# King Fahd University of Petroleum and Minerals
## Prep-Year Math Program

**Code: Master**

**Prep-Year Math I**
**FINAL EXAM**
**Semester I, 2003-04**
**Saturday, January 10, 2004**
**Net Time Allowed: 180 minutes**

**Code: Master**

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**Student's Name:** ..........................................................................

**ID #:** ................................ **Section #:** ........................

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**Important Instructions:**

1. All types of CALCULATORS, PAGERS, and OR MOBILES ARE NOT ALLOWED to be with you during the examination.
2. Use an HB $2\frac{1}{2}$ pencil.
3. Use a good eraser. Do not use the eraser attached to the pencil.
4. Write your name, ID number and Mathematics Section on the examination paper and on the upper left corner of the answer sheet.
5. When bubbling your ID number and Math Section number, be sure that bubbles match with the number that you write.
6. The test Code Number is already typed and bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.
9. Check that the exam paper contains **35** questions.

1. An equation of a circle that has a diameter with endpoints $(-2, 7)$ and $(-6, -3)$ is

(a) $(x+4)^2+(y-2)^2=29$

(b) $(x-2)^2+(y-5)^2=23$

(c) $(x+4)^2+(y-2)^2=35$

(d) $(x+2)^2+(y+5)^2=29$

(e) $(x-4)^2+(y+2)^2=31$

2. The sum of all the rational zeros of the polynomial

$$P(x)=6x^4+5x^3+7x^2+5x+1$$

is equal to

(a) $-\frac{5}{6}$

(b) $-\frac{13}{6}$

(c) $-\frac{7}{6}$

(d) $0$

(e) $-\frac{11}{6}$

3. If $x - i$ is a factor of the polynomial $P(x) = 7x^{171} + 8x^{172} - 9x^{173} + kx^{174}$ where $i = \sqrt{-1}$, then the constant $k$ is equal to

(a) $8 - 16i$

(b) $13 + 7i$

(c) $-14 - 16i$

(d) $-8 + 5i$

(e) $17 - 11i$

4. The solution set of the equation

$$\sqrt{2 - x} - 1 = \sqrt{3 - x}$$

contains

(a) no real numbers

(b) two positive integers

(c) only one negative integer

(d) two negative integers

(e) only one positive integer

5. If $f(x) = \dfrac{2x}{x-1}$, $x \neq 1$, then $f^{-1}(x) =$

(a) $\dfrac{x}{x-2}$, $x \neq 2$

(b) $\dfrac{x-1}{2x}$, $x \neq 0$

(c) $\dfrac{-2x}{x-1}$, $x \neq 1$

(d) $\dfrac{2x}{x+1}$, $x \neq -1$

(e) $\dfrac{3x}{x-2}$, $x \neq 2$

6. The solution set of the equation

$$\frac{t}{t+3} + \frac{2t^2-3}{t^2-t-12} = \frac{3t}{t-4}$$

contains

(a) one negative rational number

(b) one positive integer

(c) one positive integer and one negative rational number

(d) two negative irrational numbers

(e) two positive integers

7. If $f(x) = \sqrt{4 - x}$ and $g(x) = x^2 - 25$, then the domain, in interval notation, of the function $\frac{f}{g}$ is equal to

