Rotational Inertia

Objective

To determine the rotational inertia of rigid bodies and to investigate its dependence on the distance to the rotation axis.

Introduction

Rotational Inertia, also known as *Moment of Inertia*, plays the same role in rotational motion as that of the mass in translational motion.

The rotational inertia I of a point mass m located at a distance r from a fixed axis of rotation (see Figure 1) is defined as

$$I = m\, r^2$$

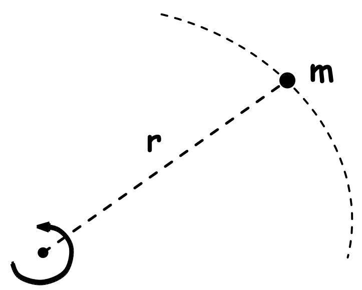

Figure 1

In this experiment, you will determine the rotational inertia of rigid bodies of different shapes by measuring their angular accelerations. You will use a rotary motion sensor, shown in Figure 2, to find the angular acceleration of the object. The sensor measures angular displacement as a function of time.

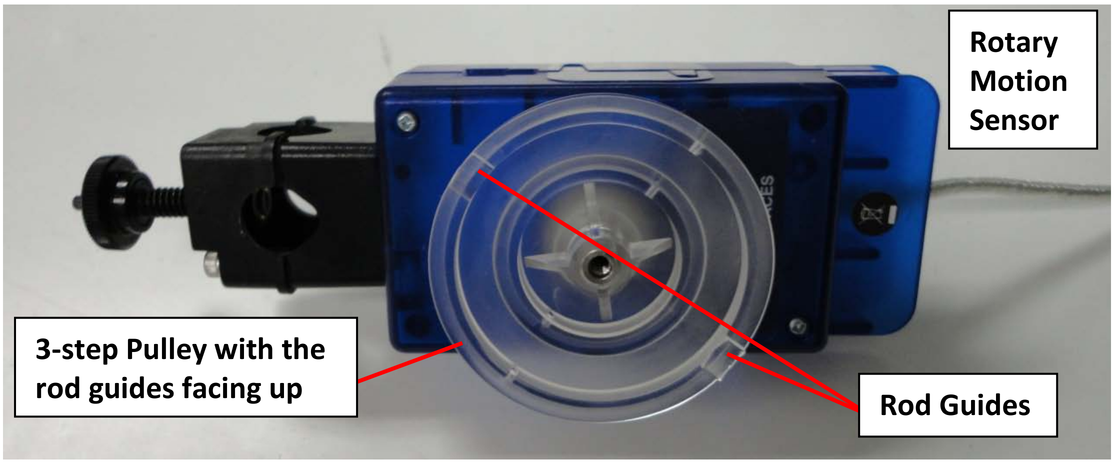

Figure 2

The apparatus you will use for this purpose is shown in Figure 3 for a rod. When hanging mass pull the string, the rod rotates about the axis of the 3-step pulley.

