

Arbitrarily oriented CT/MR slice construction matching tracked US probe

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Motivation

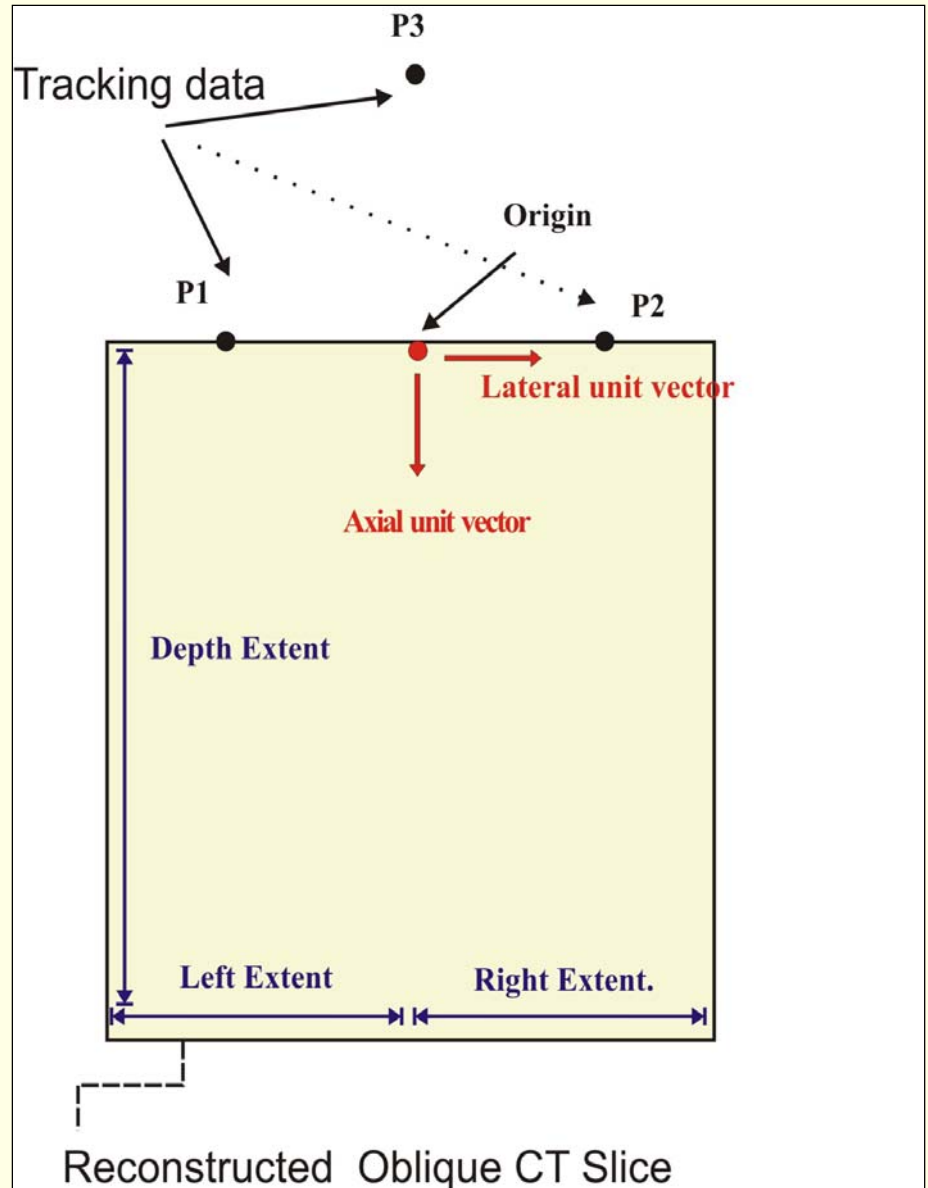
- Pedagogical tool: to associate CT/MR anatomic details with US details for training/educational settings
- Interventional tool: to add CT/MR anatomic details to the real time US images
- *visualization*:
 - Show by each 2D-US image another 2D-CT/MR image of the same anatomical slice
 - (or part of an *Augmented Reality* setting)

Registration Requirement

- Rigid Registration: either automatic or manual, relative orientation and position of 3D-US to CT/MR volume should be determined to
- Deformable Registration: organs are expected to deform as observed clinically. One valuable objective to pursue is to deform CT/MR to match the real-time US

Reconstructing Oblique Volumetric Slice

- Tracking data is cast into *origin* and two vectors *axial* and *lateral* in the sense of US terminology. The vectors are scaled to match the pixel size (typically square).
- The reconstructed image plane is **rastered**. *Horizontal lines* are picked by axial vector then each horizontal is scanned along by the lateral vector. Each step is to generate from the origin a point *p* of *x,y & z* coordinates relative to the volumetric image.
- The point is assigned a gray value using an appropriate interpolation method.



