King Fahd University of Petroleum and Minerals

Prep-Year Math Program

Math 002 Class Test I
Textbook Sections: 4.1 to 6.1
Term 151
Time Allowed: 90 Minutes

Time: 6:00 pm – 7:30 pm

Student's Name: …………… ………………………………………………
ID #:………………………. **Section:** ……. **Serial Number:** …….

Provide neat and complete solutions.
Show all necessary steps for full credit and write the answer in simplest form.

No Calculators, Cameras, or Mobiles are allowed during this exam.

Question	**Points**	Student's Score
1	6	
2	4	
3	4	
4	5	
5	5	
6	4	
7	4	
8	4	
9	4	
10	4	
11	6	
Total	**50**	___ / 50
		___ / 100

Q1. (6 points): (4.1 Exercise 76, Page 396): Given $f(x) = -\sqrt{x^2 - 16}$ for $x \geq 4$.

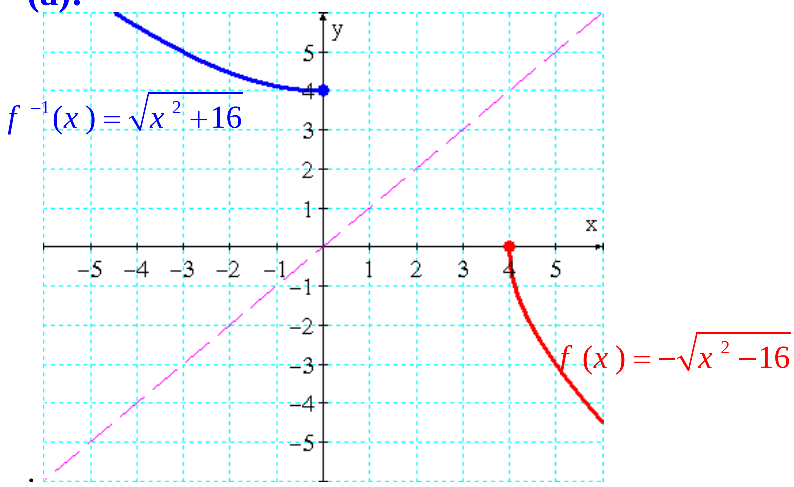
(a): Sketch the graph of f and f^{-1}

(b): State the domain and range of f and f^{-1}.

(c): $f^{-1}(x) = ?$

Solution:

(a):



(b): Given $D_f = [4, \infty)$ then $R_f = (-\infty, 0]$

So, $D_{f^{-1}} = R_f = (-\infty, 0]$ and $R_{F^{-1}} = D_f = [4, \infty)$

(c): $f^{-1}(x) = ?$ First check that F is a one-to-function:

$y = -\sqrt{x^2 - 16}$, $x \geq 4$

$x = -\sqrt{y^2 - 16}$, $y \geq 4 \Rightarrow x \leq 0$

Square both sides:

$x^2 = y^2 - 16$, $x \leq 0$, $y \geq 4$

$y^2 = x^2 + 16$, $x \leq 0$, $y \geq 4$

$\sqrt{y^2} = \sqrt{x^2 + 16}$, $x \leq 0$, $y \geq 4$

$|y| = \sqrt{x^2 + 16}$, $x \leq 0$, $y \geq 4$

$y = \sqrt{x^2 + 16}$, $x \leq 0$, $y \geq 4$

$f^{-1}(x) = \sqrt{x^2 + 16}$, $x \leq 0$, $y \geq 4$

Q2. (4 points): Find the interval where the graph of $f(x) = 3 - 2^{-x}$ is above the x-axis.

My note: 4.3 Page 84

(a): $(-\log_2 3, \infty)$

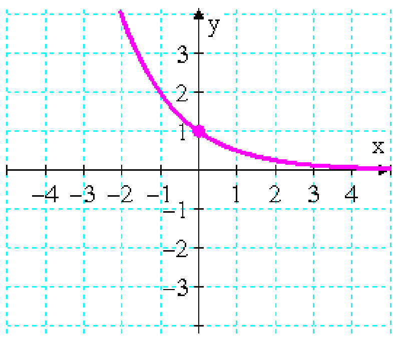
(b): $(-1, \infty)$

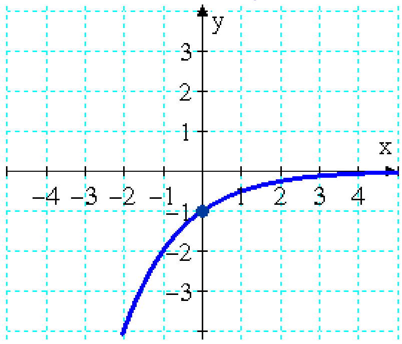
(c): $(-\infty, 3)$

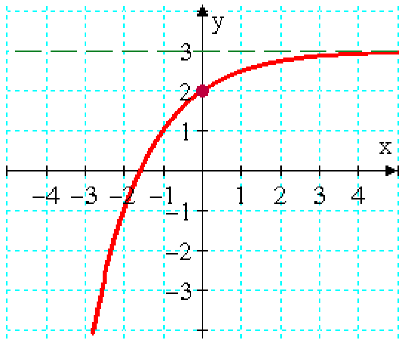
(d): $\left(-\infty, \log_2 \frac{1}{3}\right)$

