SN _____ ID _____ NAME _____

Show all necessary steps for full marks.

Question 1: (7 points): If $f(x)=2x-x^2$; $x\le 1$ then

i) find $f^{-1}(x)$

ii) sketch the graph of $f^{-1}(x)$

Solution (i):

$y=2x-x^2$; $x\le 1$

$x=-y^2+2y$; $y\le 1$

$-x=y^2-2y$; $y\le 1$

$1-x=y^2-2y+1$; $y\le 1$

$1-x=(y-1)^2$; $y\le 1$ $\Rightarrow$ $1-x\ge 0$

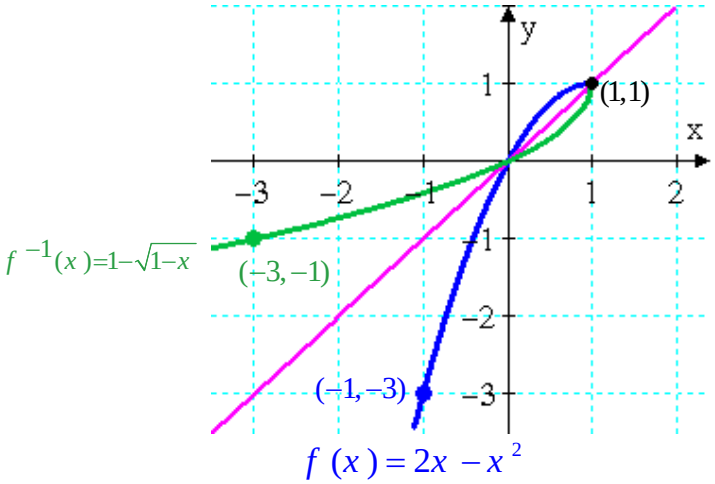
$|y-1|=\sqrt{1-x}=$; $y\le 1$; $x\le 1$

$-(y-1)=\sqrt{1-x}=$; $y\le 1$; $x\le 1$

$y-1=-\sqrt{1-x}=$; $y\le 1$; $x\le 1$

$f^{-1}(x)=1-\sqrt{1-x}$; $R_{f^{-1}}=(-\infty,1]$; $D_{f^{-1}}=(-\infty,1]$

(ii):



Question 2: (4 points): If $f(x)=\dfrac{2x}{x-1}$, $x\ne 1$, then $f^{-1}\left(\dfrac{3}{2}\right)$ is equal to

(a) -3

(b) 3

(c) $2/3$

(d) $-2/3$

(e) $3/2$

Solution: Let $x=f^{-1}\left(\dfrac{3}{2}\right)$. $\Rightarrow$ $f(x)=\dfrac{3}{2}$ $\Rightarrow$ $\dfrac{3}{2}=\dfrac{2x}{x-1}$ $\Rightarrow$ $3x-3=4x$ $\Rightarrow$ $x=-3$

$f^{-1}\left(\dfrac{3}{2}\right)=-3$

Question 3: (4 points): If $f(t)=2^{1-3t}$ is written in the form $f(t)=k\,a^t$, then find the values of a and k.

Solution: $f(t)=2^{1-3t}=2^1\cdot 2^{-3t}=2\left(2^{-3}\right)^t=2\left(\frac{1}{8}\right)^t \Rightarrow \boxed{k=2}$ and $\boxed{a=\frac{1}{8}}$

Question 4: (4.2Example 3) (6 points):

Graph each function. Give the **domain** and **range**.

(a): $f(x)=-2^x$ **(b):** $f(x)=2^{x+3}$ **(c):**: $f(x)=2^{x-2}-1$

SOLUTION In each graph, we show in particular how the point $(0, 1)$ on the graph of $y = 2^x$ has been translated.

(a) The graph of $f(x) = -2^x$ is that of $f(x) = 2^x$ reflected across the x-axis. The domain is $(-\infty, \infty)$, and the range is $(-\infty, 0)$. See **Figure 18.**

(b) The graph of $f(x) = 2^{x+3}$ is the graph of $f(x) = 2^x$ translated 3 units to the left, as shown in **Figure 19.** The domain is $(-\infty, \infty)$, and the range is $(0, \infty)$.

(c) The graph of $f(x) = 2^{x-2} - 1$ is that of $f(x) = 2^x$ translated 2 units to the right and 1 unit down. See **Figure 20.** The domain is $(-\infty, \infty)$, and the range is $(-1, \infty)$.

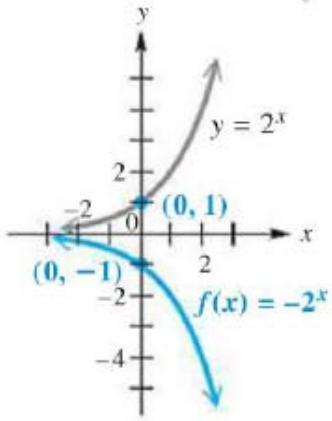


Figure 18

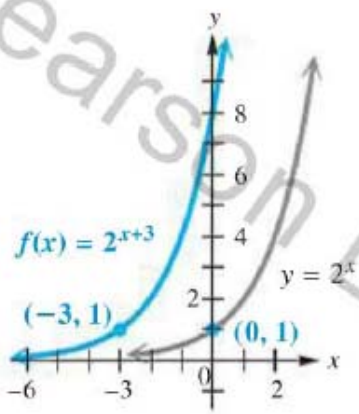


Figure 19

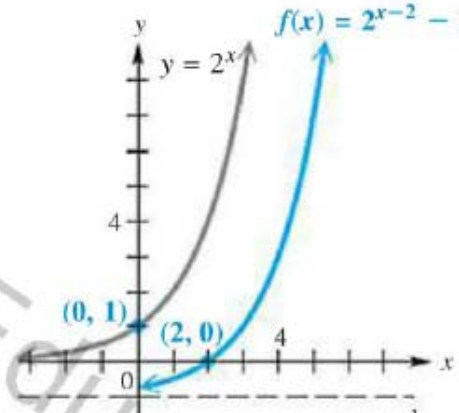


Figure 20

Question 5: (4 points): Sketch the graph of **(a):** $f(x)=\left|-e^x\right|$ **(b):** $g(x)=e^{|x|}$

Solution:

(a): $f(x)=\left|-e^x\right|=\left|e^x\right|=e^x$ because $e^x>0$ for all $x\in(-\infty,\infty)$

(b): $g(x)=e^{|x|}=\begin{cases} e^x & \text{if } x\geq 0 \\ e^{-x} & \text{if } x<0 \end{cases}$