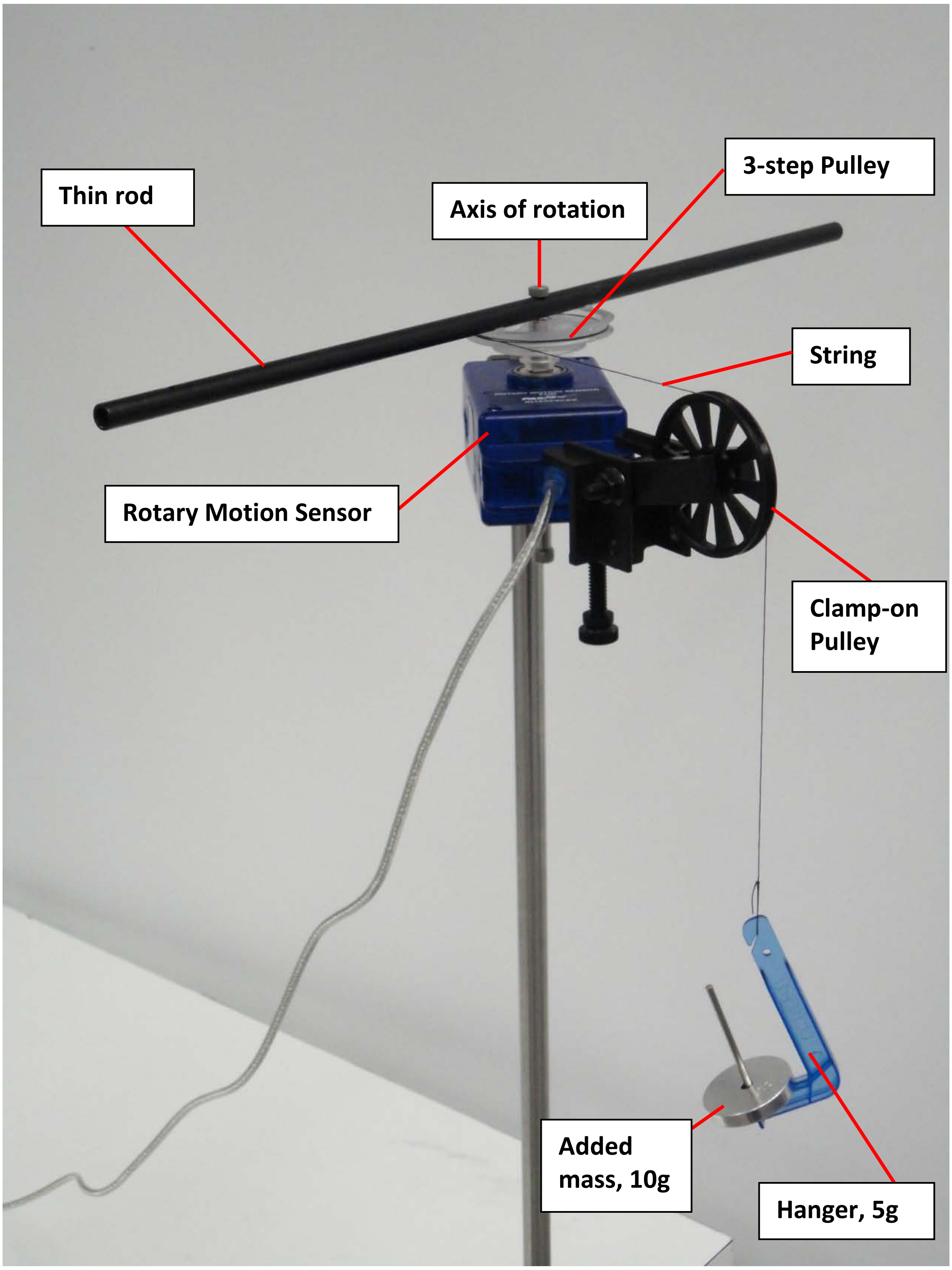

Figure 3

Ignoring friction in the pulley, it can be shown that the moment of inertia I of the rigid body with the rotating pulley system is given by (see Appendix at the end, for the derivation):

$$I = m_{hang} R^2 \left(\frac{g}{R\alpha} - 1 \right) \tag{1}$$

where m_{hang} is the hanging mass, R is the radius of the pulley used on the sensor, g is the free fall acceleration (9.80 m/s^2), and α is the angular acceleration of the rigid body.

Exercise 1 – Rotational inertia I_o of a thin rod with the rotating platform

In this exercise, you will determine experimentally the rotational inertia I_o of the rotating platform with the thin rod as follows:

1. Mount the rod on the 3-step pulley, using the center screw, as shown in Figure 4. Make sure the rod fits into the rod guides on the largest pulley.