(e): $(-\log_3 2, \infty)$

Solution: $y = \left(\frac{1}{2}\right)^x \xrightarrow{\text{reflection across } x\text{-axis}} y = -\left(\frac{1}{2}\right)^x \xrightarrow{\text{vertical shift 3 units up}} y = -\left(\frac{1}{2}\right)^x + 3$







To find x -intercept of $y = 3 - 2^{-x}$, put $y = 0$ and solve for x.

$0 = 3 - 2^{-x} \Rightarrow 2^{-x} = 3 \Rightarrow \log_2 2^{-x} = \log_2 3 \Rightarrow -x = \log_2 3 \Rightarrow x = \log_2 3$

The graph is above the x-axis on the interval $\left(-\log_2 3, \infty\right)$.

Q3. (4 points): Find the inverse of $f(x) = -\left(\frac{1}{2}\right)^{-x+1} + 2$

Solution:

$f(x) = -\left(2^{-1}\right)^{-x+1} + 2$

$y = -(2)^{x-1} + 2$

$x = -(2)^{y-1} + 2$ Interchange the variables.

$x - 2 = -(2)^{y-1}$

$-x + 2 = (2)^{y-1}$

$\log_2(-x+2) = \log_2(2)^{y-1}$

$y - 1 = \log_2(-x+2)$

$y = 1 + \log_2(-x+2)$

$f^{-1}(x) = 1 + \log_2(-x+2)$

Another Method of writing:

$y = -\left(\frac{1}{2}\right)^{-x+1} + 2$

$x = -\left(\frac{1}{2}\right)^{-y+1} + 2$ Interchange the variables.

$x - 2 = -\left(\frac{1}{2}\right)^{-y+1}$

$-x + 2 = \left(\frac{1}{2}\right)^{-y+1}$

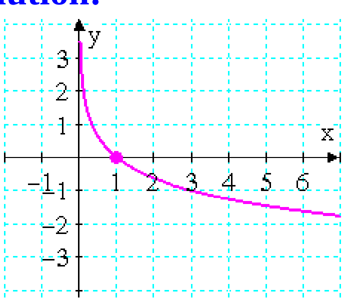
$-y + 1 = \log_{\frac{1}{2}}(-x+2)$

$y = 1 - \log_{\frac{1}{2}}(-x+2)$

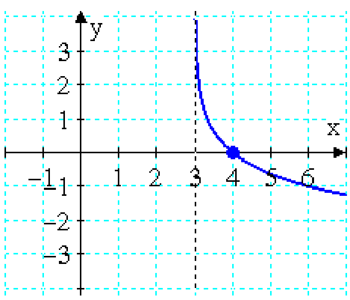
$f^{-1}(x) = 1 - \log_{\frac{1}{2}}(-x+2) = 1 + \log_2(-x+2)$

Q4. (5 points): (4.3 Similar to Textbook Exercise 44): Graph $f(x) = \left|\log_{\frac{1}{3}}(x-3)\right|$. Give the **domain** and **range**. Find the **interval(s)** where the graph is increasing or decreasing.

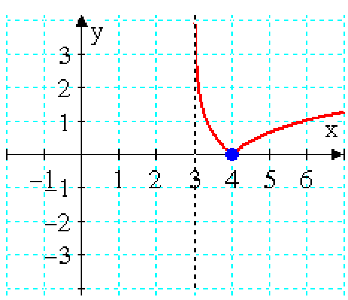
Solution:



$y = \log_{\frac{1}{3}}(x)$



$y = \log_{\frac{1}{3}}(x-3)$



$f(x) = \left|\log_{\frac{1}{3}}(x-3)\right|$ $D_f = (3, \infty)$, $R_f = [0, \infty)$

The function $f(x) = \left|\log_{\frac{1}{3}}(x-3)\right|$ is decreasing on $(3, 4]$

The function $f(x) = \left|\log_{\frac{1}{3}}(x-3)\right|$ is increasing on $[4, \infty)$

Q5. (5 points): (4.3 Textbook Summary Exercises **page 463** Q# 8, 9, 10, 16 and 17): Find the domain of each function. **Write answers using interval notation.**

(a): $f(x)=\sqrt[3]{x^3+7x-4}$, **(b):** $f(x)=\log_5(16-x^2)$, **(c):** $f(x)=\log\left(\frac{x+7}{x-3}\right)$,

(d): $f(x)=\ln\left|x^2-5\right|$, **(e):** $f(x)=e^{x^2+x+4}$

Solution: (a):

8. $f(x)=\sqrt[3]{x^3+7x-4}$

The domain is the set of all real numbers such that x^3+7x-4 is real. Domain: $(-\infty,\infty)$

(b):

9. $f(x)=\log_5\left(16-x^2\right)$

The domain is the set of all real numbers such that $16-x^2>0 \Rightarrow 16>x^2 \Rightarrow 4>x$ and $-4<x$.

