King Fahd University of Petroleum and Minerals

Prep-Year Math Program

Math 002 Class Test I
Textbook Sections: 2.8 ad 4.1 to 5.3
Term 172
Time Allowed: 90 Minutes

Student's Name: …………… ………………………………………………
ID #:………………………. **Section:** ……. **Serial Number:** …….

Provide neat and complete solutions.
Show all necessary steps for full credit and write the answer in simplest form.

No Calculators, Cameras, or Mobiles are allowed during this exam.

Question	**Points**	Student's Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	5	
7	5	
8	5	
9	5	
10	5	
Total	**50**	____ 50
		____ 100

Q1. (5 points): If $f(x) = -x^2 + 4x,\ x \le 2$, then find

(a): $f^{-1}(x) = ?$ **(b):** $D_{f^{-1}} = ?$ **(c):** $R_{f^{-1}} = ?$

Solution:

$f(x) = -x^2 + 4x,\ x \le 2$

$y = -x^2 + 4x,\ x \le 2$

$x = -y^2 + 4y,\ y \le 2$

$y^2 - 4y = -x,\ y \le 2$

$y^2 - 4y + 4 = 4 - x,\ y \le 2$

$(y-2)^2 = 4 - x$, where $4 - x \ge 0,\ y \le 2$

$\sqrt{(y-2)^2} = \sqrt{4-x},\ -x \ge -4,\ y \le 2$

$|y-2| = \sqrt{4-x},\ x \le 4,\ y \le 2$

$-(y-2) = \sqrt{4-x},\ x \le 4,\ y \le 2$

$y - 2 = -\sqrt{4-x},\ x \le 4,\ y \le 2$

$y = 2 - \sqrt{4-x},\ x \le 4,\ y \le 2$

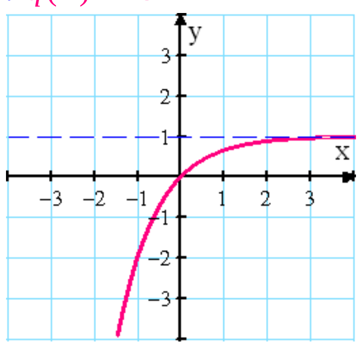
Answer: (a): $f^{-1}(x) = 2 - \sqrt{4-x}$ **(b):** $D_{f^{-1}} = (-\infty, 4]$ **(c):** $D_{f^{-1}} = (-\infty, 2]$

Q2. (5 points): Graph and find the range of the following functions;

(a): $q(x) = -3^{-x} + 1$ **(b):** $k(x) = -3^{|x|}$

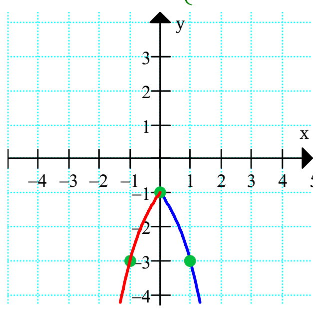
Solution:

(a): $q(x) = -3^{-x} + 1$



$Range = (-\infty, 1)$

(b): $k(x) = -3^{|x|} = \begin{cases} -3^{x} & x \ge 0 \\ -3^{-x} & x < 0 \end{cases}$



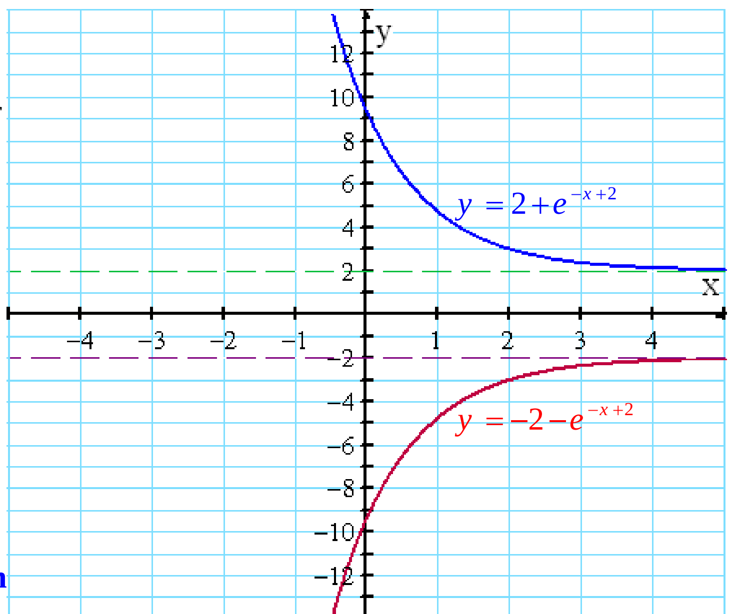
$Range = (-\infty, -1]$

Q3. (5 points): Given $f(x) = \left|-2 - e^{-x+2}\right|$. Find the following

(a) Sketch the graph of f

(b) Find the range of f

(c) Find the y- intercept of the graph of f



Solution:

(a) : $f(x) = \left|-2 - e^{-x+2}\right| = 2 + e^{-x+2}$

(b): The range of f is $(2, \infty)$

(c): The y- intercept of the graph of f is $\left(0, 2 + e^2\right)$

Q4. (5 points): If the graph of the function $f(x)=\log_5(x-20)$ intersects the graph of the function $g(x)=\log_5\left(\frac{1}{x}\right)+3$ at the point (a,b), then $a+b=?$

Solution: At the intersection point functions have same value.

$$f(x)=g(x)$$
$$\log_5(x-20)=\log_5\left(\frac{1}{x}\right)+3$$
$$\log_5(x-20)=\log_5 x^{-1}+3$$
$$\log_5(x-20)=-\log_5 x+3$$
$$\log_5(x-20)+\log_5 x=3$$
$$\log_5\left[(x-20)x\right]=3$$
$$x^2-20x=5^3$$
$$x^2-20x-125=0$$
$$(x+5)(x-25)=0$$
$$\boxed{x=25} \quad \text{or} \quad x=-5 \text{ rejected}$$

$f(25)=\log_5(25-20)=\log_5 5=1$

The intersection point is $(25,1)=(a,b)$

$a+b=25+1=26$

Q5. (5 points): Textbook Exercise 36: Solve $7^{x/2}=5^{1-x}$

Solution:

36. (a) $7^{x/2}=5^{1-x} \Leftrightarrow \log 7^{x/2}=\log 5^{1-x} \Leftrightarrow \left(\frac{x}{2}\right)\log 7=(1-x)\log 5 \Leftrightarrow \left(\frac{x}{2}\right)\log 7=\log 5-x\log 5 \Leftrightarrow$

$\left(\frac{x}{2}\right)\log 7+x\log 5=\log 5 \Leftrightarrow x\left(\frac{1}{2}\log 7+\log 5\right)=\log 5 \Leftrightarrow x=\frac{\log 5}{\frac{1}{2}\log 7+\log 5}$

Question 6: Each tire of a car has a radius of 40 cm. If the tires are rotating at 500 revolutions per minute, find the speed of the car in kilometers per hour.

Solution:

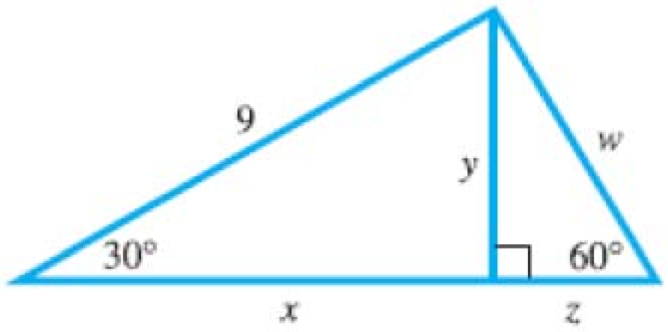
$$r=40\text{ cm}=40\left(\frac{1}{100}\text{m}\right)=\frac{4}{10}\text{ m}=\frac{4}{10}\left(\frac{1}{1000}\text{km}\right)$$

$$w=500\frac{rev}{\min}=500(2\pi)\frac{radian}{\min}=500(2\pi)\frac{radina}{\frac{1}{60}\text{hr}}=500(2\pi)60\frac{radian}{\text{hr}}$$

$$v=rw=\left(\frac{4}{10000}\text{ km}\right)500(2\pi)60\frac{radian}{\text{hr}}=\frac{4}{10}(5)(2\pi)(6)\frac{\text{km}}{\text{hr}}=24\pi\frac{\text{km}}{\text{hr}}$$

Answer: $24\pi\frac{\text{km}}{\text{hr}}$

Q7. (5 points): Find the exact value of each labeled part with a variable in the following figure



Solution:

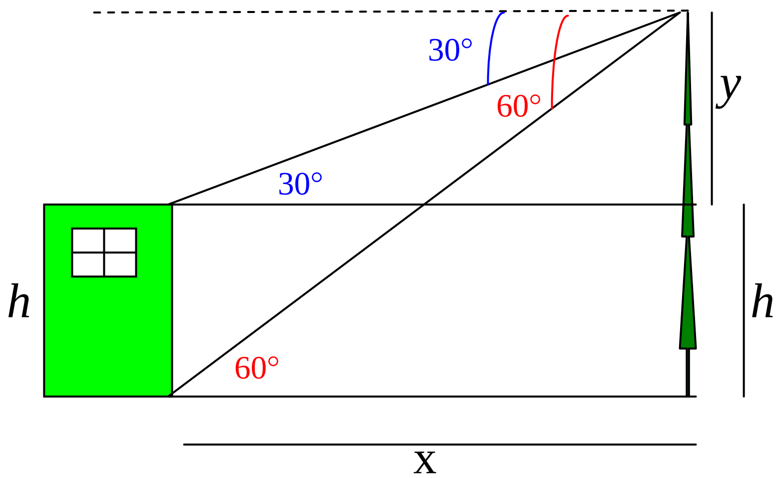
$\cos 30° = \frac{x}{9}$ $\frac{\sqrt{3}}{2} = \frac{x}{9} \Rightarrow \boxed{x = \frac{9\sqrt{3}}{2}}$	$\sin 30° = \frac{y}{9}$ $\frac{1}{2} = \frac{y}{9} \Rightarrow \boxed{y = \frac{9}{2}}$	$\tan 60° = \frac{y}{z}$ $\sqrt{3} = \frac{\frac{9}{2}}{z} \Rightarrow z\sqrt{3} = \frac{9}{2} \Rightarrow z = \frac{9}{2\sqrt{3}} \Rightarrow \boxed{z = \frac{3\sqrt{3}}{2}}$