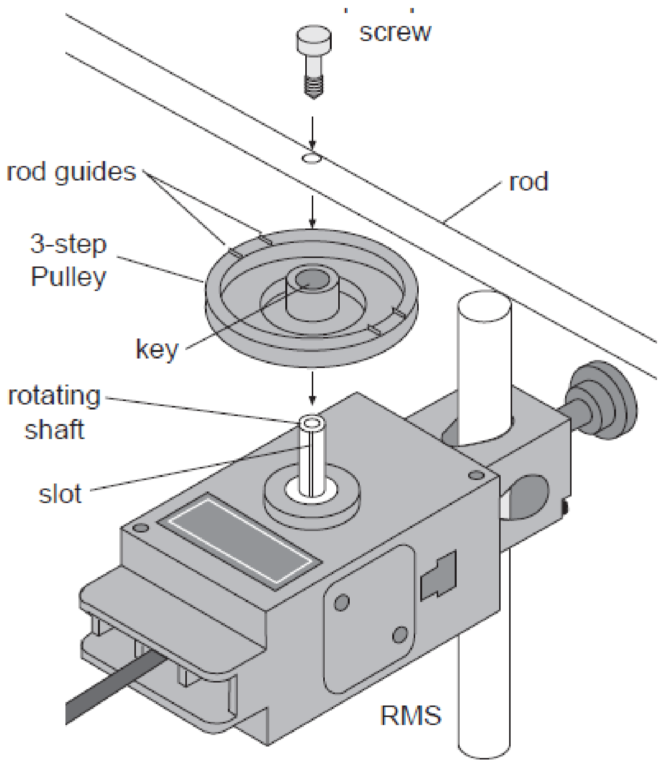

Figure 4

2. Download the file **rotation.ds** from the link in phys101 homepage and save it in the desktop. This file has been preconfigured for optimum experimental parameters such as angular position and time scales.

3. From the **Start** button, go to **All Program → DataStudio → English**. This will open **DataStudio**. Click on ***Open Activity***, navigate to the folder desktop and to the file **rotation.ds**, and open it. This will open the angular position versus time graph.

4. Make sure the rotary motion sensor is connected to a computer via an USB interface module.

5. Adjust the height and angle of the clamp-on pulley so that the string runs horizontally in a line tangent to the point where it leaves the 3-step pulley and straight down the middle of the groove on the clamp-on pulley. (See Figure 5). Also note that you have to wind the string only in one of the directions to get this right.

Figure 5

6. Wind the string around the largest pulley (R = 2.4 cm) on the rotary motion sensor. Keep the system at rest in this position with your fingers.

7. Attach a 5-g mass hanger to the free end of the string. Add an additional 10-g mass to make the total m_{hang} = 15 g.

8. Release the system from rest and then click the ***Start*** button in **DataStudio**. It is ok to be little late to click the ***Start*** button after the release, but do not click the ***Start*** button before releasing the system. Click the ***Stop*** button well before the hanging mass reaches the lowest point.

9. Your data will look like the red curve in Figure 6. It doesn't matter if you get the curve running downward or upward. Click on the ***Fit*** button in **DataStudio** and then select ***Quadratic Fit***. The blue curve in Figure 6 shows the FIT, however the FIT also will be in red (NOT blue) in your case; it is shown in blue here in Figure 6 for clarity. The fitting parameter A is related to the angular acceleration α of the rigid body, through $\alpha = 2\,|A|$.

Recall that the angular position θ of a particle moving along an arc with a constant angular acceleration α is given by the following equation

$$\vec{\theta} = \tfrac{1}{2}\,\vec{\alpha}\,t^2 + \vec{\omega}_0\,t + \vec{\theta}_0$$

where t is time, θ_0 and ω_0 are the angular position and angular velocity at t = 0, respectively. Therefore, a quadratic fit of the form

$$y = A\,x^2 + B\,x + C$$

to the angular position versus time graph will give $A = ½\,\alpha$, $B = \omega_0$, and $C = \theta_0$.