Domain: $(-4, 4)$

(c):

10. $f(x)=\log\left(\frac{x+7}{x-3}\right)$

$\frac{x+7}{x-3}>0 \quad \Rightarrow$ Domain: $(-\infty,-7)\cup(3,\infty)$

(d):

16. $f(x)=\ln\left|x^2-5\right|$

The domain is the set of all real numbers such that $\left|x^2-5\right|\neq 0 \Rightarrow x^2-5\neq 0 \Rightarrow x^2\neq 5 \Rightarrow$

$x\neq\sqrt{5}$ or $x\neq-\sqrt{5}$

Domain: $\left(-\infty,-\sqrt{5}\right)\cup\left(-\sqrt{5},\sqrt{5}\right)\cup\left(\sqrt{5},\infty\right)$

(e):

17. $f(x)=e^{x^2+x+4}$

The domain is the set of values such that x^2+x+4 is real. Domain: $(-\infty,\infty)$

Q6. (4 points): (4.4 Additional Exercises #17): If $A=\left(\sqrt[3]{2}\right)^{\log_2 27}$ and $B=\left(\log_2 81\right)\left(\log_{\sqrt{3}}16\right)$, then $B-A=?$

Solution: $A=\left(\sqrt[3]{2}\right)^{\log_2 27}=\left(2^{\frac{1}{3}}\right)^{\log_2 3^3}=\left(2^{\frac{1}{3}}\right)^{3\log_2 3}=2^{\log_2 3}=3$

$$B=\left(\log_2 81\right)\left(\log_{\sqrt{3}}16\right)=\left(\log_2 3^4\right)\left(\frac{\log_2 16}{\log_2\sqrt{3}}\right)=\left(4\log_2 3\right)\left(\frac{\log_2 2^4}{\log_2 3^{1/2}}\right)$$

$$=\left(4\log_2 3\right)\left(\frac{4}{\frac{1}{2}\log_2 3}\right)=\left(4\log_2 3\right)\left(\frac{4(2)}{\log_2 3}\right)=4(4)(2)=32$$

$B-A=32-3=29$

Q7. (4 points): (5.1 Textbook Exercise 36 and 87):

(a): Find $47°23' - 73°48' = ?$

(b): Find the angle of least positive measure coterminal with $-5280°$

Solution:

(a):

$$\begin{aligned} 47°23' - 73°48' &= 47° + 23' - (73° + 48') \\ &= 47° + 23' - 73° - 48' \\ &= -73° + 47° - 48' + 23' \\ &= -26° - 25' \\ &= -(26° + 25') \\ &= -26°25' \end{aligned}$$

Another Method of Writing:

$$47°\,23' - 73°\,48' = -(73°\,48' - 47°\,23')$$

Since $73°\,48' - 47°\,23' = 26°\,25'$, we have

$$\begin{aligned} 47°\,23' - 73°\,48' &= -(73°\,48' - 47°\,23') \\ &= -26°\,25' \end{aligned}$$

(b):

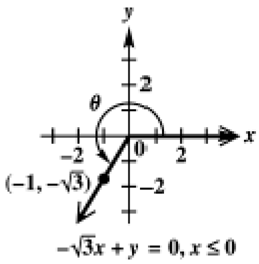
$-5280°$ is coterminal with

$-5280° + 15 \cdot 360° = -5280° + 5400° = 120°.$

Q8. (4 points): (5.2 Textbook Exercise 41): Find the six trigonometric function values of the angle θ in standard position, if the terminal side of θ is defined by $-\sqrt{3}x + y = 0, x \le 0$.