(a) $(-\infty, -5) \cup (-5, 4]$

(b) $(-\infty, -5) \cup (-5, 5) \cup (5, \infty)$

(c) $(-5, 4) \cup (4, 5)$

(d) $(-\infty, 5)$

(e) $(-\infty, -5) \cup (-5, 4) \cup (4, 5)$

8. The graph of $|y - x| = |x^2 + 1|$ is symmetric with respect to

(a) the origin only

(b) the $x$-axis only

(c) the $y$-axis only

(d) the $x$-axis and the origin

(e) the $y$-axis and the origin

9. If $x^4 + 8x^3 - 9x^2 + 10x + 100$ is divided by $x + 9$, then the quotient $Q(x)$ and the remainder $R$ are given by

(a) $Q(x) = x^3 - x^2 + 10\,, \quad R = 10$

(b) $Q(x) = x^3 - x^2 + 2x + 10\,, \quad R = 10$

(c) $Q(x) = x^3 + x^2 - 25\,, \quad R = -4$

(d) $Q(x) = x^3 - x^2 + 10\,, \quad R = 80$

(e) $Q(x) = x^3 - x^2 + 15x + 10\,, \quad R = 10$

10. The expression $\dfrac{4x^2 + 12xy + 9y^2}{4x^2 + 6xy + 9y^2} \div \dfrac{4x^2 - 9y^2}{8x^3 - 27y^3}$ simplifies to

(a) $2x + 3y$

(b) $2x - 3y$

(c) $4x^2 - xy + 9y^2$

(d) $4x^2 + xy + 9y^2$

(e) $\dfrac{2x + 3y}{2x - 3y}$

11. The largest negative integer that is a lower bound of the zeros of

$$P(x) = 4x^4 - 12x^3 - 3x^2 + 12x - 7$$

is equal to

(a) $-2$

(b) $-1$

(c) $-3$

(d) $-4$

(e) $-5$

12. Let $f(x) = \begin{cases} \left[\frac{1}{3}x\right], & \text{if } x < 0 \\ |4 - 3x|, & \text{if } x \geq 0 \end{cases}$, where $[y]$ is the greatest integer less than or equal to $y$. Then the value of $f(-5) + f(5)$ is equal to

(a) 9

(b) 11

(c) $-14$

(d) 13

(e) $\frac{29}{3}$

13. If the graph of $f(x) = |x|$ is reflected across the $x$-axis, then translated one unit left and two units up, then the equation of the new graph is

(a) $g(x) = -|x + 1| + 2$

(b) $g(x) = |-x + 1| + 2$

(c) $g(x) = |-x - 1| - 2$

(d) $g(x) = -|x - 1| - 2$

(e) $g(x) = -|x + 2| + 1$

14. The far-left and far-right behavior of the graph of the polynomial

$$P(x) = -2(x - 3)(x + 1)^2(2 - x)$$

is as follows:

(a) up to the left and up to the right

(b) down to the left and up to the right

(c) up to the left and down to the right

(d) down to the left and down to the right

(e) none of the above

15. The asymptotes of the graph of the function

$$f(x) = \frac{x^2 - x - 2}{x^2 + x - 6}$$

are

(a) one vertical, and one horizontal

(b) two vertical, and one horizontal

(c) one vertical, one slant, and one horizontal

(d) two vertical, and one slant

(e) two vertical, and neither horizontal nor slant

16. The expression $\left[\dfrac{\sqrt[3]{y^9}\sqrt{x^{18}}}{8y^{-3}x^{-6}}\right]^{-\frac{1}{3}}$, where $x > 0$ and $y > 0$, simplifies to

(a) $\dfrac{2}{x^5y^2}$

(b) $\dfrac{y^2}{2x^5}$

(c) $\dfrac{2x^5}{y^2}$

(d) $2x^5y^2$

(e) $\dfrac{2y^2}{x^5}$

17. A polynomial $P(x)$ of lowest degree and with real coefficients that has the zeros $i = \sqrt{-1}$ of multiplicity 2 and 3 of multiplicity 1, is given by

(a) $P(x) = x^5 - 3x^4 + 2x^3 - 6x^2 + x - 3$

(b) $P(x) = x^5 - 3x^4 + 2x^3 - 6x^2 - x - 3$

(c) $P(x) = x^5 - 3x^4 - 2x^3 + 6x^2 + x - 3$

(d) $P(x) = x^5 + 3x^4 + 2x^3 - 6x^2 - x - 3$

(e) $P(x) = x^5 + 3x^4 + 2x^3 + 6x^2 + x - 3$

18. The number of **possible** positive and the number of **possible** negative real zeros of the polynomial

$$P(x) = 5x^4 + 12x^3 + 6x^2 - 3x - 1$$

are

(a) one positive, and either three or one negative zeros

(b) one positive, and three negative zeros

(c) one positive, and one negative zeros

(d) one negative, and either three or one positive zeros

(e) one negative, and three positive zeros

19. The value of the constant $k$ for which the lines $(k+1)x - 3y + 1 = 0$ and $x + 2ky + 1 = 0$ are perpendicular is equal to