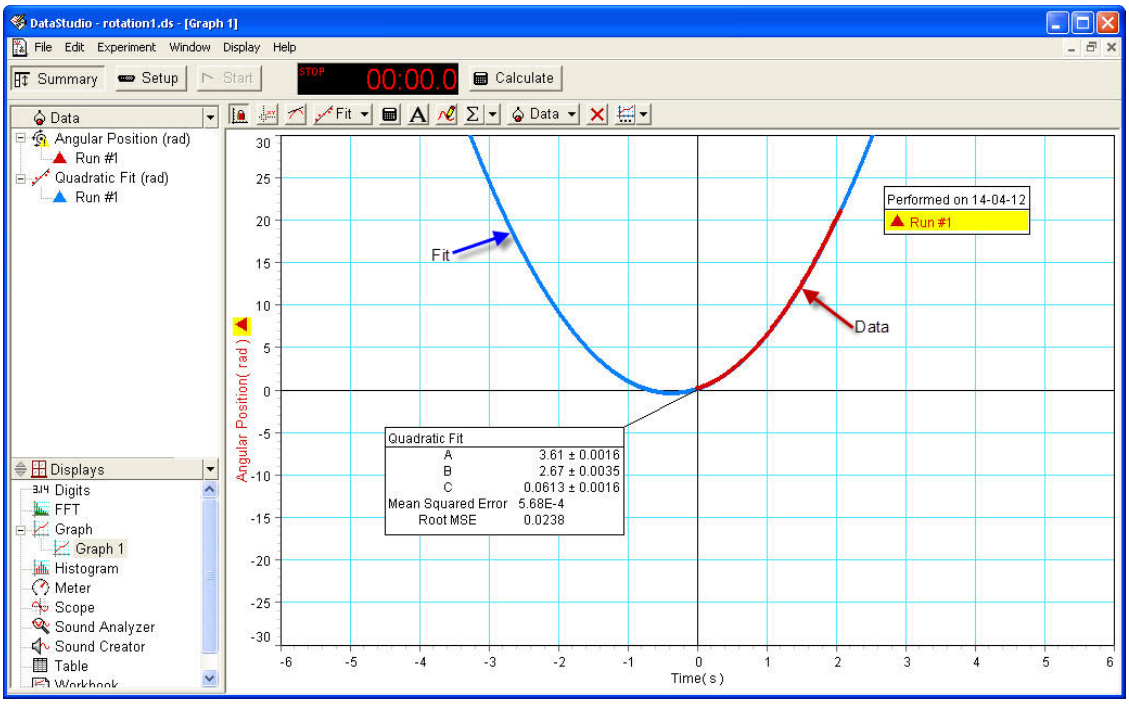

Figure 6

10. From the menu **Display** choose **Export Picture..** to copy your angular position versus time graph. Paste this graph in your report using **Insert → Picture** in Microsoft Word.

11. Record the values of Radius of the 3-step pulley used on the sensor, R and the total hanging mass, m_{hang} in the report.

12. Record the values of $|A|$ in Table 1 of your report and calculate directly in Table 1 the value of α and then the rotational inertia of the rod I_o from the values of α, m_{hang}, and R, using EXCEL Formulas (Equation (1)). Take the value of g to be 9.80 m/s^2 correct to 3 significant figures.

Exercise 2 – Rotational inertia of a dumbbell

A dumbbell is shown in Figure 7. It consists of a thin rod and two mass pieces of mass m each, attached to the rod symmetrically.

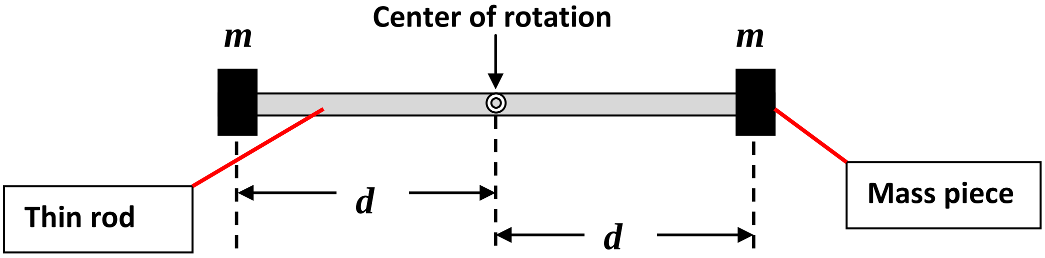

Figure 7

If we assume the two mass pieces to be point masses, the rotational inertia of the dumbbell, about an axis through its center and perpendicular to its length, can be written as

$$I = I_o + 2md^2 \qquad (2)$$

where d is the distance to the center of mass of the mass piece from the axis of rotation and I_o is the rotational inertia of the thin rod with the rotating platform, about the axis of rotation.

<u>OPTIONAL READING</u>

NOTE that an exact treatment (without making the point mass approximation) gives I as:

$$I = \left\{ I_o + \frac{1}{6} m[3(r_o^2 + r_i^2) + h^2] \right\} + 2md^2 = I_o' + 2md^2 \, ,$$

where I_o' includes contribution from the rotational inertia of the mass pieces about its own perpendicular axis when correctly treated as extended objects (not point particles); h is the height and r_i and r_o are the inner and outer radius of the mass piece. However, I_o' is still a constant that doesn't depend on d and the extra term $\frac{1}{6} m[3(r_o^2 + r_i^2) + h^2]$ is at least one order of magnitude smaller than I_o.

In this exercise, you will verify the dependence of I on d as in the equation: $I = I_0 + 2md^2$. You will measure I experimentally, as you did in Exercise 1, for varying values of d.