Interpolation Link

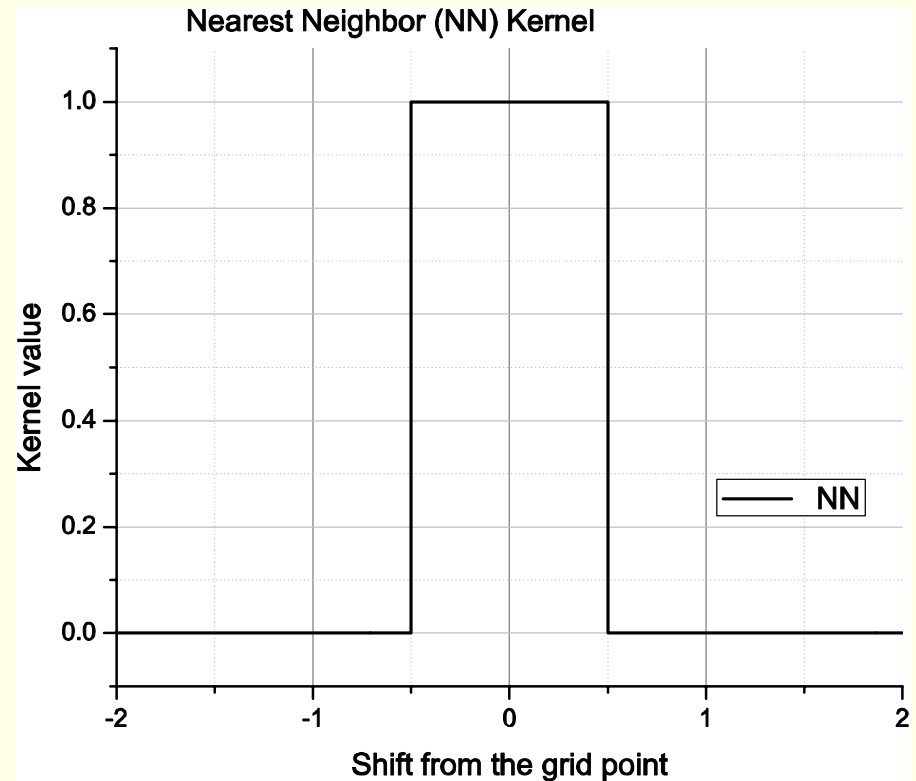
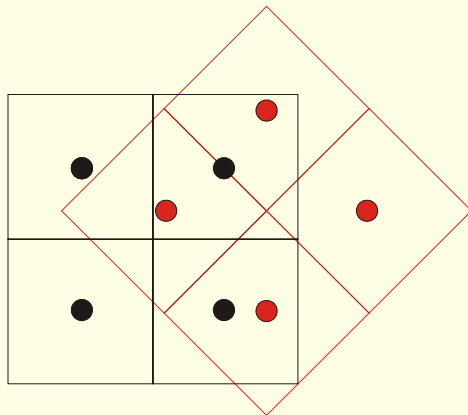
- The reconstructed CT/MR slices are not acquired physically oblique but they are *generated (reconstructed)* as per users specification in terms:
 - voxel dimension
 - slice position & orientation
 - Spatial extents (depth and width)
- Interpolation is used to assign gray values for the positions/points based on values of neighboring points of the source volume image

Proposed Interpolation Method: *Overlapping Spheres*

- Outline:
 - Method and Motivation
 - Fourier Analysis and comparisons
 - Tests and results

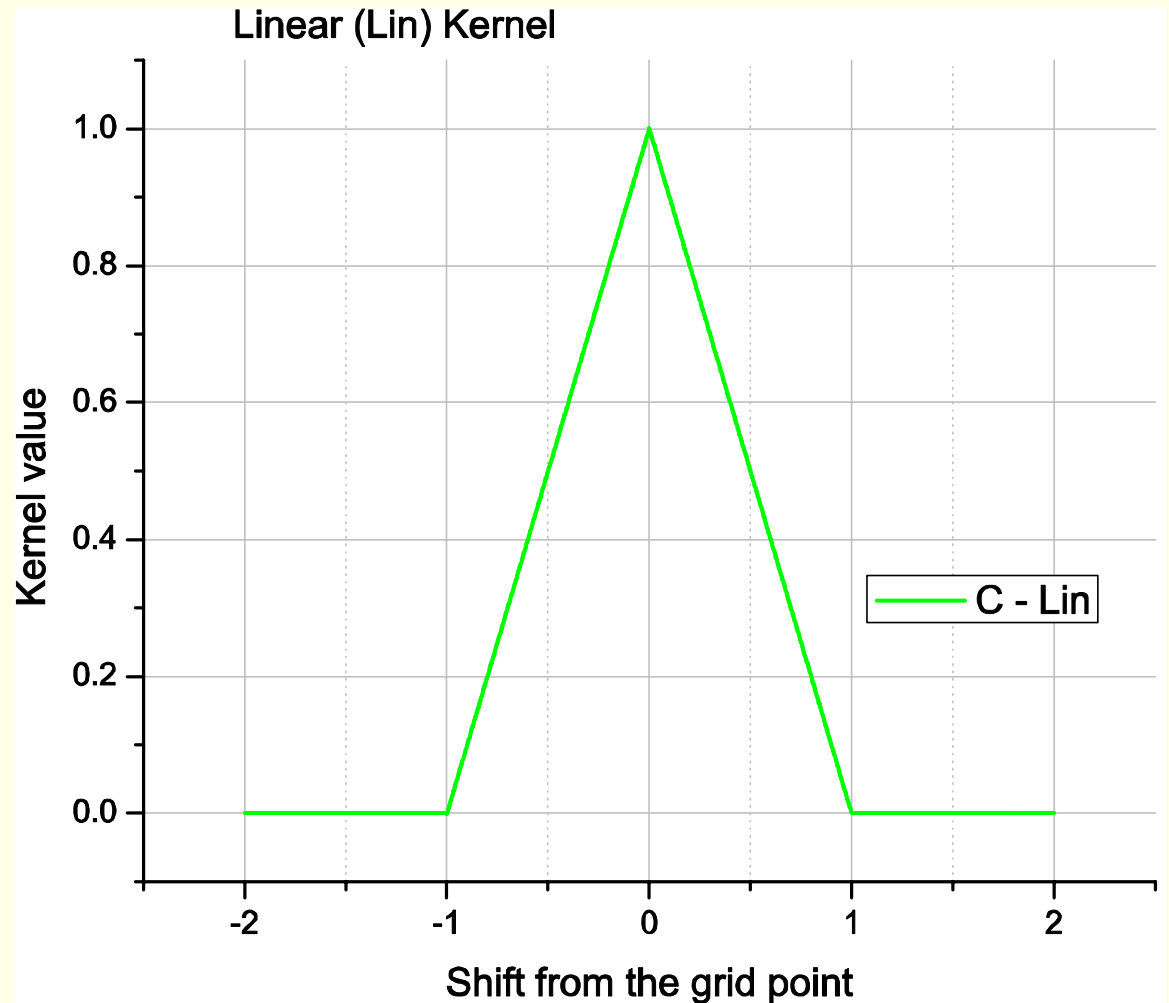
Existing Methods: Nearest Neighbor (NN)

- Simple & fast
- Inaccurate and most violating of sampling theorem
- Bellow, the same voxel is sampled twice despite that the new grid has the same frequency as the old one



