

Question1

a) Expand the logarithm $\log \sqrt[5]{\frac{x^2 z^2}{(y^4 + 2)}}$

Solution

$$\frac{1}{5} [\log x^2 + \log z^2 - \log(y^4 + 2)] = \frac{2}{5} \log x + \frac{2}{5} \log z - \frac{1}{5} \log(y^4 + 2)$$

- b) If x and y are any positive real number, then write the logarithmic expression $-2 + \log_3 x^2 + \log_{\frac{1}{3}} xy$ as a single logarithm of base 3.

Solution

$$\begin{aligned}\text{The given expression} &= -2 \log_3 3 + \log_3 x^2 + \frac{\log_3 xy}{\log_3 1/3} \\ &= \log_3 3^{-2} + \log_3 x^2 - \log_3 xy \\ &= \log_3 \frac{3^{-2} x^2}{xy} \\ &= \log_3 \left(\frac{x}{9y} \right)\end{aligned}$$

Question2:

If $\log 2 = .3$ and $\log 3 = .5$, then find the value of

- 1) $\log 6$
 - 2) $\log 5$
 - 3) $\log_5 600$
- See the solution in the notes**

Question3:

Find the exact value of

$$1) \frac{\log_5 32}{\log_5 2}$$

$$2) \ln(\ln e) + e^{1-2\ln 3} \quad \text{See the solution in the notes}$$

$$3) (\sqrt{3})^{\frac{1}{\log_4 3}}$$

Question4:

Solve the following equations

$$1) \left(\frac{1}{9}\right)^{(3-x)} = (81)^{2x-5}$$

Solution

$$\left(\frac{1}{9}\right)^{(3-x)} = (81)^{2x-5}$$

$$\Rightarrow 9^{-(3-x)} = 9^{2(2x-5)}$$

$$\Rightarrow 9^{-3+x} = 9^{4x-5}$$

$$\Rightarrow -3 + x = 4x - 5$$

$$\Rightarrow -3x = -2$$

$$\Rightarrow x = 2/3$$

$$2) \log_3(-x) + \log_3(6-x) = 3$$

Solution

$$\log_3(-x) + \log_3(6-x) = 3$$

$$\Rightarrow \log_3(-x)(6-x) = 3$$

$$\Rightarrow -6x + x^2 = 3^3 = 27$$

$$\Rightarrow x^2 - 6x - 27 = 0$$

$$\Rightarrow (x-9)(x+3) = 0$$

$$\Rightarrow x = 9 \quad \text{or} \quad x = -3$$

Check : $x = 9$: Rejected

$$x = -3 : ok$$

$$\therefore S.S. = \{-3\}$$

Question5:

TRUE or FALSE

$$1) \log x^2 = (\log x)^2 \quad \text{False}$$

$$2) \log(3+3) = \log 3 + \log 3 \quad \text{False}$$

$$3) \log(1+2+3) = \log 1 + \log 2 + \log 3 \quad \text{True}$$

$$4) \text{If } x \text{ is any nonzero real number, then } \log x^2 = 2 \log x \quad \text{False}$$

$$5) \text{If } \log_{\frac{1}{2}} x > 0, \text{ then } x \in (0,1) \quad \text{True}$$