King Fahd University of Petroleum and Minerals
Prep-Year Math Program
Math 002 - Term 062
Recitation Hour (4.2 & 4.3)

Question1:

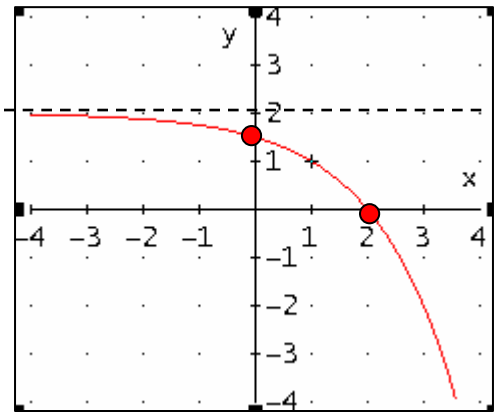
For the function

$$f(x) = -\left(\frac{1}{2}\right)^{-x+1} - 2$$

1) sketch the graph of $f(x)$

Solution

- H.A.: $y = -2$
- $x = 0 \Rightarrow y = \frac{3}{2}$
- $y = 0 \Rightarrow -\left(\frac{1}{2}\right)^{-x+1} + 2 = 0 \Rightarrow 2^{x-1} = 2 \Rightarrow x - 1 = 1 \Rightarrow x = 2$



2) find the $x-$intercept and the $y-$intercept

Solution

- $y = 0 \Rightarrow -\left(\frac{1}{2}\right)^{-x+1} + 2 = 0 \Rightarrow 2^{x-1} = 2 \Rightarrow x - 1 = 1 \Rightarrow x = 2$

$\therefore$ The $x-$intercept: $(2,0)$

- $x=0 \Rightarrow y=\frac{3}{2}$

$\therefore$ The $y-$ intercept: $(0,\frac{3}{2})$

3) find the range

Solution

$R=(-\infty,2)$

4) find the asymptote(s)

Solution

H.A.: $y=-2$

5) find the inverse function $f^{-1}(x)$

Solution

$$x=-\left(\frac{1}{2}\right)^{-y+1}+2$$

$$\Rightarrow -\left(\frac{1}{2}\right)^{-y+1}=x-2$$

$$\Rightarrow \left(\frac{1}{2}\right)^{-y+1}=-x+2$$

$$\Rightarrow -y+1=\log_{1/2}(-x+2)$$

$$\Rightarrow y=1-\log_{1/2}(-x+2)$$

$$\therefore f^{-1}(x)=1-\log_{1/2}(-x+2)$$

Question2:

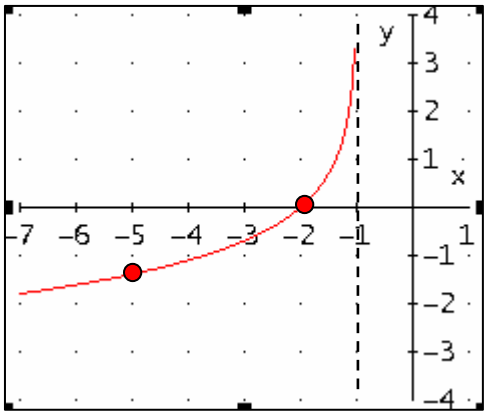
For functions

$$f(x) = -\log_2(-x-3)$$

1) sketch the graph of $f(x)$

Solution

- V.A.: $-x-3=0 \Rightarrow x=-3$
- $x=-4 \Rightarrow y=0$
- $x=-5 \Rightarrow y=-\log_2 2=-1$



2) find , if any, the $x-$intercept and the $y-$intercept

Solution

- $y=0 \Rightarrow -\log(-x-3)=0 \Rightarrow \log(-x-3)=0 \Rightarrow -x-3=10^0=1$
  $\Rightarrow x=-4$

$\therefore$ The $x-$intercept: $(-4,0)$

- $x=0 \Rightarrow y=-\log(-3)$!!!!

$\therefore$ No $y-$intercept

3) find the domain

Solution

$$-x-1>0 \Rightarrow -x>1 \Rightarrow x<-1$$

$$\therefore D=(-\infty,-1)$$

4) find the asymptote(s)

Solution

V.A.: $x=-3$

5) find the inverse function $f^{-1}(x)$

Solution

$$x=-\log(-y-3)$$

$$\Rightarrow -x=\log(-y-3)$$

$$\Rightarrow -y-3=10^{-x}$$

$$\Rightarrow y=-10^{-x}-3$$

$$\therefore f^{-1}(x)=-10^{-x}-3$$

Question3:

If the graph of the exponential function $f(x)=b^x$ passes through the point $(-3,\frac{1}{64})$, then find $f(2)$.

Solution

The point $(-3,\frac{1}{64})$ on the graph

$$\Rightarrow \log_b \frac{1}{64}=-3 \Rightarrow \frac{1}{64}=b^{-3} \Rightarrow 4^{-3}=b^{-3} \Rightarrow b=4.$$

$$\therefore f(x)=\log_4 x.$$

$$\therefore f(2)=\log_4 2=\frac{1}{2}$$

Question4:

If a bacteria population starts with 100 bacteria and doubles every three hours, then the number of bacteria after t hours is $N(t) = 100 \cdot 2^{t/3}$.
When will the population reach 6400?

Solution

$$128000 = 100 \cdot 2^{t/3}$$

$$\Rightarrow 128 = 2^{t/3}$$

$$\Rightarrow 2^7 = 2^{t/3}$$

$$\Rightarrow 7 = \frac{t}{3}$$

$$\Rightarrow t = 21 \text{hours}$$