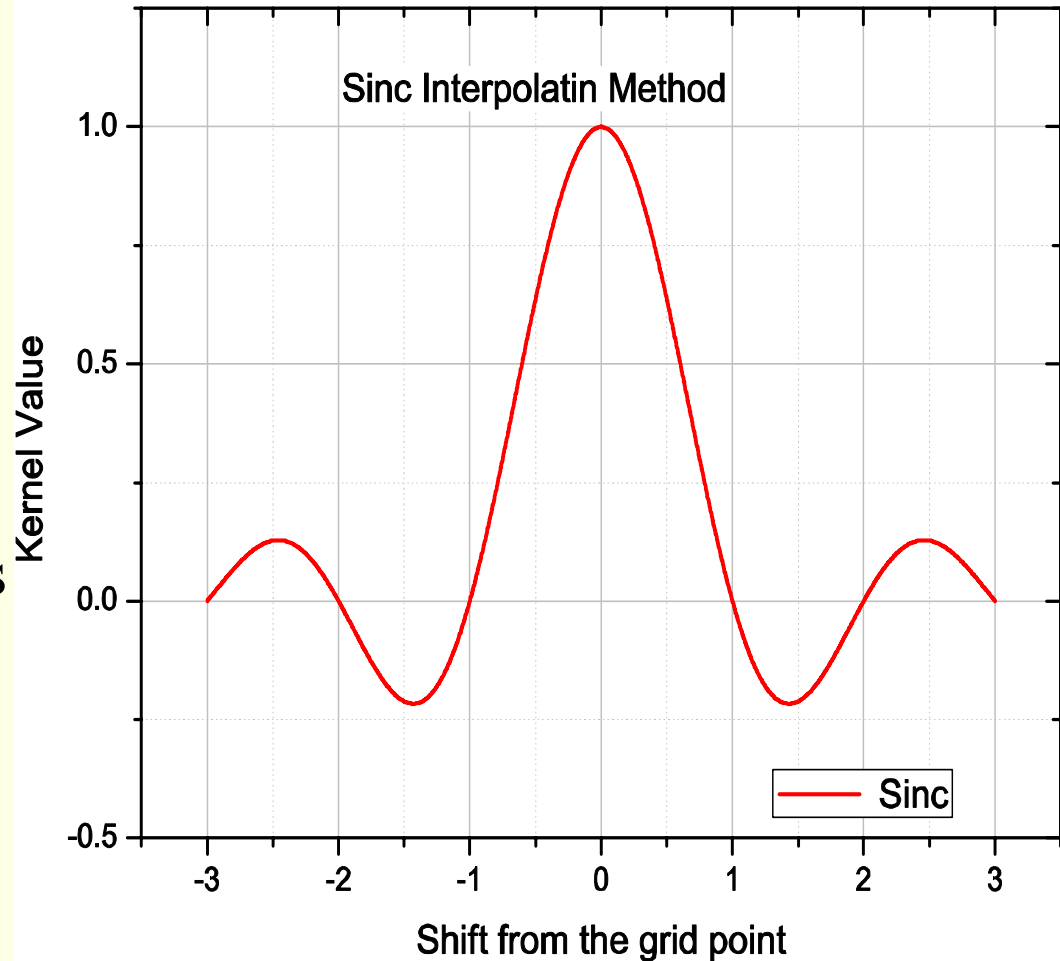
Existing Methods: Linear interpolation (Lin)

- Fast and well tested
- Good accuracy
- Far from perfect in view of sampling theorem



