Serial #: ________ ID ______________ NAME ____

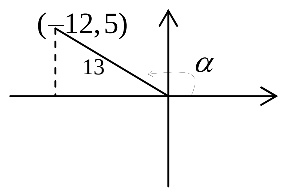
Show all necessary steps for full marks.

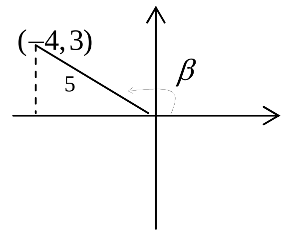
Q1. (7 points) Given $\sec\alpha = -\frac{13}{12}, \alpha$ in quadrant II, and $\sin\beta = \frac{3}{5}, \beta$ in quadrant II, find $\sec(\alpha - \beta)$.

Solution:

$\sec\alpha = -\frac{13}{12} \Rightarrow \cos\alpha = -\frac{12}{13}$ for α in Quadrant II

$\cos\alpha = -\frac{12}{13}$, $\alpha \in \text{QII} \overset{\text{figure}}{\Rightarrow} \sin\alpha = \frac{5}{13}$





$\sin\beta = \frac{3}{5} \Rightarrow \overset{\text{figure}}{\Rightarrow} \cos\beta = \frac{-4}{5}$

$\sec(\alpha-\beta) = 1/\cos(\alpha-\beta) = 1/[\cos\alpha\cos\beta + \sin\alpha\sin\beta]$

$$= 1/\left[\left(-\frac{12}{13}\right)\left(\frac{-4}{5}\right) + \left(\frac{5}{13}\right)\left(\frac{3}{5}\right)\right] = 1/\left[\frac{48}{65} + \frac{15}{65}\right] = 1/\left[\frac{63}{65}\right] = \frac{65}{63}$$

Q2. (7 points) Find the exact value of $\tan 427.5^\circ =$

A) $\sqrt{2}+1$ B) $\frac{1-\sqrt{3}}{2}$ C) $\sqrt{2}-1$ D) $\sqrt{3}+2$ E) $\frac{2-\sqrt{2}}{2}$

Solution:

$$\tan 427.5^\circ = \tan(427.5^\circ - 360^\circ) = \tan 67.5^\circ = \tan\frac{135^\circ}{2}$$

$$= \frac{\sin 135^\circ}{1+\cos 135^\circ} = \frac{\frac{\sqrt{2}}{2}}{1-\frac{\sqrt{2}}{2}} = \frac{\sqrt{2}}{2-\sqrt{2}} = \frac{\sqrt{2}}{2-\sqrt{2}} \cdot \frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{2\sqrt{2}+2}{4-2} = \sqrt{2}+1$$

OR

$$\tan 427.5^\circ = \tan(427.5^\circ - 360^\circ) = \tan 67.5^\circ = \tan\frac{135^\circ}{2} = \frac{1-\cos 135^\circ}{\sin 135^\circ} = \frac{1-\frac{-\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \sqrt{2}+1$$

Q3. (6 points) Given the function $f(x) = 2\sin\frac{x}{3} - 2\sqrt{3}\cos\frac{x}{3}$

a) Rewrite $f(x)$ in the form $f(x) = k\sin(bx+\alpha)$

b) Find the amplitude, the phase shift, the period, and the range for the graph of $f(x)$.

Solution:

(a): $f(x) = a\sin\frac{x}{3} + b\cos\frac{x}{3} = k\sin\left(\frac{x}{3}+\alpha\right)$

$a = 2$, $b = -2\sqrt{3} \Rightarrow (2, -2\sqrt{3})$ is in Quadrant IV.

$k = \sqrt{a^2+b^2} = \sqrt{2^2+(-2\sqrt{3})^2} = \sqrt{4+12} = 4$

$$\left.\begin{aligned}\sin\alpha &= \frac{b}{k} = \frac{-2\sqrt{3}}{4} = -\frac{\sqrt{3}}{2}\\ \cos\alpha &= \frac{a}{k} = \frac{2}{4} = \frac{1}{2}\end{aligned}\right\} \Rightarrow \alpha \text{ is in Quadrant IV and } \alpha = -\frac{\pi}{3} \text{ OR } \alpha = \frac{5\pi}{3}$$

$$f(x) = 4\sin\left(\frac{x}{3} - \frac{\pi}{3}\right) \quad \text{OR} \quad f(x) = 4\sin\left(\frac{x}{3} + \frac{5\pi}{3}\right)$$

(b):

Amplitude $= 4$

Phase shift $= -\dfrac{-\frac{\pi}{3}}{\frac{1}{3}} = \pi$ units to the right OR Phase shift $= -\dfrac{\frac{5\pi}{3}}{\frac{1}{3}} = -5\pi$ $|-5\pi|$ units to the right

Period $= \dfrac{2\pi}{1/3} = 6\pi$

Range $= [-4, 4]$