1. Measure the total mass $2m$ of both mass pieces, together with the thumb screws, using a triple beam balance.

2. Set the two mass pieces flush with the ends of the rod and lock it with the thumbscrew. This will place the two mass pieces at equal distance d = 18.0 cm from the center of rotation, as shown in Figure 8. Note that the distance from the center to one of the edges is 19.0 cm and the height of the mass pieces h = 2.0 cm. Therefore aligning the end of the mass piece exactly to the edge of the rod will make d =18.0 cm.

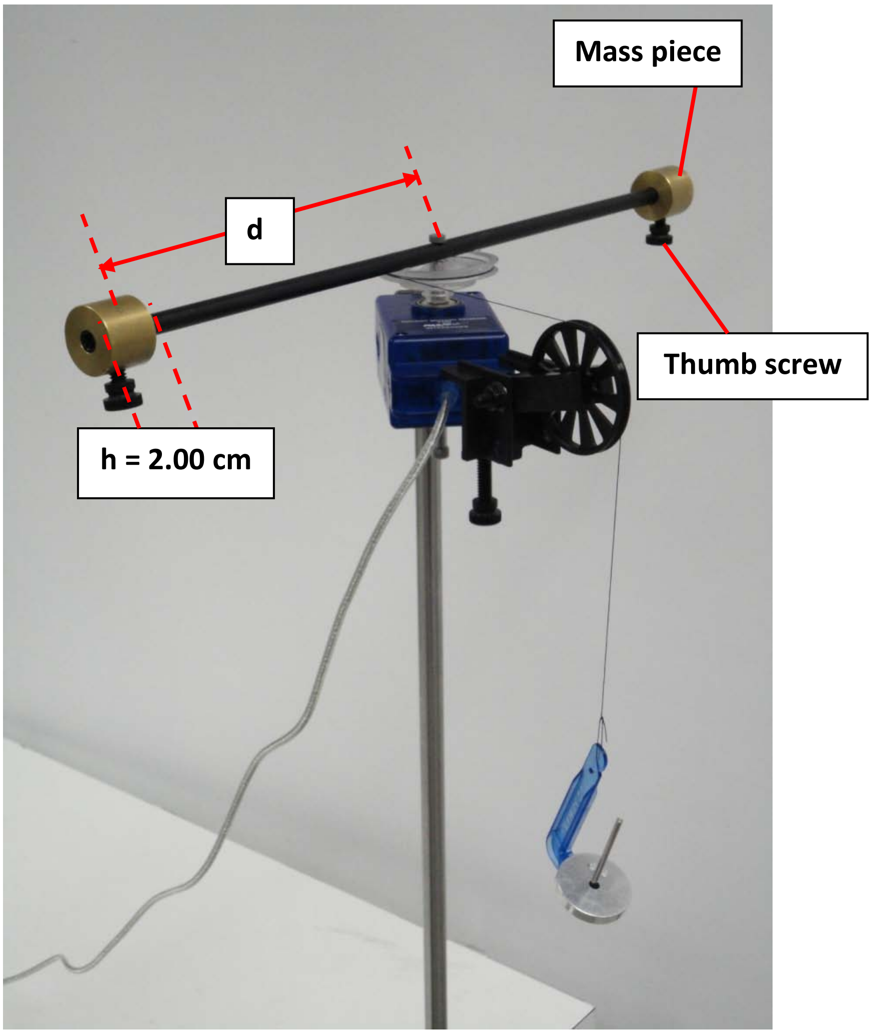

Figure 8

3. Wind the string around the largest pulley (R = 2.4 cm) on the rotary motion sensor. Keep the system at rest in this position with your fingers.

4. Attach a 5-g mass hanger with 10-g added mass to the free end of the string. That is, m_{hang} = 15 g.

5. Release the system from rest and then click the ***Start*** button in **DataStudio**. Do not start before releasing. Click the ***Stop*** button well before the hanging mass reaches the lowest point.

6. Your data will look like the red curve in Figure 6. Click on the ***Fit*** button in **DataStudio** and then select ***Quadratic Fit***. The blue curve in Figure 7 shows the fit. The fitting parameter *A* is related to the angular acceleration α of the rigid body, through $\alpha = 2\,|A|$.