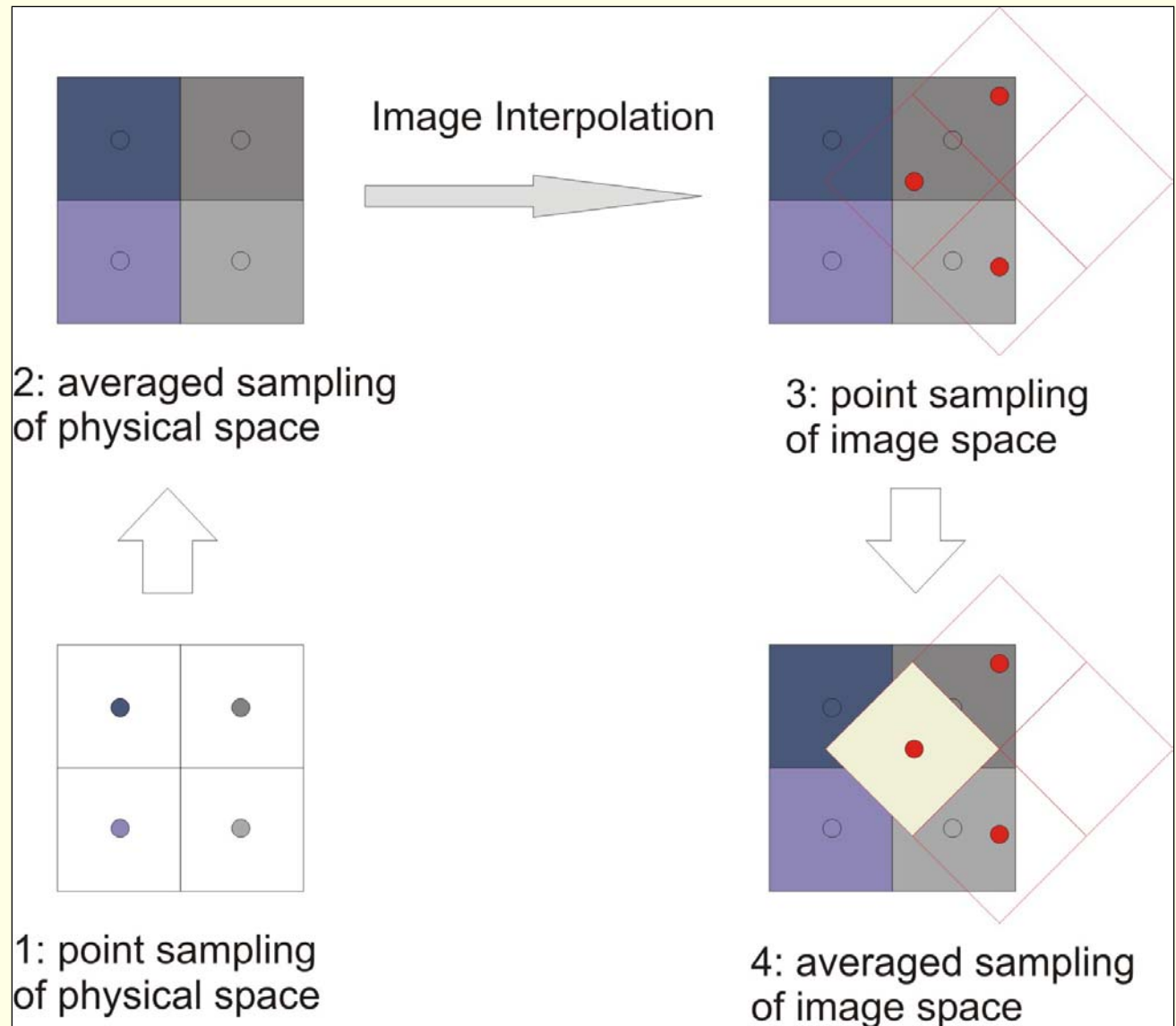
Existing Methods: Sinc

- Sinc function = $\text{Sin}(\pi x) / (\pi x)$
- Expensive computation wise: it includes all points in the image.
- Even if truncated to 3 points on each side it needs $3^3 = 27$ points
- Most compliant with sampling theory as its Fourier transform is like a rectangle function



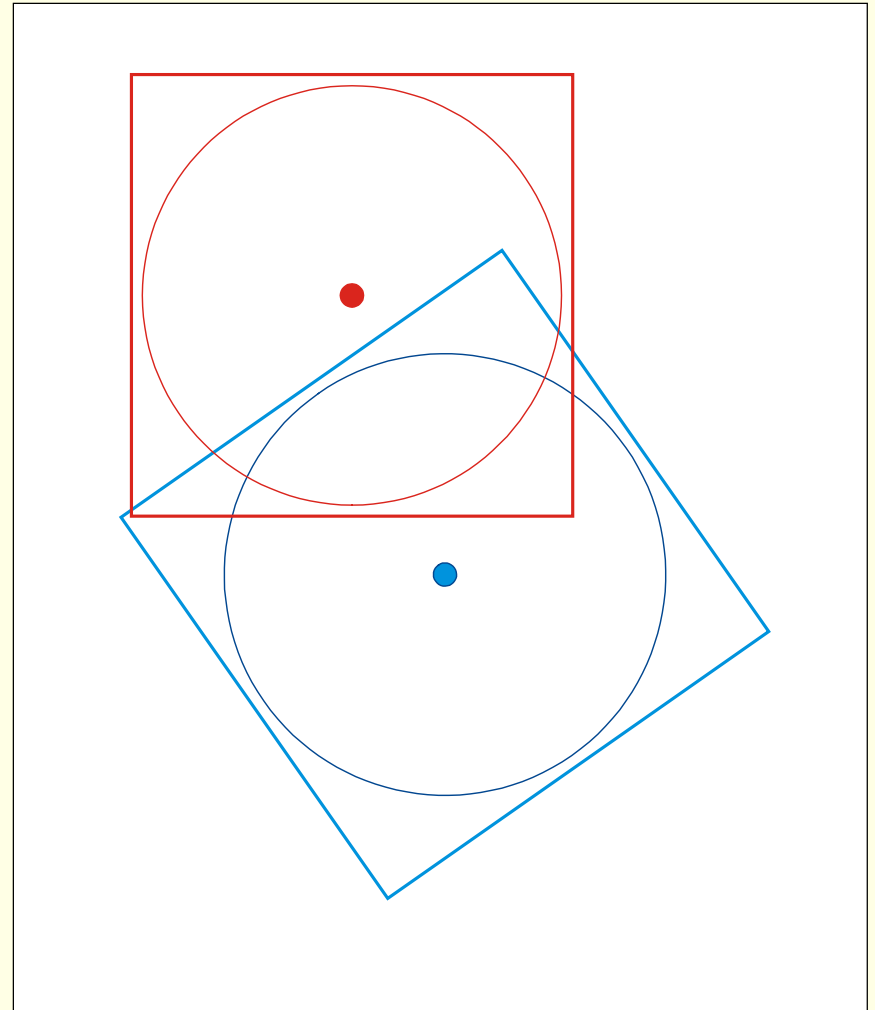
Exact Volume of overlap Motivation

- Image acquisition process from a physical space is more like *averaging* region of space and assigning value rather than *point sampling* of space
- In image interpolation, generating an image from an image space, it is more realistic to average space rather than point-sample it where exact area/volume of overlap is preferred