Solution:

41. Since $x \le 0$, the graph of the line $-\sqrt{3}x + y = 0$ is shown to the left of the y-axis. A point on this line is $(-1, -\sqrt{3})$ since $-\sqrt{3}(-1) - \sqrt{3} = \sqrt{3} - \sqrt{3} = 0$ The corresponding value of r is $r = \sqrt{(-1)^2 + (-\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$.



$$\sin\theta = \frac{y}{r} = \frac{-\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}$$

$$\cos\theta = \frac{x}{r} = \frac{-1}{2} = -\frac{1}{2}$$

$$\tan\theta = \frac{y}{x} = \frac{-\sqrt{3}}{-1} = \sqrt{3}$$

$$\cot\theta = \frac{x}{y} = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\sec\theta = \frac{r}{x} = \frac{2}{-1} = -2$$

$$\csc\theta = \frac{r}{y} = \frac{2}{-\sqrt{3}} = -\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

Q9. (4 points): (5.3 Additional Exercise 27): If $\cot\theta = \frac{1}{2}$ where $\pi < \theta < \frac{3\pi}{2}$, then $\sin\theta - \cos\theta = ?$

Solution: $\pi < \theta < \frac{3\pi}{2} \Rightarrow \theta$ is in quadrant III

$\cot\theta = \frac{1}{2} = \frac{-1}{-2}$ because

Let $x = -1$ and $y = -2$. Then $r = \sqrt{(-1)^2 + (-2)^2} = \sqrt{5}$

$$\sin\theta - \cos\theta = \frac{y}{r} - \frac{x}{r} = \frac{-2}{\sqrt{5}} - \frac{-1}{\sqrt{5}} = \frac{-2}{\sqrt{5}} + \frac{1}{\sqrt{5}} = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$$

Another Method:

Q22.

If $\cot\theta = \frac{1}{2}$ where $\pi < \theta < \frac{3\pi}{2}$, then $\sin\theta - \cos\theta =$ $-\sin\theta' + \cos\theta'$

$= -\frac{2}{\sqrt{5}} + \frac{1}{\sqrt{5}} = \frac{-1}{\sqrt{5}}$

$= -\frac{\sqrt{5}}{5}$

A) $-\frac{\sqrt{5}}{5}$

B) $-\frac{2\sqrt{5}}{5}$

C) $\frac{3\sqrt{5}}{5}$

D) $\frac{2\sqrt{5}}{5}$

E) $-\frac{3\sqrt{5}}{5}$

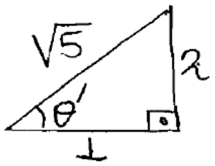
In quadrant III

$\sin\theta = -\sin\theta'$

$\cos\theta = -\cos\theta'$

Where θ' is the reference angle

$\cot\theta = \cot\theta' = \frac{1}{2} \Rightarrow$

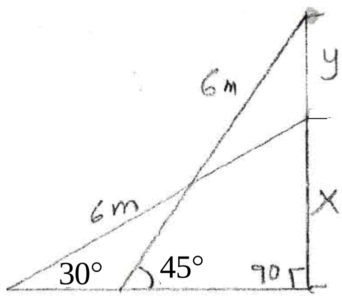


$\sin\theta' = \frac{2}{\sqrt{5}}$

$\cos\theta' = \frac{1}{\sqrt{5}}$

Q10. (4 points): (5.4 Recitation Q#3):

A ladder of 6 meters length is placed against a wall forms an angle of 30° with the ground. If the foot of the ladder is moved towards the wall, the angle changed to 45°. Find the exact distance moved by the top of the ladder on the wall.



Solution:

$\sin 30^\circ = \frac{x}{6} \Rightarrow \boxed{x = 3}$

$\sin 45^\circ = \frac{x+y}{6} \Rightarrow x + y = 3\sqrt{2} \Rightarrow y = 3\sqrt{2} - 3 = 3\left(\sqrt{2} - 1\right)$

Q11. **(6 points):** (6.1 Recitation Q# 3): Find the reference angle of the following angles

(a): $\theta = \frac{9\pi}{5}$ **(b):** $\theta = 10$

Solution: (a): $\theta = \frac{9\pi}{5}$ is in quadrant IV because $\frac{9\pi}{5} = \frac{9\pi}{5}\frac{180°}{\pi} = 9(36°) = 324°$

$\theta' = 2\pi - \frac{9\pi}{5} = \frac{\pi}{5}$

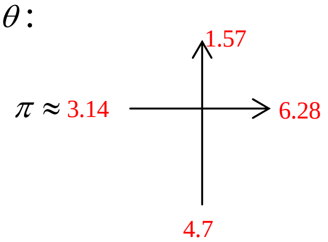
Another Method: We show that θ is in Quadrant IV:

$\theta = \frac{9\pi}{5} = \frac{18\pi}{10} \Rightarrow \frac{3\pi}{2} = \frac{15\pi}{10} < \frac{18\pi}{10} < \frac{20\pi}{10} = 2\pi \Rightarrow \theta' = 2\pi - \frac{9\pi}{5} = \frac{\pi}{5}$

(b): First, we find the smallest positive coterminal angle of θ:

$\alpha = 10 - 2\pi \approx 10 - 2(3.14) \approx 10 - 6.28 \approx 3.72 \in QIII$.

Then the reference angle of θ is



$$\theta' = \alpha' = \alpha - \pi$$
$$= 10 - 2\pi - \pi$$
$$= 10 - 3\pi$$