Serial #: _____ ID _____ NAME _____
Show all necessary steps for full marks.

Question 1: (5 points) (7.4 Textbook Exercise 42): Given $2\sin^2 x - \sin x - 1 = 0$
(a): Solve the equation over the interval $[0, 2\pi)$.
(b): Find all solutions of the equation.

Solution
(a): $(2\sin x + 1)(\sin x - 1) = 0$
$2\sin x + 1 = 0$ or $\sin x - 1 = 0$

$\sin x = -\frac{1}{2}$ or $\sin x = 1$

$x = \frac{7\pi}{6}$ or $x = \frac{11\pi}{6}$ $x = \frac{\pi}{2}$ Since $0 \le \frac{\pi}{2} < \pi$, but $\frac{7\pi}{2}$ is not in the requested domain.

$SS = \left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

(b): $\boxed{x = \frac{\pi}{2} + 2k\pi}$, $\boxed{x = \frac{7\pi}{6} + 2k\pi}$, $\boxed{x = \frac{11\pi}{6} + 2k\pi}$

Question 2: (5 points) Given $2\cos 2x - \sqrt{2} = 0$,
Find ll solutions over the interval $[0, 2\pi)$.

Solution:

$$\cos 2x = \frac{\sqrt{2}}{2} \Rightarrow 2x = \frac{\pi}{4} + 2n\pi \quad , \quad 2x = \frac{7\pi}{4} + 2n\pi$$

$$\Rightarrow x = \frac{\pi}{8} + n\pi \quad , \quad x = \frac{7\pi}{8} + n\pi$$

This is a cosine function with period 4π, so the solution set is $SS = \left\{\frac{\pi}{8} + n\pi \ , \ \frac{7\pi}{8} + n\pi\right\}$.

If $n = 0 \overset{\text{part } (a)}{\Rightarrow} x = \frac{\pi}{8} \in [0, 2\pi)$, $x = \frac{7\pi}{8} \in [0, 2\pi)$

If $n = 1 \overset{\text{part } (a)}{\Rightarrow} x = \frac{\pi}{8} + \pi = \frac{9\pi}{8} \in [0, 2\pi)$, $x = \frac{7\pi}{8} + \pi = \frac{15\pi}{8} \in [0, 2\pi)$

So, the solution set over the interval $[0, 2\pi)$ is $SS = \left\{\frac{\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{15\pi}{8}\right\}$.

Question 3: (5 points) (9.1 Textbook Exercise 22): Sketch the vector $u = \langle -8, -1 \rangle$ with initial point $(4,3)$, and find the terminal point.

Solution: The terminal point is the point which is found by shifting the point $(4,3)$ eight units to the left and one unit downward. $(x_2, y_2) = (-8+4, -1+3) = (-4, 2)$

OR By another Method:

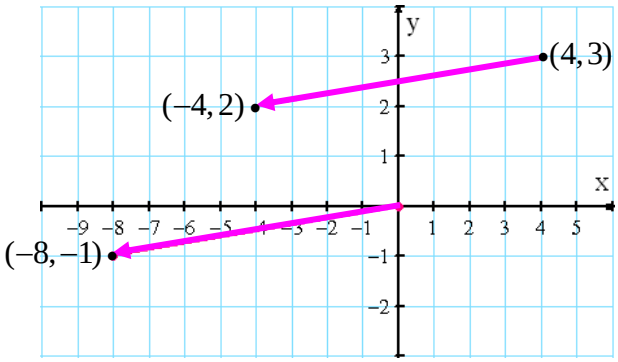
$v = (x_2 - x_1, y_2 - y_1)$

$(-8, -1) = (x_2 - 4, y_2 - 3)$

$-8 = x_2 - 4$ and $-1 = y_2 - 3$

$x_2 = -8 + 4$ and $y_2 - 1 + 3$

$(x_2, y_2) = (-4, 2)$



Question 4: (5 points) (9.2 Textbook Exercise 11):

Given $\mathbf{u} = -5\mathbf{j}$ and $\mathbf{v} = -\mathbf{i} - \sqrt{3}\mathbf{j}$. Find the following

(a): $\mathbf{u} \cdot \mathbf{v} = ?$ **(b):** Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

Solution (a): $\boldsymbol{u} = \langle 0, -5 \rangle$ and $\boldsymbol{v} = \langle -1, -\sqrt{3} \rangle$

$\mathbf{u} \cdot \mathbf{v} = \langle 0, -5 \rangle \cdot \langle -1, -\sqrt{3} \rangle = 0(-1) + (-5)(-\sqrt{3}) = 5\sqrt{3}$

(b): $\cos\theta = \dfrac{\boldsymbol{u} \cdot \boldsymbol{v}}{\|\boldsymbol{u}\| \cdot \|\boldsymbol{v}\|} = \dfrac{5\sqrt{3}}{5 \cdot 2} = \dfrac{\sqrt{3}}{2} \Rightarrow \theta = 30°$