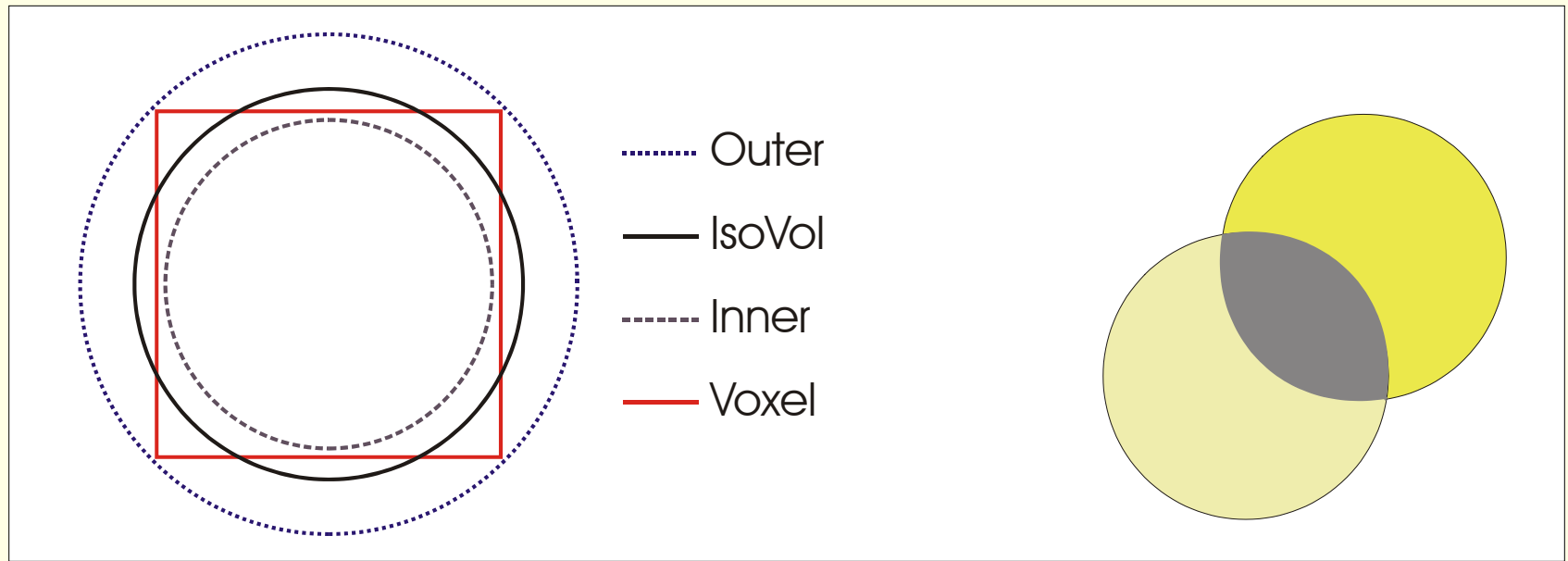
Overlapping Spheres

- The exact area of overlap requires computing the area of the polygon in the adjacent figure
- A simplified way is to compute the overlap area of the two spheres/disks iso-centric to the voxels



Proposed Method: Sphere isocenteric with voxel center

- Partial volume of overlap determines the voxel weight in interpolation
- Weights are summed to unity to preserve the mean brightness
- Volume of overlap is function of distance and radii only
 - *Greatly simplify exact volume of overlap*
 - *No directional dependency*



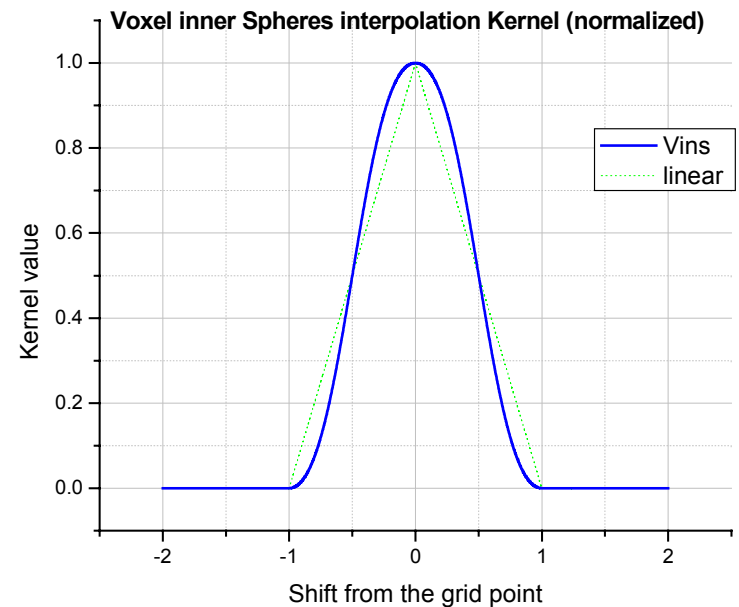
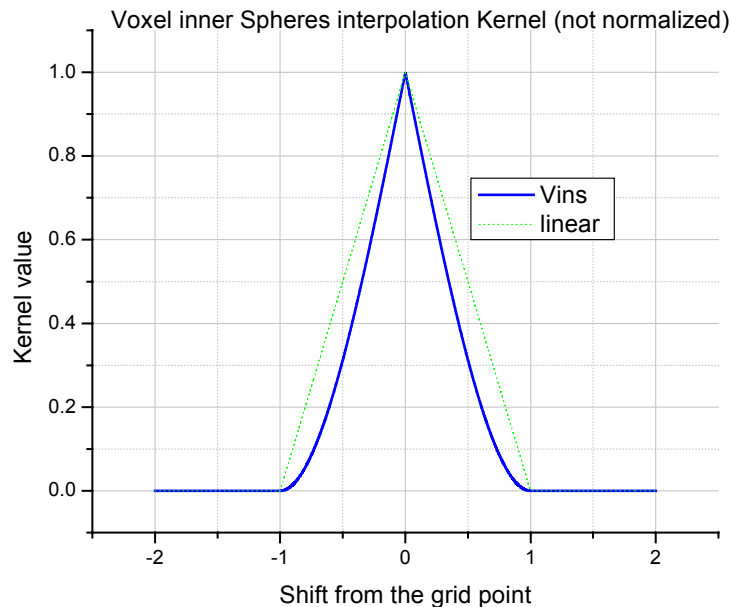
Spheres

- VIVS (Voxel iso-Volumetric Sphere)
 - does blurring and is not desired as interpolator
 - Non inner Spheres may be useful for tunable blurring
higher r for more cross talk or blurring
- VinS (Voxel inner Sphere)
 - is able to reproduce the image faithfully
 - needs to be modified if the voxel is not cubic (kernel type of interpolation)
 - Computing based on Kernel LUT does the trick

Spheres Methods

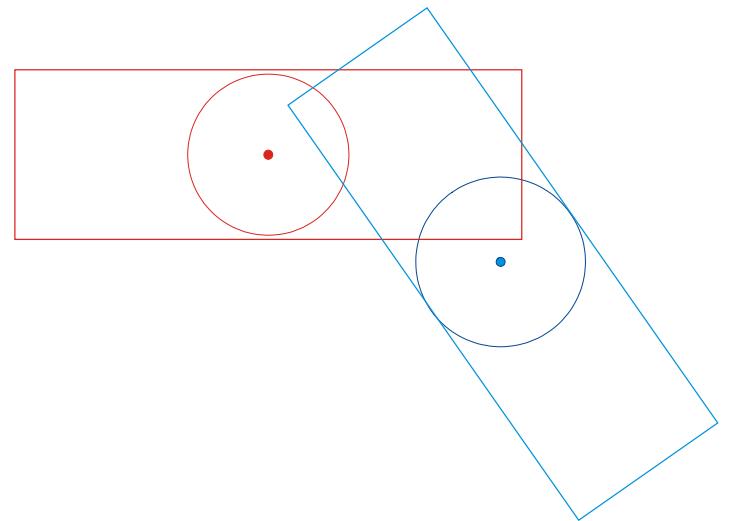
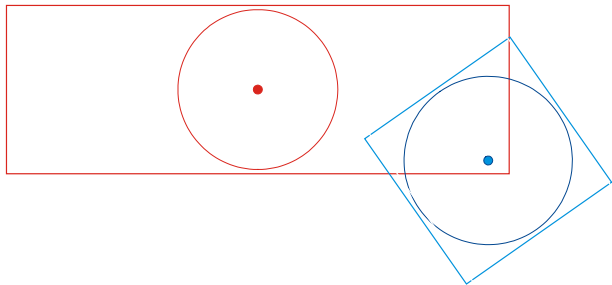
- VIVS: blurry as it causes “cross talk” with neighboring voxels
- VinS: able to reproduce the image with certain rigid transformation (voxel step translations or 90° rotations)

