Serial #: _____ ID _____ NAME _____

Show all necessary steps for full marks.

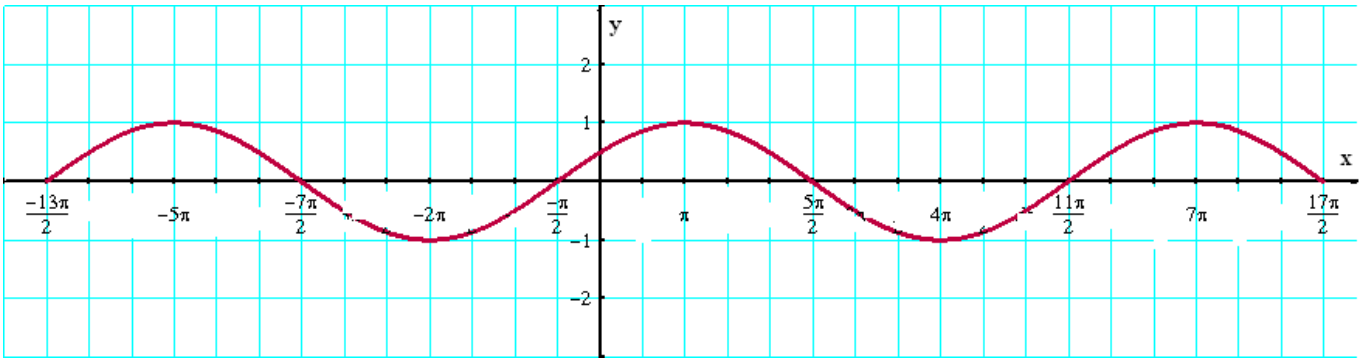
Question 1: (7 points) Given $y = \cos\left(\frac{x}{3} - \frac{\pi}{3}\right)$,

(a): Sketch the graph of function over the interval $\left[-\frac{13\pi}{2}, \frac{17\pi}{2}\right]$

(b): Find the intervals where the function is below the x-axis.

(c): Find the intervals where the function is decreasing over the interval $\left[-\frac{13\pi}{2}, \frac{17\pi}{2}\right]$

Solution (a): $0 \le \frac{x}{3} - \frac{\pi}{3} \le 2\pi \Rightarrow 0 \le x - \pi \le 6\pi \Rightarrow \pi \le x \le 7\pi$



(b): The graph is below the x-axis on $\left(-\frac{7\pi}{2}, -\frac{\pi}{2}\right)$ and on $\left(\frac{5\pi}{2}, \frac{11\pi}{2}\right)$

(c) The graph is decreasing on $(-5\pi, -2\pi)$, $(\pi, 4\pi)$ and $\left(7\pi, \frac{17\pi}{2}\right)$

Question 2: (3 points): If $f(x) = a\sin bx$, $b > 0$, is a sine function with period 4 and $f(1) = 2$, then find $f\left(\frac{2}{3}\right) = ?$

Solution:

$Period = \frac{2\pi}{b}$

$4 = \frac{2\pi}{b} \Rightarrow 4b = 2\pi \Rightarrow \boxed{b = \frac{\pi}{2}}$

$f(x) = a\sin bx$

$f(x) = a\sin\left(\frac{\pi}{2}x\right) \Rightarrow f(1) = a\sin\left(\frac{\pi}{2}\right) \Rightarrow 2 = a(1) \Rightarrow \boxed{a = 2}$

$f(x) = 2\sin\left(\frac{\pi}{2}x\right)$

$f\left(\frac{2}{3}\right) = 2\sin\left(\frac{\pi}{2}\cdot\frac{2}{3}\right) = 2\sin\left(\frac{\pi}{3}\right) = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$

Question 3: (5 points): Given the function $f(x)=2\tan\left(\frac{x}{2}-\frac{\pi}{2}\right)$ where $x\in(-2\pi,2\pi)$

(I): Sketch the graph of $f(x)$ over the given interval.

(II): Find x-intercepts of f over the given interval..

(III): Find all vertical asymptotes of f over the interval $[-2\pi,2\pi]$.

