Serial #: _____ ID _____ NAME _____

Show all necessary steps for full marks.

Q1. (4 points)(6.3Additional Exercise # 14):

If $f(x) = a\sin bx$, $b > 0$, is a sine function with period 4 and $f(1) = 2$, then find $f\left(\frac{2}{3}\right) = ?$

Solution:

$$Period = \frac{2\pi}{b}$$

$$4 = \frac{2\pi}{b} \Rightarrow 4b = 2\pi \Rightarrow \boxed{b = \frac{\pi}{2}}$$

$$f(x) = a\sin bx$$

$$f(x) = a\sin\left(\frac{\pi}{2}x\right) \Rightarrow f(1) = a\sin\left(\frac{\pi}{2}\right) \Rightarrow 2 = a(1) \Rightarrow \boxed{a = 2}$$

$$f(x) = 2\sin\left(\frac{\pi}{2}x\right)$$

$$f\left(\frac{2}{3}\right) = 2\sin\left(\frac{\pi}{2}\cdot\frac{2}{3}\right) = 2\sin\left(\frac{\pi}{3}\right) = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$$

Q2. (6 points) (6.3 Additional Exercise 5) (a): Graph $f(x) = 2\sin\left(-\frac{2}{3}x\right)$ where $x \in [-3\pi, 3\pi]$

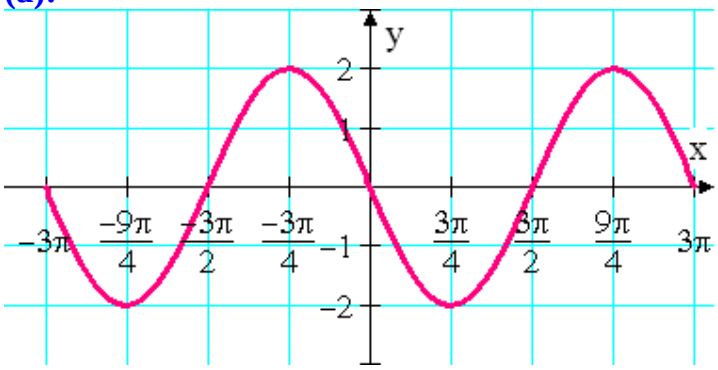
(b): Determine the intervals where the function is increasing.

(c): Determine the intervals where the function is decreasing.

Solution:

$$p = \frac{2\pi}{\left|-\frac{2}{3}\right|} = 3\pi$$

(a):



(b): $\left[-\frac{9\pi}{4}, -\frac{3\pi}{4}\right]$ and on $\left[\frac{3\pi}{4}, \frac{9\pi}{4}\right]$

(c): $\left[-3\pi, -\frac{9\pi}{4}\right]$, $\left[-\frac{3\pi}{4}, \frac{3\pi}{4}\right]$ and on $\left[\frac{9\pi}{4}, 3\pi\right]$

Q3. (2 points) If the rang of the function $f(x) = d + a\sin(bx + c)$ is $\left[\frac{2}{3}, 5\right]$, find the amplitude.

Solution:

$$Amplitude = \frac{Max - \min}{2} = \frac{5 - \frac{2}{3}}{2} = \frac{3\left(5 - \frac{2}{3}\right)}{3(2)} = \frac{15 - 2}{6} = \frac{13}{6}$$

Another Method:

$$-1 \le \sin(bx + c) \le 1$$

$$\Rightarrow \; -|a| \le |a|\sin(bx + c) \le |a|$$

$$\Rightarrow \; d - |a| \le d + |a|\sin(bx + c) \le d + |a|$$

$$\text{Range} = \left[\frac{2}{3}, 5\right] \Rightarrow d - |a| = \frac{2}{3} \text{ and } d + |a| = 5 \Rightarrow d = |a| + \frac{2}{3} \text{ and } d = -|a| + 5$$

$$\Rightarrow \; |a| + \frac{2}{3} = -|a| + 5 \Rightarrow 2|a| = 5 - \frac{2}{3} \Rightarrow 2|a| = \frac{15 - 2}{3} \Rightarrow |a| = \frac{13}{6} \Rightarrow \text{Amplitutde} = \frac{13}{6}$$

Q4. (8 points):

(a): Sketch the graph of $y = 2\cos\left(\frac{1}{3}x - \frac{2\pi}{3}\right) + 2$, $x \in [2\pi, 8\pi]$

(b): Determine the interval where the function is increasing.

(c): Determine the interval where the function is decreasing.

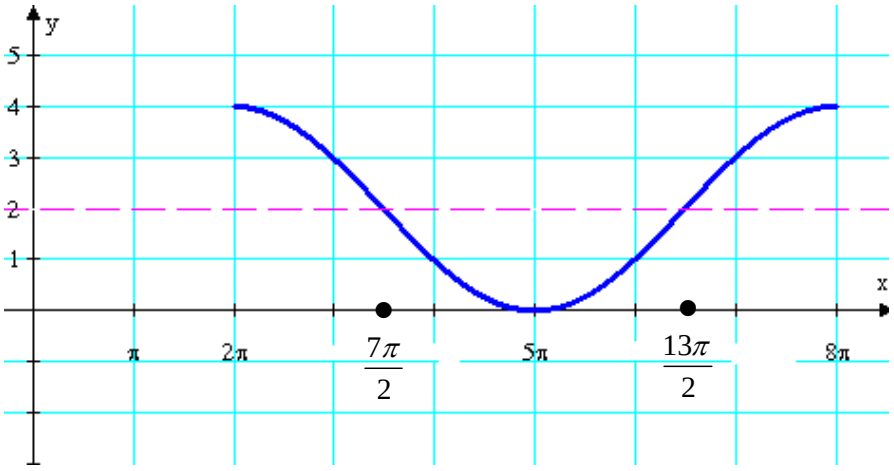
Solution:

(a):

$$0 \le \frac{1}{3}x - \frac{2\pi}{3} \le 2\pi$$

$$0 \le x - 2\pi \le 6\pi$$

$$2\pi \le x \le 8\pi$$



(b): The function is increasing on $[5\pi, 8\pi]$

(c): The function is decreasing on $[2\pi, 5\pi]$