King Fahd University of Petroleum and Minerals
Diploma Math Program
Math 004 - Term 042
Class Test

Name: **ID:** **Sec.:**

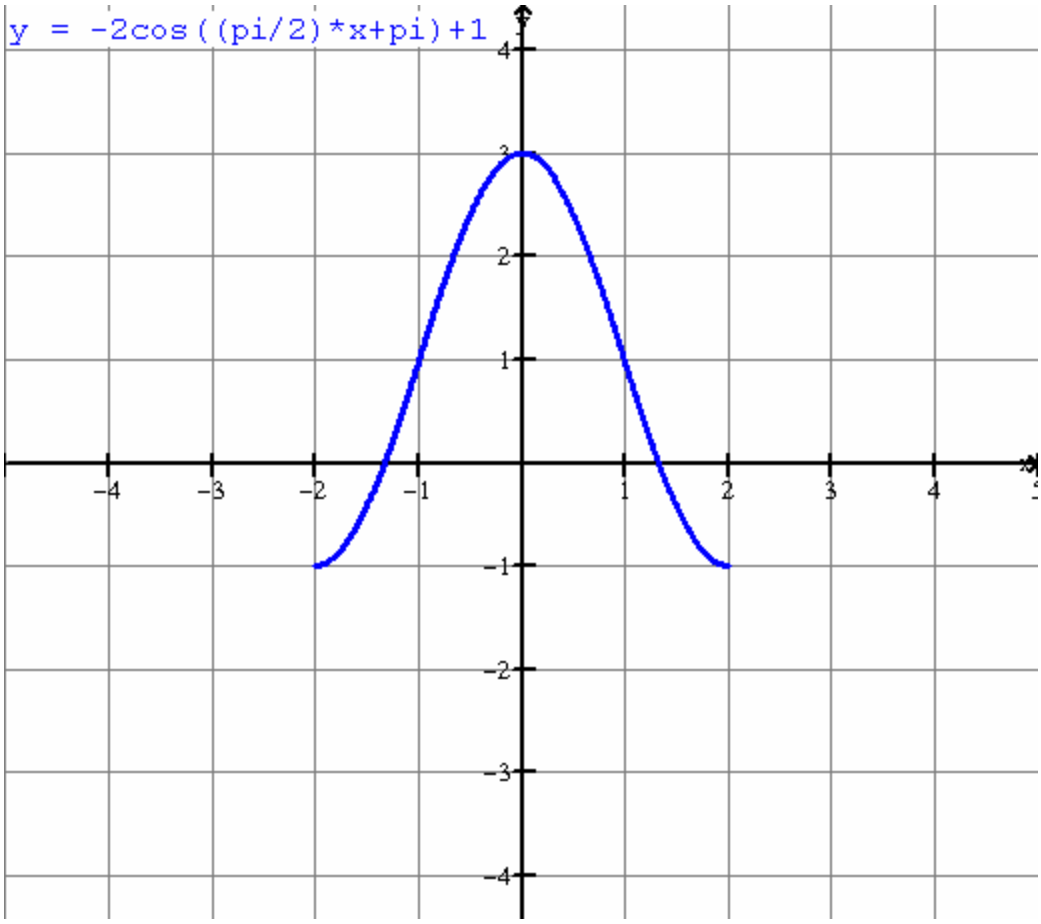
Question1

For the function $y = -2\cos(\frac{\pi}{2}x + \pi) + 1$

a) Complete the following table (write the rule when that necessary). **(2 Pts)**

the phase shift	the range
$\frac{\pi}{2}x + \pi = 0$ $\Rightarrow x = -2\,(2\,\text{units left})$	$R = [-\lvert a\rvert + d, \lvert a\rvert + d]$ $= [-2+1, 2+1] = [-1, 3]$

b) Graph the function over one complete period **(2 Pts)**



Question2

If $\sin 40° + \cos 40° = k \cos t$, where k and t are real numbers, then , find all values of k and t.