Solution (I):

$$-\frac{\pi}{2}<\frac{x}{2}-\frac{\pi}{2}<\frac{\pi}{2}$$

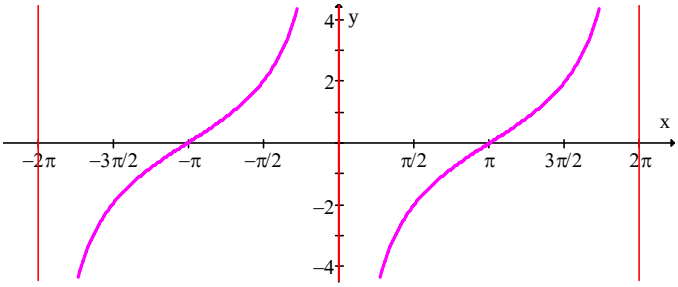
$$-\pi<x-\pi<\pi$$

$$0<x<2\pi$$

Beginning point = 0

Ending point = 2π

Middle point = π



(II): Zeros: $x_0+np=\pi+n(2\pi)=(1+2n)\pi$, $n=0,\pm1,\pm2,.....$

x −intercepts: $\boxed{x=-\pi}$, $\boxed{x=\pi}$

(III): Vertical Asymptotes: $x=x_0-\frac{1}{2}P+np=\pi-\frac{1}{2}P+np=\pi-\frac{1}{2}(2\pi)+n(2\pi)=2n\pi$

$\boxed{x=-2\pi}$, $\boxed{x=0}$, $\boxed{x=2\pi}$

Question 4: (5 points): Let $f(x)=3-2\csc\left(2x+\frac{\pi}{6}\right)$, where $-\frac{7\pi}{12}<x<\frac{5\pi}{2}$.

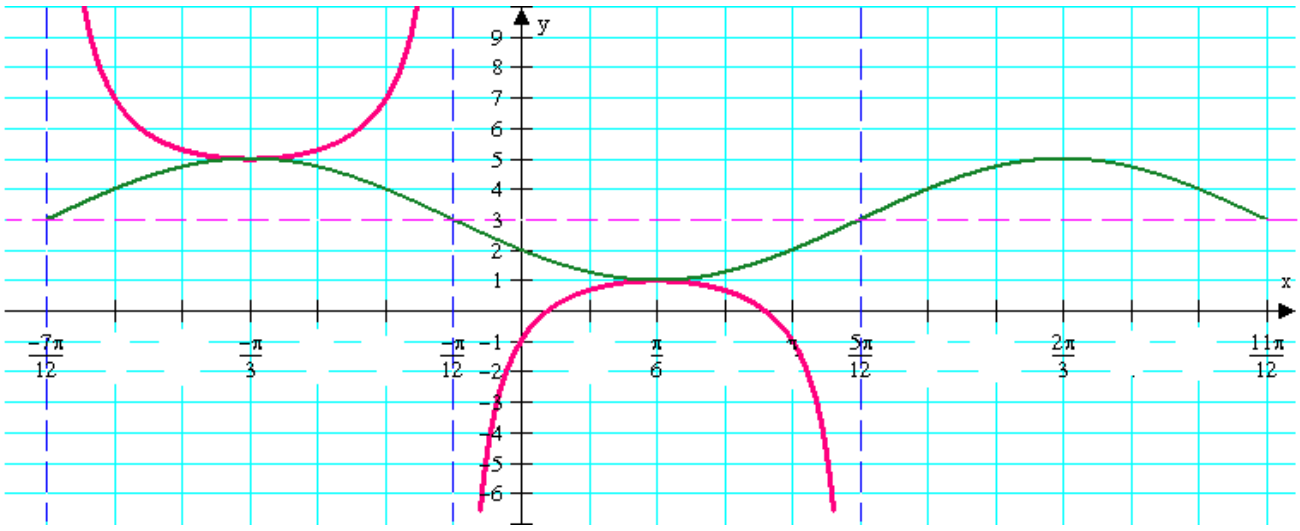
(a): Graph the function f.

(b): Determine the intervals where the function f is decreasing.

(a): $0\le 2x+\frac{\pi}{6}\le 2\pi \Rightarrow -\frac{\pi}{6}\le 2x\le 2\pi-\frac{\pi}{6} \Rightarrow -\frac{\pi}{6}\le 2x\le\frac{11\pi}{6} \Rightarrow -\frac{\pi}{12}\le x\le\frac{11\pi}{12}$

Phase shift is $-\frac{\pi}{12}$

The next key point is $-\frac{\pi}{12}+\frac{1}{4}P=-\frac{\pi}{12}+\frac{1}{4}\pi=-\frac{\pi}{12}+\frac{3\pi}{12}=\frac{2\pi}{12}=\frac{\pi}{6}$



(b): The function f is decreasing on the intervals $\left(-\frac{7\pi}{12},-\frac{\pi}{3}\right]$ and $\left[\frac{\pi}{6},\frac{5\pi}{12}\right)$.