7. Repeat Steps 3 to 6 for *d* = 17.0, 16.0, 15.0, and 14.0 cm as well. Use the tail of the digital caliper to push the two mass pieces 10.0 mm inside from the edges of the rod (see Figure 9), making *d* = 17.0 cm and so on.

Figure 9

8. Record the values of *d*, $|A|$, α, and calculate the corresponding *I* from Equation (1), directly in Table 2 of your report. Note that we are determining the value of I experimentally using Equation (1), NOT by its definition as given by Equation (2).

9. Open EXCEL and copy the values of *I* and d^2 from Table 2 in a new file. Plot *I* versus d^2 and find its slope from the linear trendline equation. Make sure you have plotted *I* on the y axis and d^2 on the x axis. *Why is this important?*

10. Observe that the experimental determination of *I* from Equation (1) is accordance with the definition of *I* in Equation (2). Since $I = I_0 + 2md^2$, *I* versus d^2 graph should give a straight line with the slope equal to *2m*.

11. Find the percent difference between your measured value of 2*m* and the slope of your *I* versus d^2 graph.

12. Copy your graph, and paste it in your report. Record the results in your report.

Exercise 3 – Rotational inertia of a ring

In this exercise, you will measure the rotational inertia of a uniform ring about its axis and compare it with the theoretical value, which is given by

$$I(Ring) = \frac{1}{2}M(R_1^2 + R_2^2) \tag{3}$$

Department of Physics
Dhahran 31261

where, M, R_1, and R_2 are the mass, inner radius and outer radius of the ring, respectively (see Figure 10).

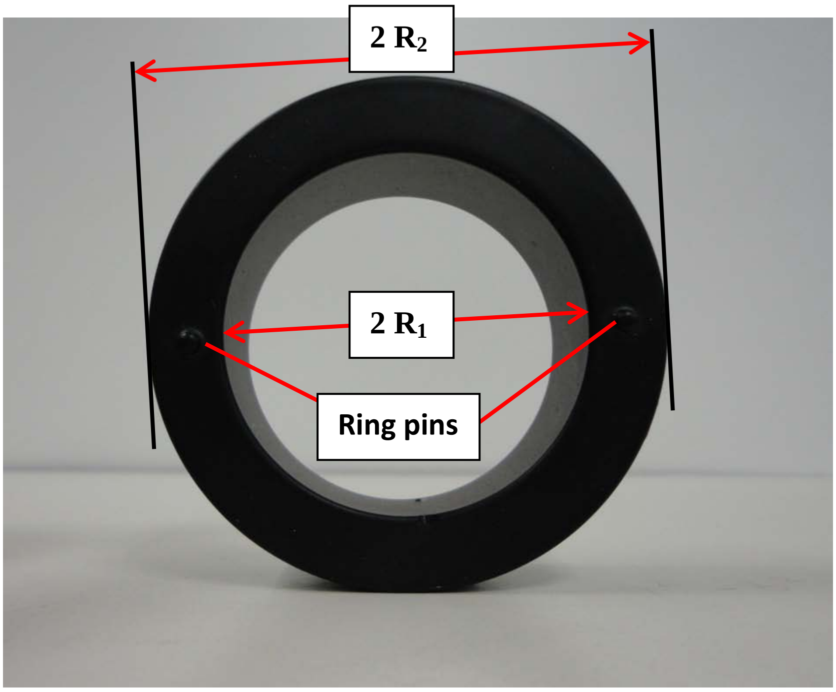

Figure 10

Since the ring cannot sit on the 3-step pulley by itself, you will use a disk in addition. If the disk and the ring are placed on top of the pulley such that their centers of mass coincide, the measured moment of inertia of the system is the sum of moments of inertia of the pulley, disk and the ring.

1. Measure the mass M of the ring using a triple-beam balance. Record the values in Table 3 of your report.
2. Measure using the Digital Caliper the inner diameter of the ring, $2R_1$, and outer diameter of the ring, $2R_2$. Record these values in Table 3 of your report.
3. Calculate *I* (*Ring*) directly in Table 3 using EXCEL formulas. This is the calculated value ((Equation (3)) of the rotational inertia of the ring, I_{cal}.

Now you need to measure *I (Ring)* experimentally.

4. Delete all the data runs from the **DataStudio** file and get it ready for Exercise 3.
5. Remove the rod from the 3-step pulley. Then mount the disk on the 3-step pulley (See Figure 11). Use the same screw removed from the rod to secure the disk to the pulley.

