King Fahd Univesity of Petroleum & Minerals
Department of Mathematical Sciences

Math 101-Term 043
Exam III
Duration: 75 Minutes

Master

Name: ______________________________ Serial #:____________

ID #:______________________________Section:______________

Question #	**Mark**	**Total Mark**
Part I		8
Part II		
Q1		2
Q2		5
Q3		3
Total Marks		**18**

Part I Multiple Choice Questions

The correct answer is a

1. If $f(x) = x + \cos x$, then $(f^{-1})'(\pi - 1) =$

$a)$ 1
$b)$ 0
$c)$ -2
$d)$ 2
$e)$ $\frac{1}{2}$

2. If $f(x) = \frac{x^3}{3} - x^2 - 3x + 5$, then on which of the following intervals $f(x)$ is one-to-one?

$a)$ $(-1, 3)$
$b)$ $(-3, 1)$
$c)$ $(-3, \infty)$
$d)$ $(1, \infty)$
$e)$ $(-\infty, 3)$

3. $\lim_{x \to 0} \frac{4^{\sin x} - 1}{8^{\tan x} - 1} =$

$a)$ $\frac{2}{3}$
$b)$ $\frac{1}{2}$
$c)$ 0
$d)$ 2
$e)$ ∞

4. If $f(x)=\log_{16}(\log_3 x)$,then $f'(e)$ is equal to

$a)\ \dfrac{1}{4e\ln 2}$

$b)\ \dfrac{1}{4e\ln 6}$

$c)\ \dfrac{1}{e\ln 48}$

$d)\ \dfrac{\ln 3}{4e\ln 2}$

$f)\ \dfrac{1}{4e\ln 48}$

5. $\lim\limits_{x\to 1^+}\left(\dfrac{1}{x-1}\right)^{\ln x}$

$a)1$

$b)e$

$c)\dfrac{1}{e}$

$d)e^2$

$e)e^{-2}$

6. $y=\dfrac{e^x\sqrt{x^5+4}}{(x+1)^4(x^2+1)^2}$, then y' at $x=0$ is equal to

$a)-6$

$b)\ -3$

$d)\ -4$

$e)\ -2$

$f)\ -8$

7. $\frac{d}{dx}\left[\sin(\tan^{-1}\frac{x^2}{2})\right] =$

$a)\ \frac{8x}{(x^4+4)^{3/2}}$

$b)\ \frac{12x}{(x^4+4)^{3/2}}$

$c)\ \frac{4x-1}{(x^4+4)^{3/2}}$

$d)\ \frac{1}{x(x^4+4)^{3/2}}$

$e)\ \frac{x}{8(x^4+4)^{3/2}}$

8. Test the function $f(x) = -\frac{5}{6}x^{2/3} + 100$ for relative extrema and cusps

$a)$ one relative maximum and a cusp s

$b)$ one relative minimum and no cusps

c) one relative minimum and a cusp

d) neither relative extrema nor cusp

e) one relative maximum and no cusps

Part II Written Questions

Question1 (2points)

If $f(x) = ax + bx^{1/2}$ has a minimum value at the point $(9,6)$, then find the values of a and b.

Solution

Since (9, 6) is a point on the curve $\Rightarrow f(9) = 6 \Rightarrow 6 = 6a + 3b \quad \Rightarrow \quad 2 = 2a + b$ **.5 pt**

$$f'(x) = a + \frac{b}{2\sqrt{x}}$$

Since f has a minimum value at the point (9, 6) and $f'(9)$ exists, then

$f'(9) = 0 \Rightarrow a + \frac{b}{2\sqrt{9}} = 0 \Rightarrow a + \frac{b}{6} = 0 \Rightarrow \quad 6a + b = 0$ **1 pt**

Solving the above equations $a = \frac{-2}{3}$ and $b = 4$ **.5 pt**

Question2 (5points)

Consider the function $f(x) = x + \frac{1}{x}$.

a) Find the critical numbers, if any exists.
b) Find the increasing and decreasing intervals.
c) Find all relative extrema.
d) Find the concavity intervals
e) Find the inflection points, if any exists.
f) Find all asymptotes
g) Sketch the graph of f using the information above. Label your diagram with the information above.

Solution

a)

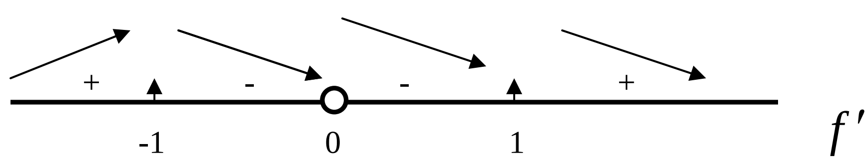
$$f'(x) = 1 - \frac{1}{x^2} = \frac{1-x^2}{x^2}$$

$$f' = 0 \Rightarrow 1 - x^2 = 0 \Rightarrow x = \pm 1$$

$f'\ DNE \Rightarrow x = 0$ not critical number since it is not in the domain

$\therefore$ Critical Numbers $= \{-1, 1\}$

b)



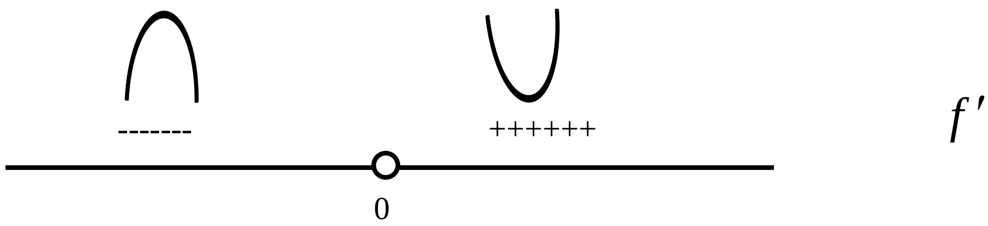
Increasing on $(-\infty, -1] \cup [1, \infty)$