$\cos 60° = \frac{z}{w}$

$\frac{1}{2} = \frac{\frac{3\sqrt{3}}{2}}{w} \Rightarrow w = 2 \cdot \frac{3\sqrt{3}}{2} \Rightarrow \boxed{w = 3\sqrt{3}}$

Answer: $\boxed{x = \frac{9\sqrt{3}}{2}}$ $\boxed{y = \frac{9}{2}}$ $\boxed{z = \frac{3\sqrt{3}}{2}}$ $\boxed{w = 3\sqrt{3}}$

Q8. (5 points):
If from the top of a tower 200 feet high, the angles of depression of the top and bottom of a building opposite to the tower are observed to be $30°$ and $60°$, respectively, then find the height of the building. (Show your work)



Solution: $\tan 60° = \frac{200}{x}$

$\sqrt{3} = \frac{200}{x} \Rightarrow \boxed{x = \frac{200}{\sqrt{3}}}$

$\tan 30° = \frac{y}{x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{y}{x} \Rightarrow x = \sqrt{3}y \Rightarrow \frac{200}{\sqrt{3}} = \sqrt{3}y \Rightarrow \frac{200}{3} = y$

$h = 200 - y = 200 - \frac{200}{3} = \frac{400}{3}$ feet

Answer: B) $\frac{400}{3}$

Q9. (5 points): Write each of the following in terms of the same trigonometric function of a reference angle.

(a): $\sin\frac{16\pi}{9}=?$ **(b):** $\csc\left(-\frac{43\pi}{5}\right)=?$

Solution: (a): $\frac{3\pi}{2}<\frac{16\pi}{9}<\frac{18\pi}{9} \Rightarrow \theta=\frac{16\pi}{9}\in QIV$

$\Rightarrow \theta'=2\pi-\frac{16\pi}{9}=\frac{2\pi}{9}$

$\Rightarrow \sin\theta=-\sin\theta'$

$\Rightarrow \sin\frac{16\pi}{9}=-\sin\frac{2\pi}{9}$

(b): $\theta=-\frac{43\pi}{5}$

$-\frac{43\pi}{5}+10\pi=\frac{-43\pi+50\pi}{5}=\frac{7\pi}{5}$ is the smallest positive coterminal angle of $\theta=-\frac{43\pi}{5}$.

$\sin\left(-\frac{43\pi}{5}\right)=\sin\left(\frac{7\pi}{5}\right)$

$\pi<\frac{7\pi}{5}<\frac{3\pi}{2} \Rightarrow \frac{7\pi}{5}\in QIII$

$\theta'=\frac{7\pi}{5}-\pi=\frac{2\pi}{5}$

$\csc\theta=-\csc\theta'$

$\csc\left(-\frac{43\pi}{5}\right)=-\csc\frac{2\pi}{5}$

Q10. (5 points): If the equation of terminal side of θ in standard position is $x+2y=0$, $x\geq 0$, then $3\csc\theta+4\sec\theta=?$

Solution:

12) If the equation of the terminal side of θ in standard position is

$x+2y=0$, $x\geq 0$, then $3\csc\theta+4\sec\theta=$

A) $-\sqrt{5}$

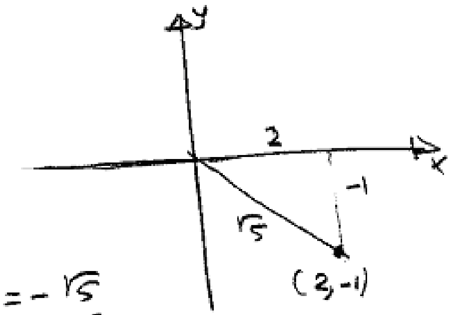
B) 1

C) $\sqrt{5}$

D) $-5\sqrt{5}$

E) $5\sqrt{5}$

$y=-\frac{x}{2}$



$\csc\theta=-\sqrt{5}$

$\sec\theta=\frac{\sqrt{5}}{2}$

$3\csc\theta+4\sec\theta=-3\sqrt{5}+4\frac{\sqrt{5}}{2}$

$=-3\sqrt{5}+2\sqrt{5}$

$=-\sqrt{5}$