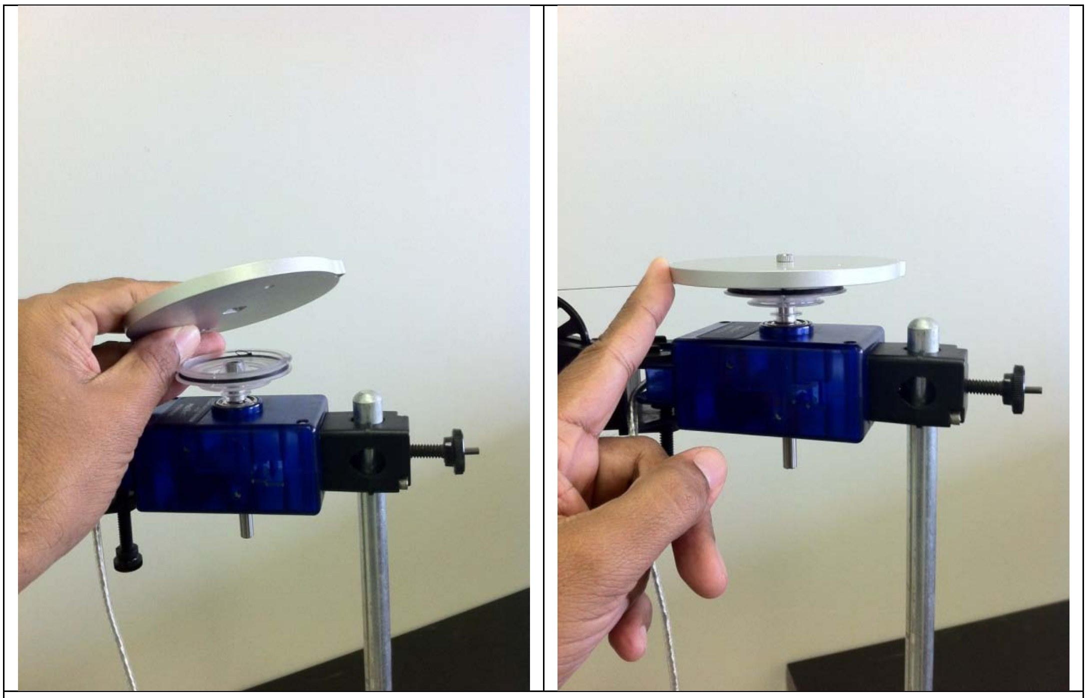

Figure 11

6. Wind the string around the largest pulley (R = 2.4 cm) on the rotary motion sensor. Keep the system at rest in this position with your fingers. Make sure the string is horizontal and tangent to both pulleys.

7. Attach a 5-g mass hanger to the free end of the string. Add an additional 10-g mass to make the total m_{hang} = 15 g.

8. Release the system from rest and then click the ***start*** button in **DataStudio**. Click the ***stop*** button just before the hanging mass reaches the lowest point.

9. Fit the data with ***Quadratic Fit*** as you did in Exercise 1. The fitting parameter A is related to the angular acceleration α of the rigid body, via $\alpha = 2\,|A|$.

10. Record the values of $|A|$, α, and calculate the corresponding *I (Disk+Pulley)* from Equation (1) in Table 4.

11. Now, place the ring on the disk. Make sure the ring pins falls into the guiding holes in the disk (See Figure 12). This ensures their centers of mass coincide with the axis of rotation.

