Serial # ________ ID ______________ NAME ____

Show all necessary steps for full marks.

Q1. (5 points)(9.5 Textbook Exercise 15): Find the solution set of the system of equations:

$$3x^2 + 2y^2 = 5$$
$$x - y = -2$$

Solution:

15. $3x^2 + 2y^2 = 5\ (1)$

$x - y = -2\ (2)$

Solving equation (2) for y, we have $y = x + 2$.

Substitute this result into equation (1).

$$3x^2 + 2(x+2)^2 = 5$$
$$3x^2 + 2(x^2 + 4x + 4) = 5$$
$$3x^2 + 2x^2 + 8x + 8 = 5$$
$$5x^2 + 8x + 3 = 0$$
$$(5x+3)(x+1) = 0 \Rightarrow x = -\tfrac{3}{5} \text{ or } x = -1$$

If $x = -\frac{3}{5}$, then $y = -\frac{3}{5} + 2 = -\frac{3}{5} + \frac{10}{5} = \frac{7}{5}$. If $x = -1$, then $y = -1 + 2 = 1$.

Solution set: $\left\{\left(-\frac{3}{5}, \frac{7}{5}\right), (-1, 1)\right\}$

Q2. (5 points)(9.5 Textbook Summary Exercise 19, page 879): Find the solution set of the system of equations:

$$2x^2 + y^2 = 9$$
$$3x - 2y = -6$$

Solution:

19. $2x^2 + y^2 = 9\ \ (1)$

$3x - 2y = -6\ (2)$

Solving equation (2) for x, we have

$$3x = 2y - 6 \Rightarrow x = \frac{2y-6}{3}.$$

Substitute this result into equation (1).

$$2\left(\frac{2y-6}{3}\right)^2 + y^2 = 9$$
$$2\cdot\frac{(2y-6)^2}{9} + y^2 = 9$$
$$2\cdot(2y-6)^2 + 9y^2 = 81$$
$$2(4y^2 - 24y + 36) + 9y^2 = 81$$
$$8y^2 - 48y + 72 + 9y^2 = 81$$
$$17y^2 - 48y + 72 = 81$$
$$17y^2 - 48y - 9 = 0$$
$$(y-3)(17y+3) = 0 \Rightarrow y - 3 \text{ or } y = -\tfrac{3}{17}$$

If $y = 3$, then $x = \frac{2(3)-6}{3} = \frac{6-6}{3} = \frac{0}{3} = 0$.

If $y = -\frac{3}{17}$, then $x = \frac{2\left(-\frac{3}{17}\right)-6}{3} = \frac{2(-3)-102}{51}$ $= \frac{-6-102}{51} = \frac{-108}{51} = -\frac{36}{17}$.

Solution set: $\left\{(0, 3), \left(-\frac{36}{17}, -\frac{3}{17}\right)\right\}$

Q3. (5 points) (9.7 Recitation Q#2): If $A=\begin{bmatrix}-1 & 2 & -3\\ 6 & -1 & 2\end{bmatrix}$, $B=\begin{bmatrix}0 & -1 & 4\\ -2 & 6 & -3\end{bmatrix}$, then find the matrix X for which $4X+B=X-2A$.

Solution:

$$\begin{aligned}
4X+B &= X-2A\\
3X &= -B-2A\\
3X &= -(B+2A)\\
X &= -\frac{1}{3}(B+2A)\\
&= -\frac{1}{3}\left(\begin{bmatrix}0 & -1 & 4\\ -2 & 6 & -3\end{bmatrix}+\begin{bmatrix}-2 & 4 & -6\\ 12 & -2 & 4\end{bmatrix}\right)\\
&= -\frac{1}{3}\begin{bmatrix}-2 & 3 & -2\\ 10 & 4 & 1\end{bmatrix}\\
&= \begin{bmatrix}\frac{2}{3} & -1 & \frac{2}{3}\\ -\frac{10}{3} & -\frac{4}{3} & -\frac{1}{3}\end{bmatrix}
\end{aligned}$$

Q4. (5 points): Let $A=\begin{bmatrix}2 & 0 & 9\\ 1 & -1 & 3\\ -3 & 1 & 0\\ 0 & 2 & 4\end{bmatrix}$, $B=\begin{bmatrix}-3 & 0 & 5\\ 0 & -1 & 3\\ 3 & 0 & 4\end{bmatrix}$ and $AB=C=[c_{ij}]$. Find $c_{13}-c_{31}=?$

Hint: c_{13} = The element in the first row and third column of AB .
c_{31} = The element in the third row and first column of AB.

Solution:

$$c_{13}=\begin{bmatrix}2 & 0 & 9\end{bmatrix}\begin{bmatrix}5\\ 3\\ 4\end{bmatrix}=(2)(5)+(0)(3)+(9)(4)=46$$

$$c_{31}=\begin{bmatrix}-3 & 1 & 0\end{bmatrix}\begin{bmatrix}-3\\ 0\\ 3\end{bmatrix}=(-3)(-3)+(1)(0)+(0)(3)=9$$

$$\begin{aligned}
c_{13}-c_{31} &= 46-9\\
&= 37
\end{aligned}$$