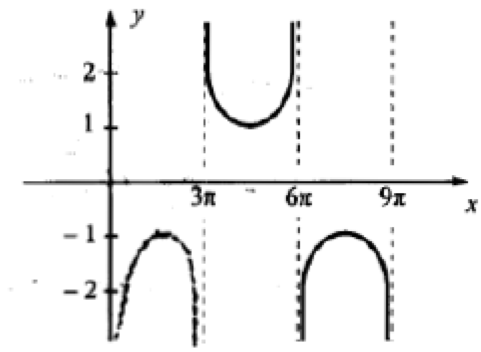
A) $y = \sec 9x$

B) $y = -\sec\frac{x}{3}$

C) $y = -\csc\frac{x}{3}$

D) $y = -\csc\frac{x}{9}$

E) $y = 2\csc\frac{x}{3}$



Solution: Clearly, it is the graph of $y = a\csc bx$ where $\boxed{a = -1}$.

$Period = 6\pi$

$\frac{2\pi}{b} = 6\pi \Rightarrow \boxed{b = \frac{1}{3}}$

$y = a\csc bx = (-1)\csc\left(\frac{1}{3}x\right) = -\csc\frac{x}{3}$

Answer: C) $y = -\csc\frac{x}{3}$

Q5. (4 points) (7.1 Recitation Q#1): If the terminal side of an angle θ intersects the unit circle at the point $\left(-\frac{4}{5}, -\frac{3}{5}\right)$, then find the exact value of $\sec(-\theta)+\tan(-\theta)$.

Solution: $(x, y)=\left(-\frac{4}{5},-\frac{3}{5}\right)$ and the radius $r=1$

$$\cos\theta=\frac{x}{1}=-\frac{4}{5} \quad \text{and} \quad \sin\theta=\frac{y}{1}=-\frac{3}{5}$$

$$\sec(-\theta)=\sec\theta=-\frac{5}{4}$$

$$\tan(-\theta)=-\tan\theta=-\frac{-\frac{3}{5}}{-\frac{4}{5}}=-\frac{3}{4}$$

$$\sec(-\theta)+\tan(-\theta)=-\frac{5}{4}-\frac{3}{4}=-\frac{8}{4}=-2$$

Answer: -2

Q6. (4 points) (7.3 Textbook Exercises 89): Suppose $\cos s=-\frac{1}{5}$, $\sin t=\frac{3}{5}$ where s and t are in quadrant II. Find $\cos(s-t)=?$ and $\sec(s-t)=?$.

Solution:

$\cos s=-\frac{1}{5}$, $\sin t=\frac{3}{5}$, s and t are in quadrant II.

$\cos s=\frac{x}{r}\Rightarrow \cos s=-\frac{1}{5}=\frac{-1}{5}\Rightarrow x=-1, r=5$.

Substituting into the Pythagorean theorem, we have $(-1)^2+y^2=5^2\Rightarrow y^2=24\Rightarrow y=\sqrt{24}$,

since $\sin x>0$. Thus, $\sin s=\frac{y}{r}=\frac{\sqrt{24}}{5}$.

$$\cos t=-\sqrt{1-\sin^2 t}=-\sqrt{1-\left(\frac{3}{5}\right)^2}$$
$$=-\sqrt{1-\frac{9}{25}}=-\sqrt{\frac{16}{25}}=-\frac{4}{5}$$

$$\cos(s-t)=\cos s\cos t+\sin s\sin t$$
$$=\left(-\frac{1}{5}\right)\left(-\frac{4}{5}\right)+\left(\frac{\sqrt{24}}{5}\right)\left(\frac{3}{5}\right)$$
$$=\frac{4}{25}+\frac{3\sqrt{24}}{25}=\frac{4}{25}+\frac{6\sqrt{6}}{25}=\frac{4+6\sqrt{6}}{25}$$

$$\sec(s-t)=\frac{1}{\cos(s-t)}=\frac{25}{4+6\sqrt{6}}=\frac{25}{2(2+3\sqrt{6})}=\frac{25(2-3\sqrt{6})}{2(4-54)}=\frac{25(2-3\sqrt{6})}{2(-50)}=-\frac{2-3\sqrt{6}}{4}=\frac{3\sqrt{6}-2}{4}$$

Q7. (4 points) (7.4 Textbook Exercise 100): Verify $\cos x = \dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$ is an identity.

Solution: $\tan\frac{x}{2} = \pm\sqrt{\frac{1-cox}{1+\cos}} \Rightarrow \tan^2\frac{x}{2} = \frac{1-cox}{1+\cos}$

100. Verify $\cos x = \dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$ is an identity.

Working with the right side, we have

$$\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{1-\frac{1-\cos x}{1+\cos x}}{1+\frac{1-\cos x}{1+\cos x}} = \frac{1-\frac{1-\cos x}{1+\cos x}}{1+\frac{1-\cos x}{1+\cos x}}\cdot\frac{1+\cos x}{1+\cos x} = \frac{(1+\cos x)-(1-\cos x)}{(1+\cos x)+(1-\cos x)} = \frac{2\cos x}{2} = \cos x$$

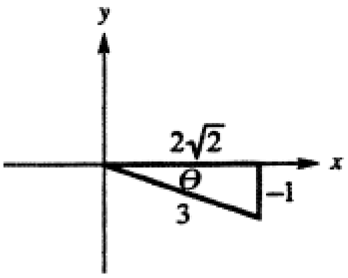
Q8. (4 points) (Ch7 Textbook Review Exercise 98): $\sec\left(2\sin^{-1}\left(-\frac{1}{3}\right)\right) = ?$

Solution:

98. $\sec\left(2\sin^{-1}\left(-\frac{1}{3}\right)\right)$

Let $\theta = \sin^{-1}\left(-\frac{1}{3}\right)$, so $\sin\theta = -\frac{1}{3}$. Since arcsine is defined only in quadrants I and IV, and $-\frac{1}{3}$ is negative, θ is in quadrant IV. Sketch θ and label a triangle with the side adjacent to θ equal to

$\theta = \sqrt{3^2-(-1)^2} = \sqrt{9-1} = \sqrt{8} = 2\sqrt{2}$.



