

King Fahd University of Petroleum and Minerals

Prep-Year Math Program

Math 002 - Term 062

Recitation Hour Problems (8.3 & 9.1)

Question1

Find the center, vertices, foci, eccentricity, and asymptotes of the hyperbola given by the equation $4x^2 - 9y^2 - 16x + 54y - 29 = 0$.

Question2

Find the equation in the standard form of the hyperbola with center at $(-3, -3)$, conjugate axis of length 6, and eccentricity 2.

Question3

Solve for x and y if: $(-3-i)x - (4+2i)y = 1-i$.

Question4

If the system of linear equations $\begin{cases} 3x - 2y + 1 = 0 \\ x + ky = 0 \end{cases}$ is inconsistent, then find k .

• Recitation 8.3 and 9.1

Q1. $4x^2 - 9y^2 - 16x + 54y - 29 = 0$

$$\Rightarrow (4x^2 - 16x) - (9y^2 - 54y) = 29$$

$$\Rightarrow 4(x^2 - 4x) - 9(y^2 - 6y) = 29$$

$$\Rightarrow 4(x^2 - 4x + 4) - 9(y^2 - 6y + 9) = 29 + 16 - 81$$

$$\Rightarrow 4(x-2)^2 - 9(y-3)^2 = -36$$

$$\Rightarrow \frac{(y-3)^2}{4} - \frac{(x-2)^2}{9} = 1$$

$\Rightarrow$ Transverse axis is parallel to the y-axis.

$\Rightarrow$ center: $(h, k) = (2, 3)$

vertices: $(h, k \pm a) = (2, 3 \pm 2)$

$$\begin{array}{l} \nearrow (2, 1) \\ \searrow (2, 5) \end{array}$$

Foci: $(h, k \pm c) = (2, 3 \pm \sqrt{13})$

asymptotes: $y - 3 = \pm \frac{2}{3}(x - 2)$

$$a^2 = 4 \Rightarrow a = 2$$

$$b^2 = 9 \Rightarrow b = 3$$

$$c^2 = a^2 + b^2 = 13$$

$$\Rightarrow c = \sqrt{13}$$

Q2. $2b = 6 \Rightarrow b = 3$

$$e = \frac{c}{a} = \frac{\sqrt{a^2 + b^2}}{a} = \frac{\sqrt{a^2 + 9}}{a} = 2 \Rightarrow \sqrt{a^2 + 9} = 2a$$

$$\Rightarrow 4a^2 = a^2 + 9$$

$$\Rightarrow a^2 = 3$$

$\therefore$ Eq.

$$\frac{(x+3)^2}{3} - \frac{(y+3)^2}{9} = 1 \quad \text{or} \quad \frac{(y+3)^2}{3} - \frac{(x+3)^2}{9} = 1$$

Q3.

$$\underline{-3x - i^2x - 4y - 2i^2y = 1 - i}$$

$$(-3x - 4y) + (-x - 2y)i = 1 - i$$

$$\Rightarrow \begin{cases} -3x - 2y = 1 \\ (-x - 2y = -1) \cdot (-1) \end{cases} \Rightarrow \begin{cases} 3x - 2y = 1 \\ x + 2y = 1 \end{cases}$$

$$4x = 2$$

$$x = \frac{1}{2}$$

$$\Rightarrow \frac{1}{2} - 2y = -1$$

$$\Rightarrow -2y = -\frac{1}{2}$$

$$\Rightarrow y = \frac{1}{4}$$

$$\therefore \text{S.S.} = \left\{ \left(\frac{1}{2}, \frac{1}{4} \right) \right\}$$

substitute in the 2nd eq.

Method 1

$$3x - 2y = 1 \Rightarrow -2y = -3x + 1 \Rightarrow y = \frac{3}{2}x - \frac{1}{2}$$

$$x + ky = 0 \Rightarrow ky = -x \Rightarrow y = -\frac{1}{k}x$$

inconsistent $\Rightarrow$ No solutions

$\Rightarrow$ the two lines are parallel

$\Rightarrow$ the two lines have the same slope.

$$\Rightarrow \frac{3}{2} = -\frac{1}{k}$$

$$\Rightarrow 3k = -2$$

$$\Rightarrow k = -\frac{2}{3}$$

Method 2

$$\begin{array}{rcl} 3x - 2y = 1 & \text{---(1)} & \\ -3(x + ky = 0) & \text{---(2)} & \end{array} \Rightarrow \begin{array}{r} 3x - 2y = 1 \\ -3x - 3ky = 0 \\ \hline (-2 - 3k)y = 1 \end{array}$$

0"

inconsistent $\Rightarrow$ No solution $\Rightarrow -2 - 3k = 0$

$$\Rightarrow k = -\frac{2}{3}$$