(a) $\frac{1}{5}$

(b) $\frac{3}{5}$

(c) $\frac{2}{5}$

(d) $-\frac{4}{5}$

(e) $\frac{6}{5}$

20. If $f(x) = \sqrt{\frac{9}{4}x + 16}$, then the value of $(f \circ f)(0) + f^2(0)$ is equal to

(a) 21

(b) 25

(c) 32

(d) 0

(e) $\frac{\sqrt{181}}{2}$

21. The polynomial $P(x) = 3x^3 + 7x^2 - 3x + 7$ has a zero between

(a) $-3$ and $-2$

(b) $1$ and $2$

(c) $-2$ and $-1$

(d) $3$ and $4$

(e) $-1$ and $0$

22. If $f(x) = x^2 + 1,\ x \leq 0,$ then $f^{-1}(5)$ is equal to

(a) $-2$

(b) $2$

(c) $\pm 2$

(d) $-\frac{1}{26}$

(e) $\frac{1}{26}$

23. If $r_1$ and $r_2$ are the roots of the equation $x^2 - 2x + 2 = 0$, then $\frac{1}{r_1} + \frac{1}{r_2} =$

(a) 1

(b) 3

(c) $\frac{2}{3}$

(d) $-1$

(e) $\frac{3}{4}$

24. The solution set, in interval notation, of the inequality

$$\frac{(5x+13)^3}{(x+4)^2(x+5)^4} \geq 0$$

is equal to

(a) $\left[-\frac{13}{5}, \infty\right)$

(b) $(-5, -4)$

(c) $(-\infty, -5) \cup (-5, -4) \cup \left[-\frac{13}{5}, \infty\right)$

(d) $(-\infty, -5) \cup (-4, \infty)$

(e) $\left(-\infty, -\frac{13}{5}\right) \cup \left(-\frac{13}{5}, \infty\right)$

25. If $(a, b)$ is the intersection point of the graphs of $f_1(x) = -3x - 7$ and $f_2(x) = 2x + 13$, then $a + b =$

(a) 1

(b) $-2$

(c) 4

(d) $-3$

(e) 3

26. If $g(x) = x^2 + 2x + 1$ and $h \neq 0$, then $\dfrac{g(1+h) - g(1)}{h}$ is equal to

(a) $h + 4$

(b) $2h + 3$

(c) $h - 3$

(d) $\dfrac{h + 5}{h}$

(e) $h + 2$

27. Which one of the following functions represents the given graph?

(a) $f(x) = \dfrac{x}{16 - x^2}$

(b) $f(x) = \dfrac{x^2}{16 - x^2}$

(c) $f(x) = \dfrac{x^2}{16x - x^3}$

(d) $f(x) = \dfrac{x^3}{16 - x^2}$

(e) $f(x) = \dfrac{1}{16 - x^2}$

28. The expression $\dfrac{1}{2 + \dfrac{3}{1 + \dfrac{4}{x}}}$ simplifies to

(a) $\dfrac{x + 4}{5x + 8}$

(b) $\dfrac{x}{x + 9}$

(c) $\dfrac{x + 5}{5x + 4}$

(d) $\dfrac{3x + 5}{x + 4}$

(e) $\dfrac{x + 4}{5x + 11}$

29. A triangle has a perimeter of 15 centimeters. Each of the two equal sides of the triangle is one-third the length of the third side. Then the **product** of the lengths of all sides of the triangle is

(a) 81

(b) 27

(c) 36

(d) 48

(e) 63

30. If $x$ is a real number, then the maximum area of a rectangle of length $3+2x$ and width $1-2x$ is equal to

(a) 4

(b) 6

(c) 9

(d) 12

(e) 8

31. If $-2$ is a zero of multiplicity 2 of $P(x) = 3x^3 + 6x^2 + Ax + B$, then the sum of the constants $A$ and $B$ is

(a) $-36$

(b) $-22$

(c) $31$

(d) $38$

(e) $25$

32. The graph of the rational function $f(x) = \dfrac{x^2 + x - 2}{x}$ lies completely above its slant asymptote on the interval

(a) $(-\infty, 0)$

(b) $(-\infty, -1) \cup (0, 1)$

(c) $(-1, 0) \cup (1, \infty)$

(d) $(-\infty, -1) \cup (1, \infty)$

(e) $(0, \infty)$

33. The solution set, in interval notation, of the inequality

$$1 < |2x - 3| \leq 5$$

is equal to

(a) $[-1, 4]$
(b) $[-2, 5]$
(c) $[-1, 5]$
(d) $[-3, 4]$
(e) $[-2, 4]$

34. $\dfrac{2}{2\sqrt{2} - 3} + \dfrac{8}{\sqrt{2}} =$

(a) $-6$
(b) $6 + 8\sqrt{2}$
(c) $4\sqrt{2}$
(d) $-4\sqrt{2}$
(e) $\dfrac{-6 + \sqrt{2}}{5}$

35. If the graph of the quadratic function $f(x) = -2x^2 + 3x + c$ intersects the $x$-axis at two different points, then $c$ is any number in the interval

(a) $\left(-\frac{9}{8}, \infty\right)$

(b) $(-3, \infty)$

(c) $\left(-3, -\frac{9}{8}\right)$

(d) $\left(-\frac{3}{2}, \infty\right)$

(e) $\left(-\frac{3}{2}, -\frac{9}{8}\right)$