Thus, $\cos\theta = \frac{2\sqrt{2}}{3}$ and

$\sec\left(2\sin^{-1}\left(-\frac{1}{3}\right)\right) = \sec 2\theta$.

$$\sec 2\theta = \frac{1}{\cos 2\theta} = \frac{1}{2\cos^2\theta - 1} = \frac{1}{2\left(\frac{2\sqrt{2}}{3}\right)^2 - 1}$$

$$= \frac{1}{2\left(\frac{8}{9}\right)-1} = \frac{1}{\frac{16}{9}-1} = \frac{9}{16-9} = \frac{9}{7}$$

Q9. (4 points) (Quiz Textbook Review Exercise 109): Solve $\cos\theta + 1 = 2\sin^2\theta$ over $[0°, 360°)$.

Solution:

6.

$$\cos\theta + 1 = 2\sin^2\theta$$
$$\cos\theta + 1 = 2(1-\cos^2\theta)$$
$$2\cos^2\theta + \cos\theta - 1 = 0$$
$$(2\cos\theta - 1)(\cos\theta + 1) = 0 \Rightarrow$$

$\cos\theta = \frac{1}{2}$ or $\cos\theta = -1$

Over the interval $[0°, 360°)$, the equation $\cos\theta = \frac{1}{2}$ has two solutions, the angles in quadrants I and IV that have a reference angle of 60°. These are 60° and 300°. Over the interval $[0°, 360°)$, the equation $\cos\theta = -1$ has one solution, 180°.

Solution set: {60°, 180°, 300°}

Q10. (4 points) **(7.7 Textbook Exercise 38):** Find the solution set of $\arccos x + 2\arcsin\frac{\sqrt{3}}{2} = \frac{\pi}{3}$.

Solution:

38. $\arccos x + 2\arcsin\frac{\sqrt{3}}{2} = \frac{\pi}{3}$

$\arccos x + 2\arcsin\frac{\sqrt{3}}{2} = \frac{\pi}{3} \Rightarrow$

$\arccos x + 2\left(\frac{\pi}{3}\right) = \frac{\pi}{3} \Rightarrow \arccos x = -\frac{\pi}{3}$

$-\frac{\pi}{3}$ is not in the range of $\arccos x$. Therefore,
the equation has no solution. Solution set: $\varnothing$

Q11. (6 points) Given the vectors $\boldsymbol{u} = 6\boldsymbol{i} + 3\sqrt{3}\boldsymbol{j}$ and $\boldsymbol{v} = 12\boldsymbol{i} + 8\sqrt{3}\boldsymbol{j}$, then

(a): Find the magnitude of $\boldsymbol{w} = \frac{1}{3}\boldsymbol{u} - \frac{1}{4}\boldsymbol{v}$

(b): Find a unit vector in the direction of $\boldsymbol{w} = \frac{1}{3}\boldsymbol{u} - \frac{1}{4}\boldsymbol{v}$.

(c): Find a vector of magnitude 8 in the opposite direction of $\boldsymbol{w} = \frac{1}{3}\boldsymbol{u} - \frac{1}{4}\boldsymbol{v}$.

Solution:

(a):

$$\begin{aligned}\boldsymbol{w} &= \frac{1}{3}\boldsymbol{u} - \frac{1}{4}\boldsymbol{v}\\ &= \frac{1}{3}\left\langle 6, 3\sqrt{3}\right\rangle - \frac{1}{4}\left\langle 12, 8\sqrt{3}\right\rangle\\ &= \left\langle 2, \sqrt{3}\right\rangle - \left\langle 3, 2\sqrt{3}\right\rangle\\ &= \left\langle -1, -\sqrt{3}\right\rangle\end{aligned}$$

$$\|\boldsymbol{w}\| = \left\|\frac{1}{3}\boldsymbol{u} - \frac{1}{4}\boldsymbol{v}\right\| = \left\|\left\langle -1, -\sqrt{3}\right\rangle\right\| = \sqrt{(-1)^2 + \left(\sqrt{3}\right)^2} = \sqrt{4} = 2$$

$$\tan\theta = \frac{-\sqrt{3}}{-1} = \sqrt{3} \quad \theta = \frac{4\pi}{3} = 240^\circ$$

(b): $\frac{1}{\|\boldsymbol{w}\|}\boldsymbol{w} = \frac{1}{2}\left\langle -1, -\sqrt{3}\right\rangle = \left\langle -\frac{1}{2}, -\frac{\sqrt{3}}{2}\right\rangle$

(c): $(-8)\frac{1}{\|\boldsymbol{w}\|}\boldsymbol{w} = (-8)\frac{1}{2}\left\langle -1, -\sqrt{3}\right\rangle = (-4)\left\langle -1, -\sqrt{3}\right\rangle = \left\langle 4, 4\sqrt{3}\right\rangle$