King Fahd University of Petroleum and Minerals

Prep-Year Math Program

Math 002 - Term 062

Recitation Hour Problems (6.6 & 7.3)

Question1

Solve the following equations

(i) $\sin\frac{x}{2}+\cos x=1,\ \text{for}\ 0\le x\le\pi.$

(ii) $\tan^2 x=3(\sec x-1)$, where $0\le x<2\pi$

(iii) $\cos 2x+3\sin x=2$, where $0\le x<2\pi$

Question2

Given the vectors $\boldsymbol{u}=\langle 10,-8\rangle$ and $\boldsymbol{v}=\langle 12,-6\rangle$.

a. Find the magnitude and the direction angle of the vector $\frac{1}{2}\boldsymbol{u}-\frac{1}{6}\boldsymbol{v}$

b. Find a vector of magnitude 6 in the opposite direction of the vector $\frac{1}{2}\boldsymbol{u}-\frac{1}{6}\boldsymbol{v}$

Question3

Given the vector $\boldsymbol{u}=\sqrt{3}i+j$, $\boldsymbol{v}=i+\sqrt{3}j$.

a. Find $\boldsymbol{u}\cdot\boldsymbol{v}$

b. Find the smallest positive angle between the vectors $\boldsymbol{u}$ and $\boldsymbol{v}$

c. Find $\text{Proj}_{\text{v}}\,\boldsymbol{u}$

• Recitation 6.6 and 7.3

Q1.

(i) $\sin\frac{x}{2} + \cos x = 1$, $0 \le x \le \pi$.

Sol.

$$\pm\sqrt{\frac{1-\cos x}{2}} + \cos x = 1$$

$$\pm\sqrt{\frac{1-\cos x}{2}} = 1 - \cos x$$

square both sides,

$$\frac{1-\cos x}{2} = 1 - 2\cos x + \cos^2 x$$

$$\Rightarrow 2\cos^2 x - 3\cos x + 1 = 0$$

$$\Rightarrow (2\cos x - 1)(\cos x - 1) = 0$$

$$\Rightarrow \cos x = \frac{1}{2} \quad \text{or} \quad \cos x = 1$$

$$\Rightarrow x = \frac{\pi}{3} \quad \text{or} \quad x = 0.$$

check (why?)

• $x = \frac{\pi}{3}$:

$$\Rightarrow \sin\frac{\pi}{6} + \cos\frac{\pi}{3} = \frac{1}{2} + \frac{1}{2} = 1 \checkmark$$

• $x = 0$:

$$\Rightarrow \sin 0 + \cos 0 = 0 + 1 = 1 \checkmark$$

$\therefore$ S.S. $= \{\frac{\pi}{3}, 0\}$

(ii) $\tan^2 x = 3(\sec x - 1)$, $0 \le x \le 2\pi$.

Sol. $1 + \tan^2 x = \sec^2 x$

$\Rightarrow \tan^2 x = \sec^2 x - 1$

$\therefore$ The given eq simplifies to,

$$\sec^2 x - 1 = 3\sec x - 3$$

$$\Rightarrow \sec^2 x - 3\sec x + 2 = 0$$

$$\Rightarrow (\sec x - 1)(\sec x - 2) = 0$$

$\Rightarrow \sec x = 1$ or $\sec x = 2$

$\Rightarrow \cos x = 1$ or $\cos x = \frac{1}{2}$

$\Rightarrow x = 0$ or $x = \frac{\pi}{3}$ or $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$.

$\therefore$ S.S. $= \{0, \frac{\pi}{3}, \frac{5\pi}{3}\}$

(iii) $\cos 2x + 3\sin x = 2$, $0 < x < 2\pi$.

Sol. $1 - 2\sin^2 x + 3\sin x = 2$

$\Rightarrow -2\sin^2 x + 3\sin x - 1 = 0$

$\Rightarrow 2\sin^2 x - 3\sin x + 1 = 0$

$\Rightarrow (2\sin x - 1)(\sin x - 1) = 0$

$\Rightarrow \sin x = \frac{1}{2}$ or $\sin x = 1$

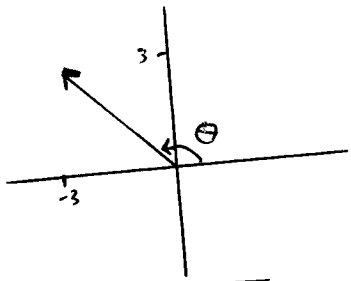
$\Rightarrow x = \frac{\pi}{6}$ or $x = \pi - \frac{\pi}{6}$ or $x = \frac{\pi}{2}$

$\therefore$ S.S. $= \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}\}$

Q2

a. $\frac{1}{2}u - \frac{1}{6}v = \langle 5, -4\rangle - \langle 2, -1\rangle = \langle 3, -3\rangle$

$\tan\theta = \frac{b}{a} = \frac{-3}{3} = -1 \Rightarrow \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$.



b. $\| \frac{1}{2}u - \frac{1}{6}v \| = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$.

∴ The required vector $= -\frac{1}{3\sqrt{2}} \cdot 6 \langle 3, -3\rangle$

$= \frac{-2}{\sqrt{2}} \langle 3, -3\rangle$

$= \left\langle \frac{-6}{\sqrt{2}}, \frac{6}{\sqrt{2}} \right\rangle$

$= \left\langle \frac{-6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}, \frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \right\rangle$

$= \left\langle \frac{-6\sqrt{2}}{2}, \frac{6\sqrt{2}}{2} \right\rangle$

$= \langle -3\sqrt{2}, 3\sqrt{2} \rangle$

Q3.

a. $u \cdot v = \sqrt{3} + \sqrt{3} = 2\sqrt{3}$.

b. $\cos\theta = \frac{u \cdot v}{\|u\| \|v\|} = \frac{2\sqrt{3}}{\sqrt{4} \cdot \sqrt{4}} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{6}$.

c. $\text{proj}_v u = \frac{u \cdot v}{\|v\|} = \frac{2\sqrt{3}}{2} = \sqrt{3}$