12. Repeat steps 6 to 10 to find *I (Ring+Disk+Pulley)*.

Department of Physics
Dhahran 31261

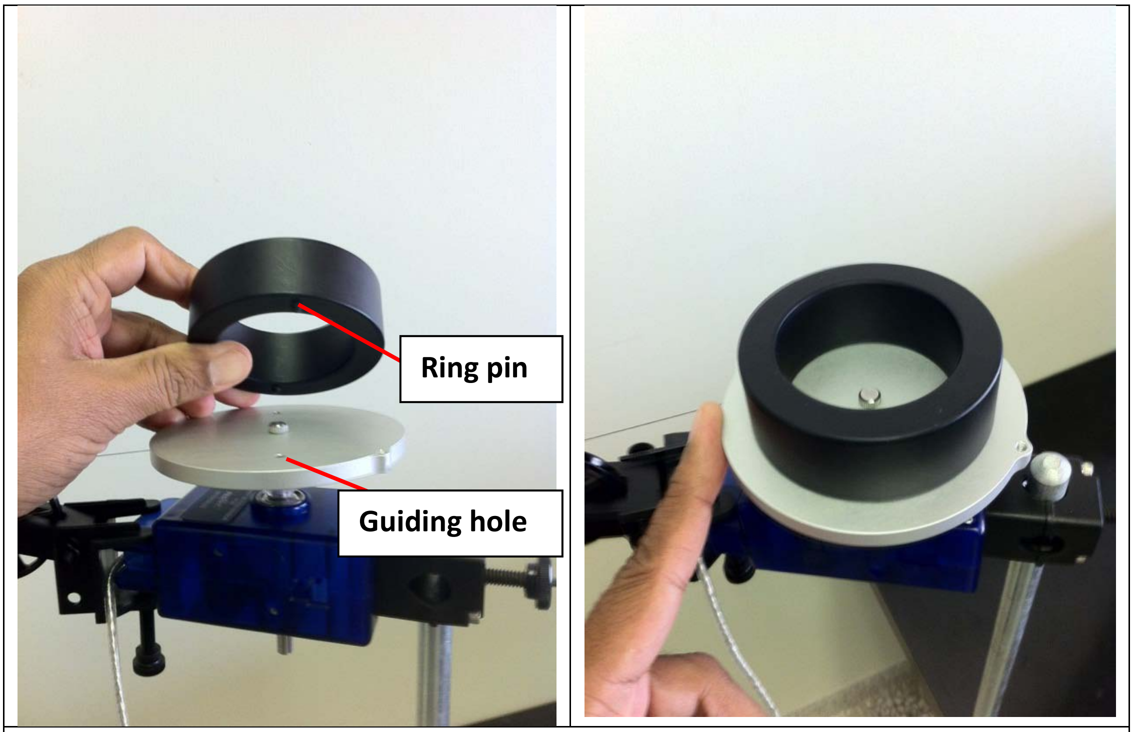

Figure 12

13. Calculate *I (Ring) = I (Ring+Disk+Pulley) – I (Disk+Pulley)* directly in Table 4 in the allocated cell.

14. Find the percent difference between the calculated and experimental values of *I (Ring)* and record it in your report.

15. Copy and paste the DataStudio graph for Disk+Ring in the report.

Appendix

Experimentally, moments of inertia of rigid bodies can be determined by applying the dynamical relations for rotational motion. The apparatus to be used for this purpose consists of a pulley of radius R, which can rotate about a fixed vertical axis through its center (see Figure 13). A mass m_{hang} is attached to a string wrapped around the periphery of the pulley. If enough weight (m_{hang} g) is used and the system is released from rest, the mass m_{hang} moves downward with acceleration *a* while the disk rotates about its axis with an angular acceleration α. The equations of motion of this dynamical system are:

$$m_{hang}\, g - T = m_{hang}\, a \qquad (1)$$

where T is the Tension in the string.

Ignoring friction in the pulley,

$$T R = I\alpha \qquad (2)$$

where I is the moment of inertia of the pulley about a vertical axis through its center.

The linear acceleration *a* and the angular acceleration α are related through

$$a = R\,\alpha \qquad (3)$$

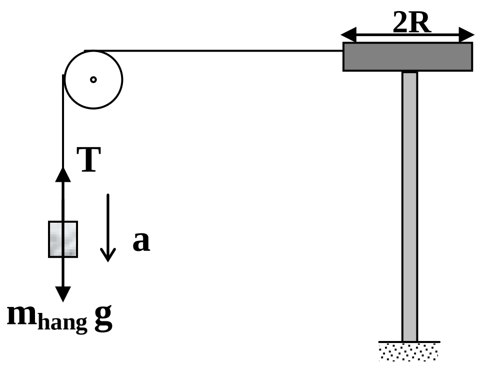

Figure 13

Solving equations (1), (2) and (3) for I, one gets

$$I = m_{hang} R^2 \left(\frac{g}{R\alpha} - 1\right) \qquad (4)$$

Now, if a rigid body is placed on the top of the pulley such that their centers of mass coincide, the measured moment of inertia of the system is the sum of moments of inertia of both the pulley and the rigid body