Decreasing on $[-1, 0) \cup (0, 1]$

c) Relative maximum at $(-1, f(-1)) = (-1, -2)$

Relative minimum at $(1, f(1)) = (1, 2)$

d) $f''(x) = 2x^{-3} = \dfrac{2}{x^3}$



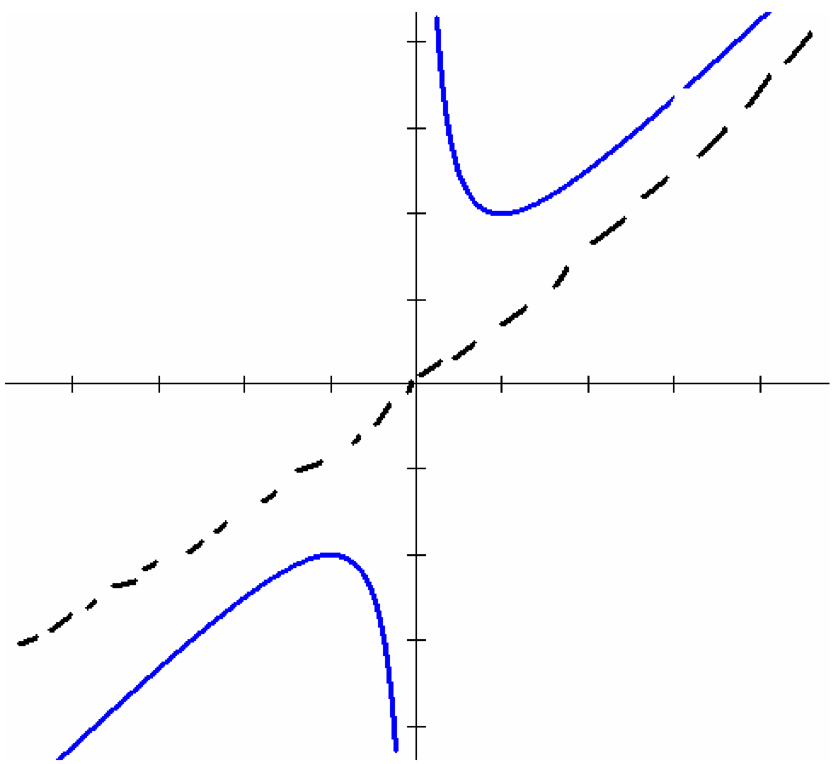
Concave down on $(-\infty, 0)$

Concave up on $(0, \infty)$

e) No inflection points

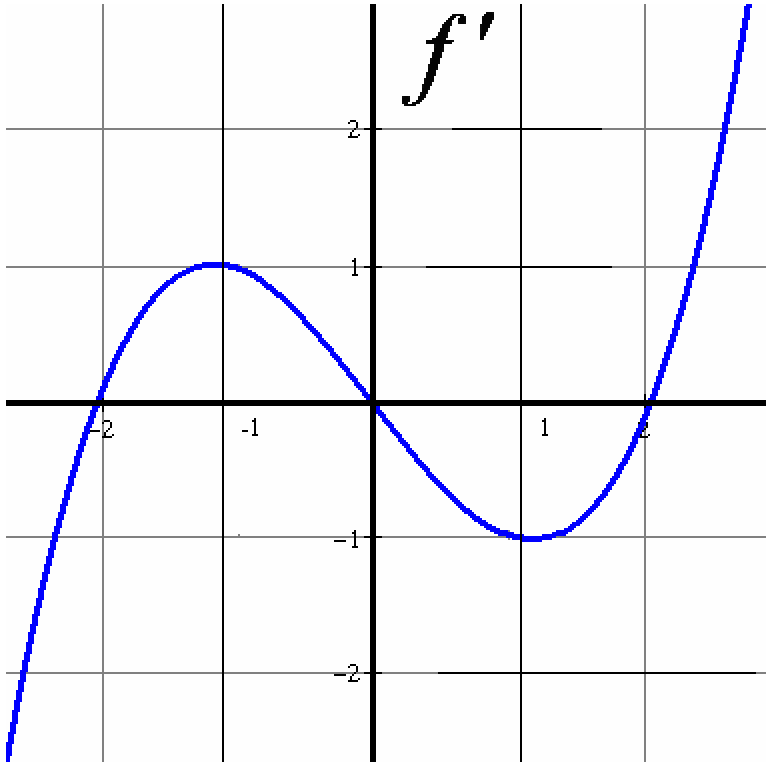
f) Oblique Asymptote: $y = x$
Vertical Asymptote $x = 0$

g)



Question3 (3 points)

The figure below shows the graph of the derivative f' of a function f



a) Find the critical numbers of f
b) Find the intervals on which f is increasing and the interval oin which f is decreasing
c) Find the intervals on which f is concave up and the intervals on which f is concave down

Solution

a) $\{-2, 0, 2\}$

b) Increasing on $[-2,0] \cup [2,\infty)$
Decreasing on $(-\infty,-2] \cup [0,2]$

c) Concave up on $(-\infty,-1) \cup (1,\infty)$
Concave down on $(-1,1)$