King Fahd University of Petroleum and Minerals
Prep -Year Math Program
Math 002 - Term 062
Recitation Hour Problems (6.1 & 6.2)

Question1:

Simplify the following expression

(i) $\dfrac{\sin x}{1-\cot x}+\dfrac{\cos x}{1-\tan x}=?$

(ii) $\dfrac{(\sec\theta-\tan\theta)^2+1}{\sec\theta\csc\theta-\tan\theta\csc\theta}=?$

Question2:

Verify the following identities:

$a)$ $\cot^2 x-\cos^2 x=\cot^2 x\cos^2 x$

$b)$ $(\cot\theta-\csc\theta)^2=\dfrac{1-\cos\theta}{1+\cos\theta}$

Question3:

(i) If $\cos\theta=0.8$, θ in quadrant IV, find the exact value of $\cos\left(\theta+\dfrac{\pi}{6}\right)$.

(ii) If $\tan(x+y)=33$ and $\tan x=3$, find the value of $\tan y=?$

Question4;

Find the exact value of the following expressions:

$a)$ $sin\,89^\circ\cos 361^\circ+\sin 1^\circ\sin 721^\circ$

$b)$ $\cos 20^\circ-\sin 70^\circ+\csc\left(\dfrac{19\pi}{12}\right)+\tan\left(-\dfrac{\pi}{12}\right)$

$c)$ $\dfrac{\tan 75^\circ-\cot 75^\circ}{1+\cot 15^\circ\cot 75^\circ}$

Question5:

If the point $\left(-1,\dfrac{3}{4}\right)$ lies on the terminal side of an angle θ, and $\sec\alpha=-\dfrac{13}{5}$, for α in quadrant II, then find the exact value of $\csc(\theta+\alpha)=?$

Recitation 6.1 and 6.2

Q1.

(i) $$\frac{\sin x}{1-\cot x} + \frac{\cos x}{1-\tan x} = \frac{\sin x}{1-\frac{\cos x}{\sin x}} + \frac{\cos x}{1-\frac{\sin x}{\cos x}}$$

$$= \frac{\sin x}{\frac{\sin x - \cos x}{\sin x}} + \frac{\cos x}{\frac{\cos x - \sin x}{\cos x}}$$

$$= \frac{\sin^2 x}{\sin x - \cos x} + \frac{\cos^2 x}{\cos x - \sin x}$$

$$= \frac{\sin^2 x - \cos^2 x}{\sin x - \cos x} \quad \text{(why?)}$$

$$= \frac{\cancel{(\sin x - \cos x)}(\sin x + \cos x)}{\cancel{\sin x - \cos x}}$$

$$= \sin x + \cos x$$

(ii) $$\frac{(\sec\theta - \tan\theta)^2 + 1}{\sec\theta\csc\theta - \tan\theta\csc\theta} = \frac{\sec^2\theta - 2\sec\theta\tan\theta + \overbrace{\tan^2\theta + 1}^{\sec^2\theta}}{\sec\theta\csc\theta - \tan\theta\csc\theta}$$ but $1+\tan^2\theta = \sec^2\theta$

$$= \frac{2\sec^2\theta - 2\sec\theta\tan\theta}{\sec\theta\csc\theta - \tan\theta\csc\theta}$$

$$= \frac{2\sec\theta\,\cancel{(\sec\theta - \tan\theta)}}{\csc\theta\,\cancel{(\sec\theta - \tan\theta)}}$$

$$= 2\cdot\frac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}}$$

$$= 2\,\frac{\sin\theta}{\cos\theta}$$

$$= 2\tan\theta$$

Q2.

a) $$L.H.S = \frac{\cos^2 x}{\sin^2 x} - \cos^2 x$$

$$= \frac{\cos^2 x - \cos^2 x \sin^2 x}{\sin^2 x}$$

$$= \frac{\cos^2 x\,(1 - \sin^2 x)}{\sin^2 x}$$

$$= \cot^2 x \cdot \cos^2 x$$

$$= R.H.S$$

b) $$L.H.S. = \left(\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)^2$$

$$= \left(\frac{\cos\theta - 1}{\sin\theta}\right)^2$$

$$= \frac{\cos^2\theta - 2\cos\theta + 1}{\sin^2\theta}$$

$$= \frac{(\cos\theta - 1)^2}{1 - \cos^2\theta}$$

$$= \frac{(\cos\theta - 1)^2}{(1 - \cos\theta)(1 + \cos\theta)}$$

$$= \frac{-(\cos\theta - 1)}{1 + \cos\theta}$$

$$= \frac{1 - \cos\theta}{1 + \cos\theta} = R.H.S$$

Q3. check your notes.

Q4.

a) $\sin 89^\circ \cos 361^\circ + \sin 1^\circ \sin 721^\circ = \cos 1^\circ \cos 1^\circ + \sin 1^\circ \sin 1^\circ$

$= \cos^2 1^\circ + \sin^2 1$

$= 1$

b) $\cos 20^\circ - \sin 70^\circ + \csc\left(\underset{\in \mathrm{IV}\,Q}{\frac{19\pi}{12}}\right) + \tan\left(\frac{-\pi}{12}\right) = \cos 20^\circ - \cos 20^\circ - \csc \frac{5\pi}{12} - \tan \frac{\pi}{12}$

$= -\csc(75^\circ) - \tan 15^\circ$

$12 \overline{)19}$, 1, 12, 7

$2\pi - \frac{19\pi}{12} = \frac{24\pi - 19\pi}{12} = \frac{5\pi}{12}$

* $\sin 75^\circ = \sin(45^\circ + 30^\circ) = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6} + \sqrt{2}}{4}$

* $\tan 15^\circ = \tan(45^\circ - 30^\circ) = \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$

$= \frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}$

$= \frac{3 - \sqrt{3}}{3 + \sqrt{3}} \cdot \frac{3 - \sqrt{3}}{3 - \sqrt{3}}$

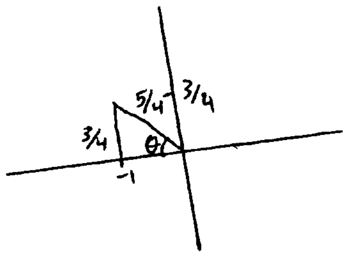
$= \frac{9 - 6\sqrt{3} + 3}{9 - 3}$

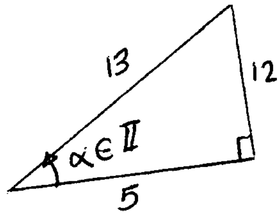
$= \frac{12 - 6\sqrt{3}}{6} = 2 - \sqrt{3}$.

$\therefore$ The given expression $= -\frac{4}{\sqrt{6} + \sqrt{2}} - (2 - \sqrt{3})$

c) check your notes:

Q5.





$$r = \sqrt{1 + \frac{9}{16}}$$
$$= \sqrt{\frac{25}{16}}$$
$$= \frac{5}{4}$$

$$\sin(\theta + \alpha) = \sin\theta\cos\alpha + \cos\theta\sin\alpha$$
$$= \frac{\frac{3}{4}}{5/4} \cdot \frac{-5}{13} + \frac{-1}{5/4} \cdot \frac{12}{13}$$
$$= \frac{-15}{65} - \frac{48}{65}$$
$$= \frac{-63}{65}$$

$$\therefore \csc(\theta + \alpha) = \frac{-65}{63}.$$