(2 Pts)

Solution

$$k = \sqrt{1+1} =$$

$$\left.\begin{aligned} \sin\alpha &= \frac{1}{\sqrt{2}} \\ \cos\alpha &= \frac{1}{\sqrt{2}} \end{aligned}\right\} \Rightarrow \alpha \in I \text{ Quadrant} \Rightarrow \alpha = 45°$$

$$\therefore \sin 40° + \cos 40° = \sqrt{2}\sin(40° + 45°) = \sqrt{2}\sin 85° = \sqrt{2}\cos 5°$$

$$\therefore k = \sqrt{2}\ , t = 5°$$

Question3

Verify the identity $\sqrt{\frac{1-\cos x}{1+\cos x}} = \csc x - \cot x\ , 0 < x < \frac{\pi}{2}$ **(2 Pts)**

Solution

$$\begin{aligned} L.H.S &= \sqrt{\frac{1-\cos x}{1+\cos x}.\frac{1-\cos x}{1-\cos x}} \\ &= \sqrt{\frac{(1-\cos x)^2}{\sin^{2x}}} \\ &= \frac{1-\cos x}{\sin x} \\ &= \frac{1}{\sin x} - \frac{\cos x}{\sin x} \\ &= \csc x - \cot x = R.H.S \end{aligned}$$

Question4

Find the exact value of the following expressions:

a. $\left(\sin 15^0 + \cos 15^0\right)^2$ **(2 Pts)**

Solution

$$\sin^2 15° + 2\sin 15°\cos 15° + \cos^2 15°$$

$$1 + \sin 30° = 1 + \frac{1}{2} = \frac{3}{2}$$

b. $\dfrac{1 - \tan 80° \cot 50°}{\tan 80° + \cot 50°}$ **(2 Pts)**

Solution

$$\frac{1 - \tan 80° \tan 40°}{\tan 80° + \tan 40°} = \cot 120° = -\cot 60° = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

c. $\dfrac{-3}{2\cos^2 75° - 1}$ **(2 Pts)**

Solution

$$\frac{-3}{\cos 150°} = \frac{-3}{-\cos 30°} = \frac{3}{\frac{\sqrt{3}}{2}} = \frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

d. $(\dfrac{\sec x}{\tan x})^2 - \dfrac{1}{\tan^2 x}$ **(2 Pts)**

Solution

$$\frac{\sec^2 x}{\tan^2 x} - \frac{1}{\tan^2 x} = \frac{\sec^2 x - 1}{\tan^2 x} = \frac{\tan^2 x}{\tan^2 x} = 1$$

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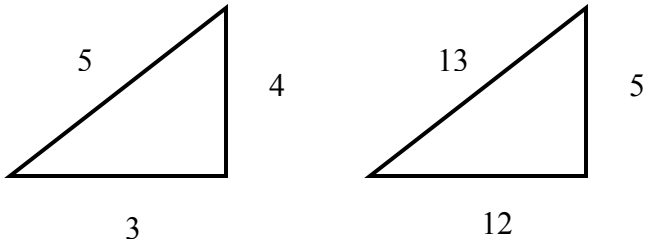
Question5

If $\sin\alpha = \frac{4}{5}, \alpha$ in quadrant II, and $\sin(\frac{\pi}{2} - \beta) = -\frac{12}{13}, \beta$ in quadrant II, then find

a. $\sec(\alpha + \beta)$. **(2 Pts)**

Solution

$$\sin(\frac{\pi}{2} - \beta) = \cos\beta = -\frac{12}{13}$$



$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta = (\frac{-3}{5}) \times (\frac{-12}{13}) - \frac{4}{5} \times \frac{12}{13} = \frac{16}{65}$$

$$\Rightarrow \sec(\alpha + \beta) = \frac{65}{16}$$

b. $\cos\frac{\alpha}{2}$ **(2 Pts)**

Solution

$$\frac{\pi}{2} < \alpha < \pi \Rightarrow \frac{\pi}{4} < \frac{\alpha}{2} < \frac{\pi}{2} \Rightarrow \frac{\alpha}{2} \in I\ Quadrant$$

$$\cos(\frac{\alpha}{2}) = \sqrt{\frac{1 + \cos\alpha}{2}} = \sqrt{\frac{1 + \frac{-3}{5}}{2}} = \sqrt{\frac{1}{10}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

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