King Fahd University of Petroleum and Minerals
Faculty of Science – Math Prep Year program
Math 002 -042
Quiz #5ℬ(6.3-6.4)

Name: **Sr#:** ID: Sec.:

Question1 **(12 points)**

Find the exact value of

a) $\log(\cos 22.5° - \sin 22.5°) + \log(\cos 22.5° + \sin 22.5°)$ **[Give the answer only in terms of log]**

Solution

$$\log(\cos 22.5° - \sin 22.5°) + \log(\cos 22.5° + \sin 22.5°) = \log(\cos^2 22.5° - \sin^2 22.5°) = \log(\cos 45°) = \log\frac{\sqrt{2}}{2}$$

b) $\sin 112.5°$

Solution

$$\sin 112.5° = \sin 67.5° = \sqrt{\frac{1-\cos 135°}{2}} = -\sqrt{\frac{1+\cos 45°}{2}} = -\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}} = -\frac{\sqrt{2+\sqrt{2}}}{2}$$

c) $2\sin^2\frac{x}{2} + \cos x - 1$

Solution

$$2\sin^2\frac{x}{2} + \cos x - 1 = 2(\frac{1-\cos x}{2}) + \cos x - 1 = 1 - \cos x + \cos x - 1 = 0$$

d) $\sin 15° \cos 75°$

Solution

$$\sin 15° \cos 75° = \sin 15° \sin 15° = \sin^2 15° = \frac{1-\cos 30°}{2} = \frac{1-\frac{\sqrt{3}}{2}}{2} = \frac{2-\sqrt{3}}{4}$$

Question2 **(5 points)**

If $\tan\frac{\theta}{2}=-\frac{5}{4},\pi<\theta<\frac{3\pi}{2}$, then find $\tan\theta$.

Solution

$$\tan\frac{\theta}{2}=\pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=-\frac{5}{4},\text{square each side}$$
$$\frac{1-\cos\theta}{1+\cos\theta}=\frac{25}{16}\Rightarrow\cos\theta=-\frac{9}{41}$$

Question3 **(3 points)**

If $f(x)=\sqrt{3}\sin x-\cos x+2$,where $0\le x<2\pi$, then find the **range** and the **phase shift** of $f(x)$.

Solution

$$k=\sqrt{1+3}=2$$
$$\left.\begin{array}{l}\sin\alpha=-\frac{1}{2}\\ \cos\alpha=\frac{\sqrt{3}}{2}\end{array}\right\}\Rightarrow\alpha\in 4^{th}\ \text{Quadrant}\Rightarrow\alpha=2\pi-\frac{\pi}{6}=\frac{11\pi}{6}$$

$$\therefore f(x)=2\sin(x+\frac{11\pi}{6})+2$$

$$*\quad R=\left[-|a|+d,|a|+d\right]=[0,4]$$

$*$ *Phase shift* :

$$x+\frac{5\pi}{6}=0\Rightarrow x=-\frac{11\pi}{6}\ (\frac{11\pi}{6}\ \textit{units left})$$