$$V = \pi (R + r - d)^2 \frac{(d^2 + 2dr + 2dR - 3r^2 - 3R^2 + 6Rr)}{12d}$$



Non cubic voxels case

- The inner spheres fail to overlap with non cubic voxels
- Outer spheres will work but at expense of blurring
- A proposed solution is to use a kernel based interpolation:
 - scale shifts in x,y&z to voxel dimension in x,y&z
 - Feed the scaled shift to the kernel's LUT to compute weights



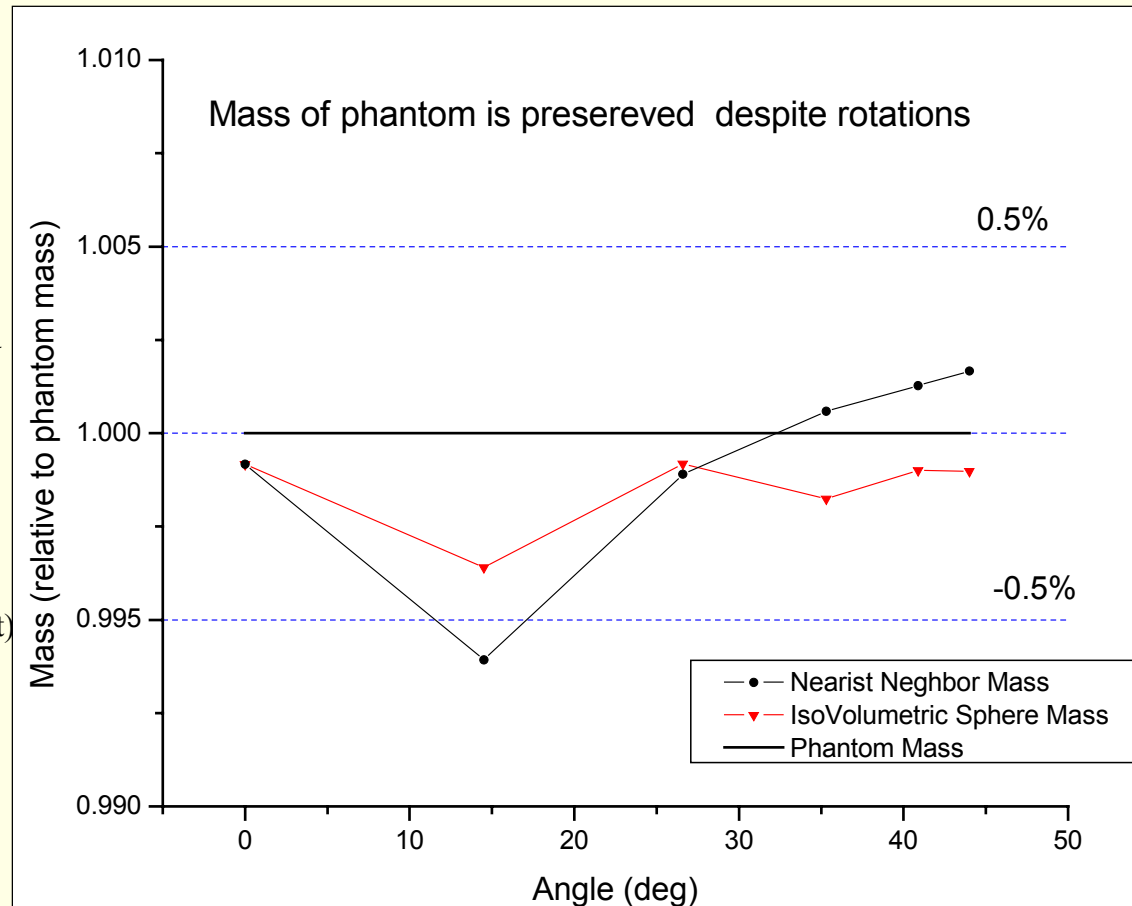
Interpolation: Methods of **Quality Assessment**

1. Basic interpolation requirement:
 - Mean brightness conservation
 - Reproducing the image at neutral transformation or integer voxel shifts or 90° multiple rotations
2. Sampling Theorem
3. Others:
 - Translation vs. MI
 - Rotation vs. MI
 - Visual assessment

Basic interpolation conditions

- The images has been reproduced with
 - $SAD = 0$ (Summation of absolute differences) for:
 - Rotations 0° & no shifts
 - Rotation 90° & no shifts
- Mean brightness* is preserved for several rotations
 - * expressed also as mass

(such method may also be described as DC-constant)



Sampling Theory

- Image is simple collection spatial frequencies with spatial extents
- Image is band limited (i.e. there is a maximum frequency)
 - This is not true physically; there are details at all scales.
 - imaging process (or detector elements) act like a low pass filter and kill higher level frequencies
- If you preserve the frequency content you reproduce the image to finest details with no loss of information

... Sampling Theory

- Taking Fourier transform of the kernel informs us about the frequency effect of the interpolation function
- Formally, the new gray values are computed using old gray values convolved by a kernel function

$$G'(x,y,z) = \text{Kernel} \otimes G(x,y,z) = \iiint \text{Kernel}(x',y',z') G(x-x',y-y',z-z') dx' dy' dz'$$

