King Fahd University of Petroleum and Minerals

Prep-Year Math Program

Math 002 - Term 132

Recitation (7.5)

Question 1: Find the exact value of $\cot\left[\frac{\pi}{2}-\sin^{-1}\left(-\frac{12}{13}\right)\right]$.

Answer: $-\frac{12}{5}$

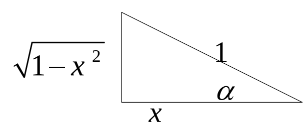
Question 2: Find the exact value of $\csc^{-1}\left(-\sqrt{2}\right)+\sin^{-1}(\sin\frac{4\pi}{5})=?$

Answer: $-\frac{\pi}{20}$

Question 3: Verify the identity $\tan\left(2\cos^{-1}x\right)=\frac{2x\sqrt{1-x^2}}{2x^2-1}$

Solution:

Let $\alpha=\cos^{-1}x \quad \Rightarrow \quad \cos\alpha = x \quad \Rightarrow \quad \cos\alpha=\frac{x}{1}$



$\tan\left(2\cos^{-1}x\right)=\tan 2\alpha$

$$=\frac{\sin 2\alpha}{\cos 2\alpha}=\frac{2\sin\alpha\cos\alpha}{2\cos^2\alpha-1}=\frac{2\left(\sqrt{1-x^2}\right)x}{2x^2-1}=\frac{2x\sqrt{1-x^2}}{2x^2-1}$$

Another Method:

$$\tan\left(2\cos^{-1}x\right)=\tan 2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}=\frac{2\frac{\sqrt{1-x^2}}{x}}{1-\frac{1-x^2}{x^2}}=\frac{2x\sqrt{1-x^2}}{x^2-(1-x^2)}=\frac{2x\sqrt{1-x^2}}{2x^2-1}$$

Question 4: Which one of the following statements is FALSE?

A) $\sin^{-1}(\sin x) = x\,,\ 0\le x\le 2\pi$

B) $\sec^{-1}x=\cos^{-1}\frac{1}{x},\ x\le -1 \text{ or } x\ge 1,$

C) $\cot^{-1}x=\frac{\pi}{2}-\tan^{-1}x, \quad -\infty<x<\infty$

D) $\csc^{-1}x=\sin^{-1}\frac{1}{x}, \quad x\le -1 \text{ or } x\ge 1$

E) $\sin^{-1}(-x)=-\sin^{-1}x, \quad -1\le x\le 1$

Answer:

(A): False. $\sin^{-1}(\sin x) = x$ if $0\le x\le 2\pi$

For example if $x=\pi$ then $\sin^{-1}(\sin\pi)\ne\pi$ because $\sin^{-1}(\sin\pi)=\sin^{-1}(0)=0$

(B): True: $\sec^{-1}x = \cos^{-1}\dfrac{1}{x},\ x \le -1$ or $x \ge 1$. **It is an identity.**

(C): True. $\cot^{-1}x = \dfrac{\pi}{2} - \tan^{-1}x$ is an identity.

(D): True. It is an identity:

(E): True. It is an identity:

Answer: **(A):** False. **(B):** True. **(C):** True. **(D):** True. **(E):** True.

Question 5 $\csc^{-1}\left(\dfrac{-2\sqrt{3}}{3}\right) + \cos^{-1}\left(\sin\dfrac{\pi}{5}\right) =$

A) $\dfrac{\pi}{20}$

B) $-\dfrac{2\pi}{15}$

C) $-\dfrac{\pi}{30}$

D) $\dfrac{2\pi}{15}$

E) $\dfrac{3\pi}{20}$

Answer: $-\dfrac{\pi}{30}$