Serial #: _________ ID ______________ NAME ____

Show all necessary steps for full marks.

Q1. (4 points) Find all vertical asymptotes of $y = 2\tan\left(3\pi x + \frac{\pi}{4}\right)$ over the interval $\left[0, \frac{5}{12}\right]$.

Solution: All vertical asymptotes are given by: $3\pi x + \frac{\pi}{4} = (2n+1)\frac{\pi}{2}$, $n = 0, \pm 1, , \pm 2, , \pm 3, \cdots$

$$3x = (2n+1)\frac{1}{2} - \frac{1}{4} = n + \frac{1}{2} - \frac{1}{4} = n + \frac{1}{4}$$

$$x = \frac{n}{3} + \frac{1}{12} \Rightarrow \boxed{x = \frac{4n+1}{12}}$$ where n is an integer

If $n = 0$ then $x = \frac{1}{12} \in \left[0, \frac{5}{12}\right]$

If $n = 1$ then $x = \frac{5}{12} \in \left[0, \frac{5}{12}\right]$

If $n = 2$ then $x = \frac{9}{12} \notin \left[0, \frac{5}{12}\right]$

The vertical asymptotes are: $\boxed{x = \frac{1}{12}}$ and $\boxed{x = \frac{5}{12}}$

Q2. (5 points): Let $f(x) = 3 - 2\csc\left(2x + \frac{\pi}{6}\right)$, where $-\frac{7\pi}{12} < x < \frac{5\pi}{12}$.

(a): Graph the function f .

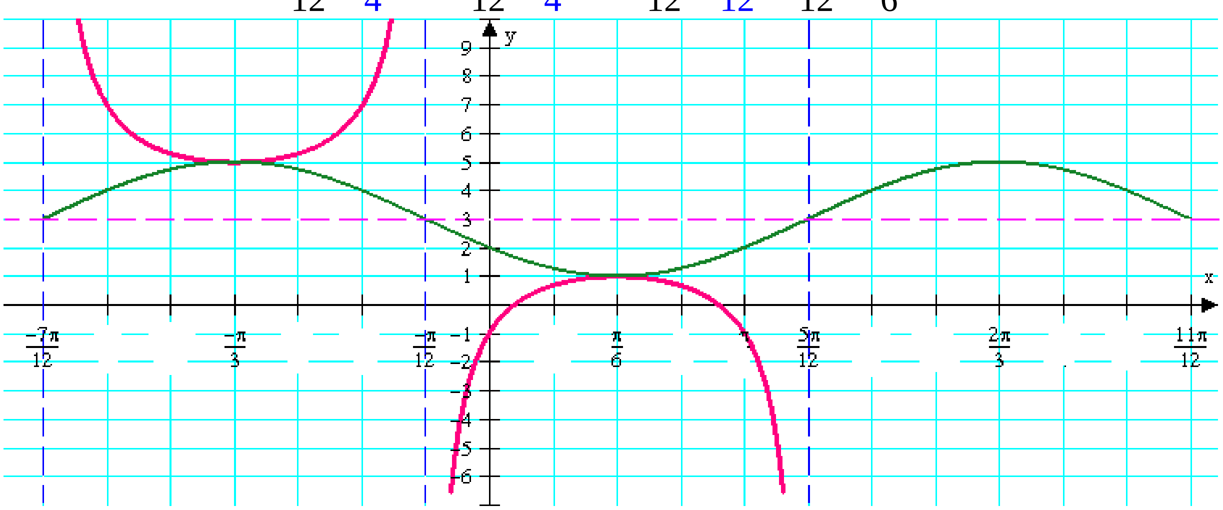
(b): Determine the intervals where the function f is decreasing.

Solution: (a):

$$0 \le 2x + \frac{\pi}{6} \le 2\pi \Rightarrow -\frac{\pi}{6} \le 2x \le 2\pi - \frac{\pi}{6} \Rightarrow -\frac{\pi}{6} \le 2x \le \frac{11\pi}{6} \Rightarrow -\frac{\pi}{12} \le x \le \frac{11\pi}{12}$$

Phase shift is $-\frac{\pi}{12}$

The next key point is $-\frac{\pi}{12} + \frac{1}{4}P = -\frac{\pi}{12} + \frac{1}{4}\pi = -\frac{\pi}{12} + \frac{3\pi}{12} = \frac{2\pi}{12} = \frac{\pi}{6}$



(b): The function f is decreasing on the intervals $\left(-\frac{7\pi}{12}, -\frac{\pi}{3}\right]$ and $\left[\frac{\pi}{6}, \frac{5\pi}{12}\right)$.

Q3. (5 points) (Additional exercise 9): If $x = 3 + 2\sin\theta,\ 0 < \theta < \frac{\pi}{6}$, then $\frac{\left[4-(x-3)^2\right]^{3/2}}{(2x-6)^2} = ?$

(Simplify your answer)

Solution: $x - 3 = 2\sin\theta,\ 0 < \theta < \frac{\pi}{6} \quad \Rightarrow \quad (x-3)^2 = (2\sin\theta)^2,\ 0 < \theta < \frac{\pi}{6}$

$$\frac{\left[4-(x-3)^2\right]^{3/2}}{(2x-6)^2} = \frac{\left[4-(2\sin\theta)^2\right]^{3/2}}{[2(x-3)]^2} = \frac{\left[4-4\sin^2\theta\right]^{3/2}}{\left[2(2\sin\theta)\right]^2} = \frac{\left[4(1-\sin^2\theta)\right]^{3/2}}{16\sin^2\theta}$$

$$= \frac{\left[4\cos^2\theta\right]^{3/2}}{16\sin^2\theta} = \frac{\left(\sqrt{4\cos^2\theta}\right)^3}{16\sin^2\theta} = \frac{|2\cos\theta|^3}{16\sin^2\theta} \quad \text{because } \cos\theta > 0 \text{ for } 0 < \theta < \frac{\pi}{6}$$

$$= \frac{(2\cos\theta)^3}{16\sin^2\theta}$$

$$= \frac{8\cos^2\theta\cos\theta}{16\sin^2\theta} = \frac{1}{2}\cot^2\theta\cos\theta$$

Q4. (6 points) (7.2 Exercises 17 and 18 and 41): Factor:

(a): $2\sin^2 x + 3\sin x + 1$

(b): $4\tan^2\beta + \tan\beta - 3$

(c): $\cot\theta + \tan\theta$

Solution:

(a):

17. $2\sin^2 x + 3\sin x + 1$

Let $a = \sin x$.

$$\begin{aligned} 2\sin^2 x + 3\sin x + 1 &= 2a^2 + 3a + 1 \\ &= (2a+1)(a+1) \\ &= (2\sin x + 1)(\sin x + 1) \end{aligned}$$

(b):

18. $4\tan^2\beta + \tan\beta - 3$

Let $a = \tan\beta$.

$$\begin{aligned} 4\tan^2\beta + \tan\beta - 3 &= 4a^2 + a - 3 \\ &= (4a-3)(a+1) \\ &= (4\tan\beta - 3)(\tan\beta + 1) \end{aligned}$$

(c):

41.

$$\begin{aligned} \cot\theta + \tan\theta &= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} \\ &= \frac{\cos^2\theta}{\sin\theta\cos\theta} + \frac{\sin^2\theta}{\sin\theta\cos\theta} \\ &= \frac{\cos^2\theta + \sin^2\theta}{\cos\theta\sin\theta} = \frac{1}{\cos\theta\sin\theta} \\ &= \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta} = \sec\theta\csc\theta \end{aligned}$$