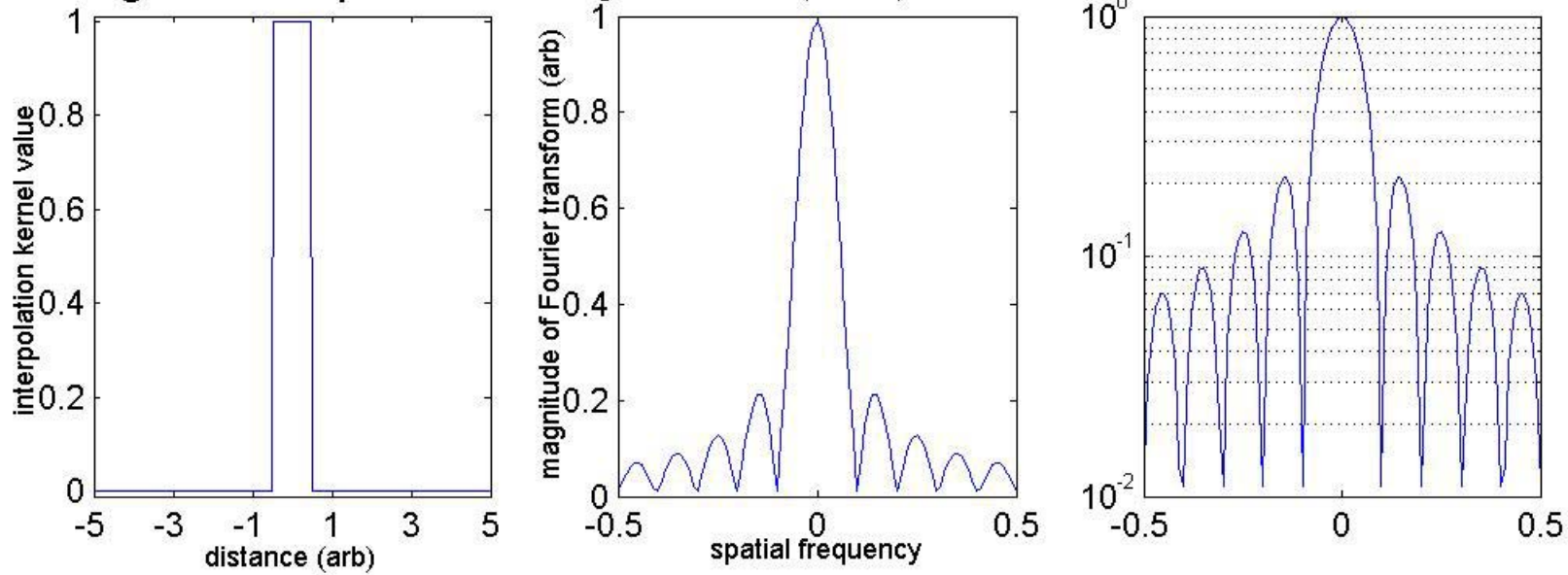
$$\text{FFT}(G'(x,y,z)) = \text{FFT}(\text{Kernel}) * \text{FFT}(G)$$

$$\text{FFT}(\text{Child_image}) = \text{FFT}(\text{Kernel}) * \text{FFT}(\text{Parent_image})$$

Good interpolator

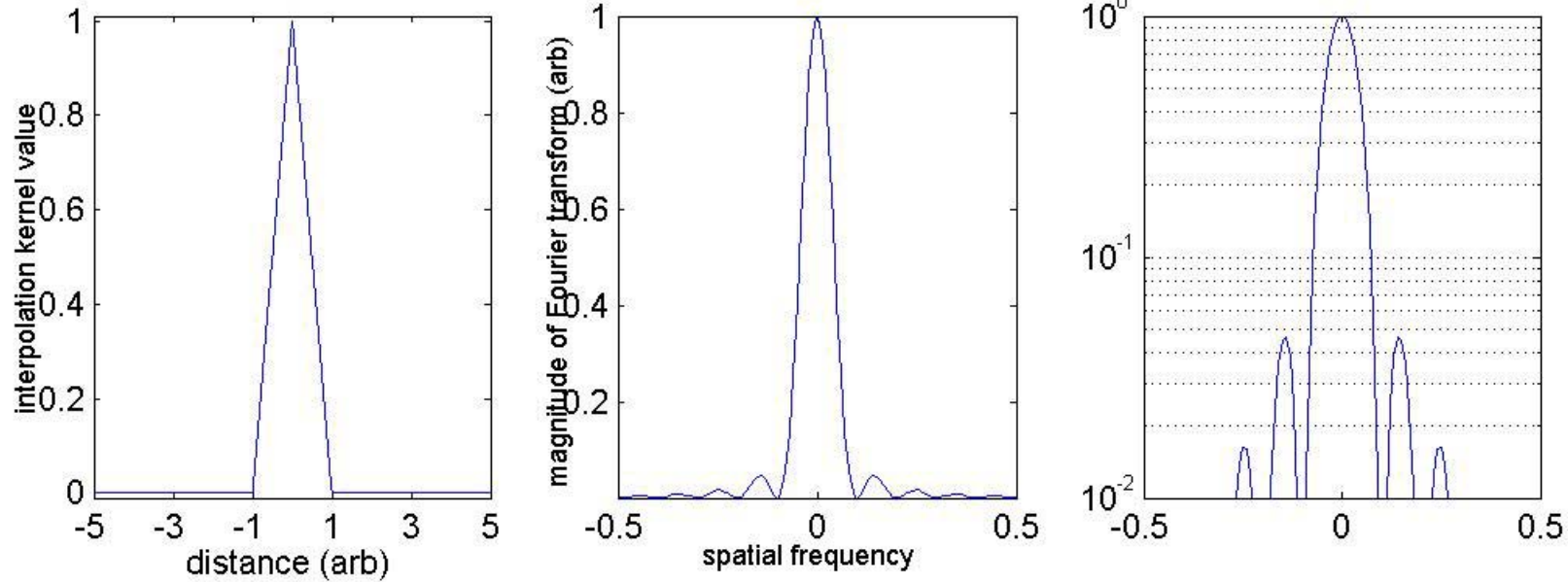
- does not degrade the frequencies within the starting band
 - does not introduce frequencies which were not present
 - does not change the weights among starting frequencies
-
- ➔ sounds like rectangular function in Fourier space (yes)
 - This is why the ideal interpolator is Sinc function
 - Sinc is infinite and takes care of all points in the image. This makes least disturbance to the frequency content. For the same reason, it is the most expensive computationally.

