SN _____ ID _____ NAME _____

Show all necessary steps for full marks.

Question 1: (7 points): (9.1 Recitation Q#1): If (a,b) is the solution of system of equation

$$\begin{aligned} 3\sqrt{2}\,x - 4\sqrt{3}\,y &= -6 \\ 2\sqrt{2}\,x + 3\sqrt{3}\,y &= 13 \end{aligned},$$

Then find $a \cdot b = ?$

Solution:

$$\begin{aligned} 3\sqrt{2}\,x - 4\sqrt{3}\,y &= -6 \quad (\text{I}) \xrightarrow{-2(\text{II})} -6\sqrt{2}\,x + 8\sqrt{3}\,y = 12 \\ 2\sqrt{2}\,x + 3\sqrt{3}\,y &= 13 \quad (\text{II}) \xrightarrow{3(\text{II})} \underline{6\sqrt{2}\,x + 9\sqrt{3}\,y = 39} \\ & \qquad\qquad 17\sqrt{3}\,y = 51 \end{aligned} \Rightarrow y = \frac{51}{17\sqrt{3}} = \frac{51\sqrt{3}}{51} = \sqrt{3}$$

$$(\text{II}) \Rightarrow 2\sqrt{2}\,x + 3\sqrt{3}\,\sqrt{3} = 13 \Rightarrow 2\sqrt{2}\,x + 9 = 13 \Rightarrow 2\sqrt{2}\,x = 4 \Rightarrow \sqrt{2}\,x = 2 \Rightarrow x = \frac{2}{\sqrt{2}} \Rightarrow \boxed{x = \sqrt{2}}$$

The solution is $\left(\sqrt{2},\sqrt{3}\right)$. The solution set is $SS = \left\{\left(\sqrt{2},\sqrt{3}\right)\right\}$

Then $a \cdot b = \sqrt{2} \cdot \sqrt{3} = \sqrt{6}$

Question 2: (7 points): (9.5 Textbook Exercise 12):

Find the solution set of the system of equations $\begin{cases} y = x^2 + 6x + 9 \\ x + 2y = -2 \end{cases}$

Solution:

12. $y = x^2 + 6x + 9$ (1)
$x + 2y = -2$ (2)

Algebraic Solution:

Solving equation (2) for y, we have $y = \frac{-x-2}{2}$.

Substitute this result into equation (1).

$$\frac{-x-2}{2} = x^2 + 6x + 9$$
$$-x - 2 = 2x^2 + 12x + 18$$
$$0 = 2x^2 + 13x + 20$$

$(x+4)(2x+5) = 0 \Rightarrow x = -4$ or $x = -\frac{5}{2}$

If $x = -4$, then $y = \frac{-(-4)-2}{2} = \frac{4-2}{2} = 1$.

If $x = -\frac{5}{2}$, then $y = \frac{-\left(-\frac{5}{2}\right)-2}{2} = \frac{5-4}{4} = \frac{1}{4}$.

Solution set: $\left\{(-4,1),\left(-\frac{5}{2},\frac{1}{4}\right)\right\}$

Question 3: (6 points): (9.7 Exercise 63): $\begin{bmatrix} -2 & -3 & -4 \\ 2 & -1 & 0 \\ 4 & -2 & 3 \end{bmatrix}\begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 3 & 2 & -2 \end{bmatrix} = ?$

Solution:

63. A 3×3 matrix multiplied by a 3×3 matrix results in a 3×3 matrix.

$$\begin{bmatrix} -2 & -3 & -4 \\ 2 & -1 & 0 \\ 4 & -2 & 3 \end{bmatrix}\begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 3 & 2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -2(0)+(-3)(1)+(-4)(3) & -2(1)+(-3)(2)+(-4)(2) & -2(4)+(-3)(-1)+(-4)(-2) \\ 2(0)+(-1)(1)+0(3) & 2(1)+(-1)(2)+0(2) & 2(4)+(-1)(-1)+0(-2) \\ 4(0)+(-2)(1)+3(3) & 4(1)+(-2)(2)+3(2) & 4(4)+(-2)(-1)+3(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 0+(-3)+(-12) & -2+(-6)+(-8) & -8+3+8 \\ 0+(-1)+0 & 2+(-2)+0 & 8+1+0 \\ 0+(-2)+9 & 4+(-4)+6 & 16+2+(-6) \end{bmatrix} = \begin{bmatrix} -15 & -16 & 3 \\ -1 & 0 & 9 \\ 7 & 6 & 12 \end{bmatrix}$$