Nearest Neighbor Interpolation : Magnited of FFT(kernel)-linear scale Magnited of FFT(kernel)-log scale

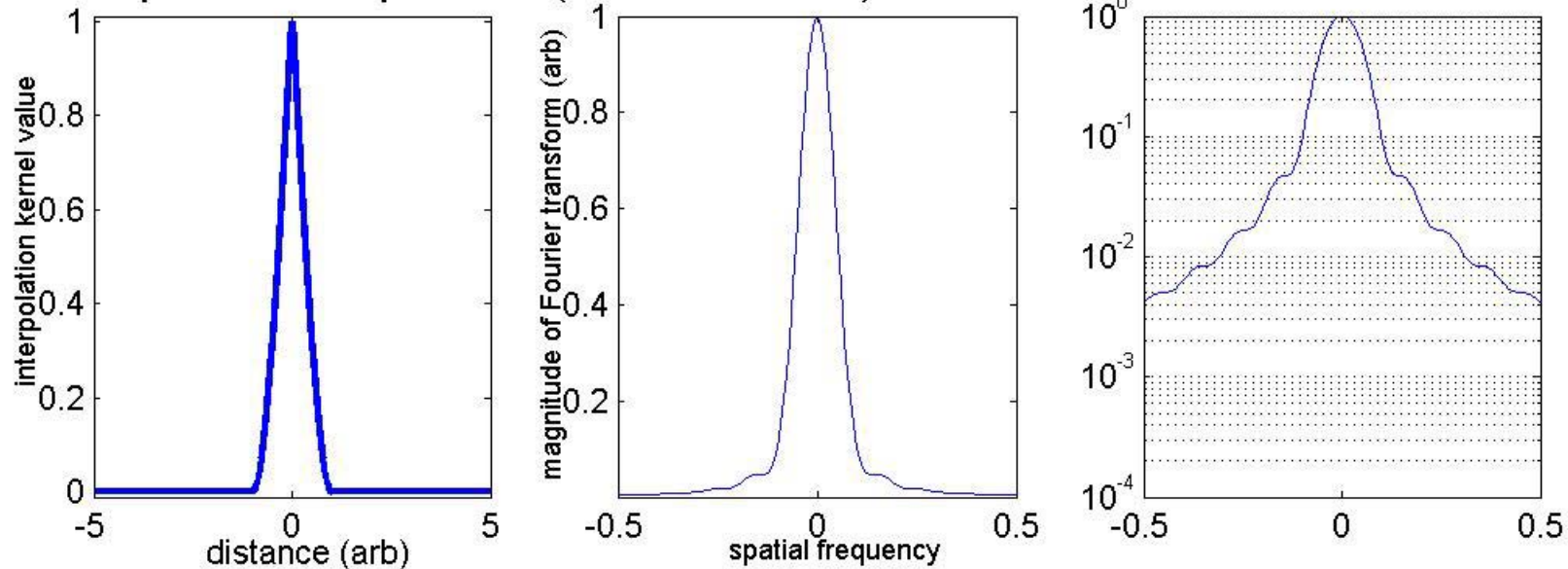


Linear Interpolation :

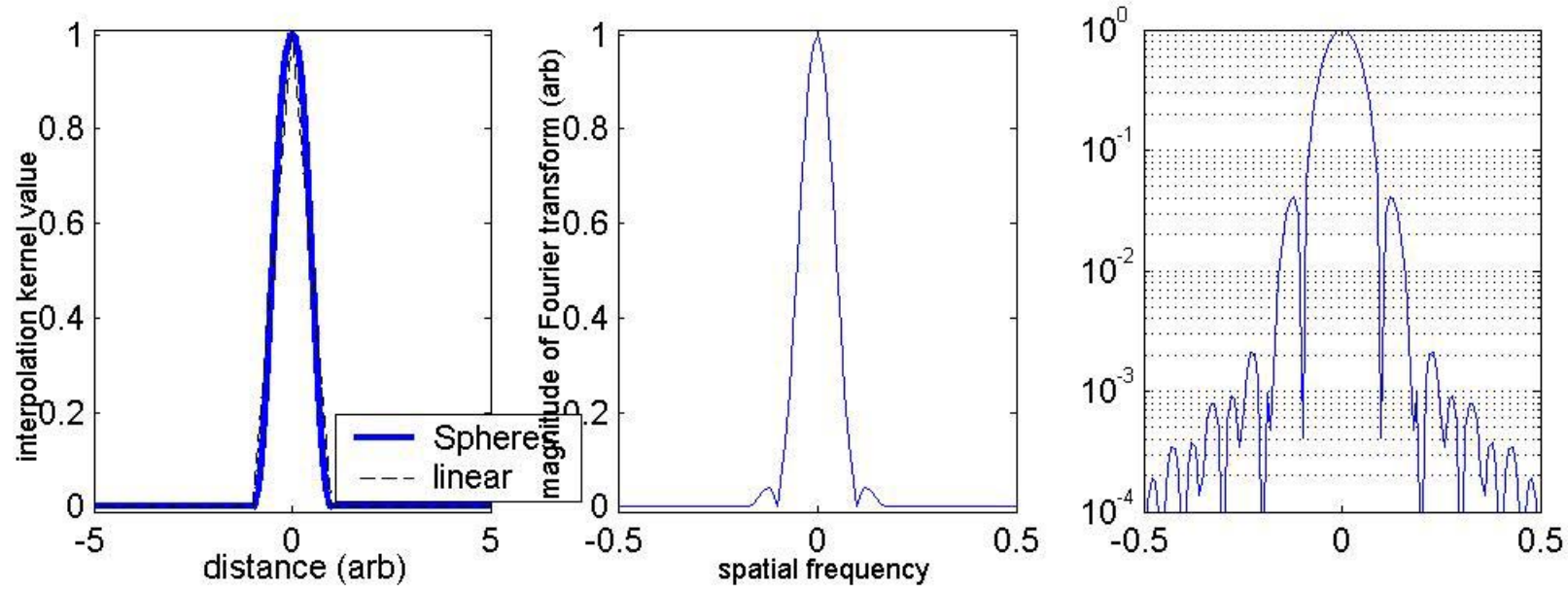
Magnited of FFT(kernel)-linear scale Magnited of FFT(kernel)-log scale

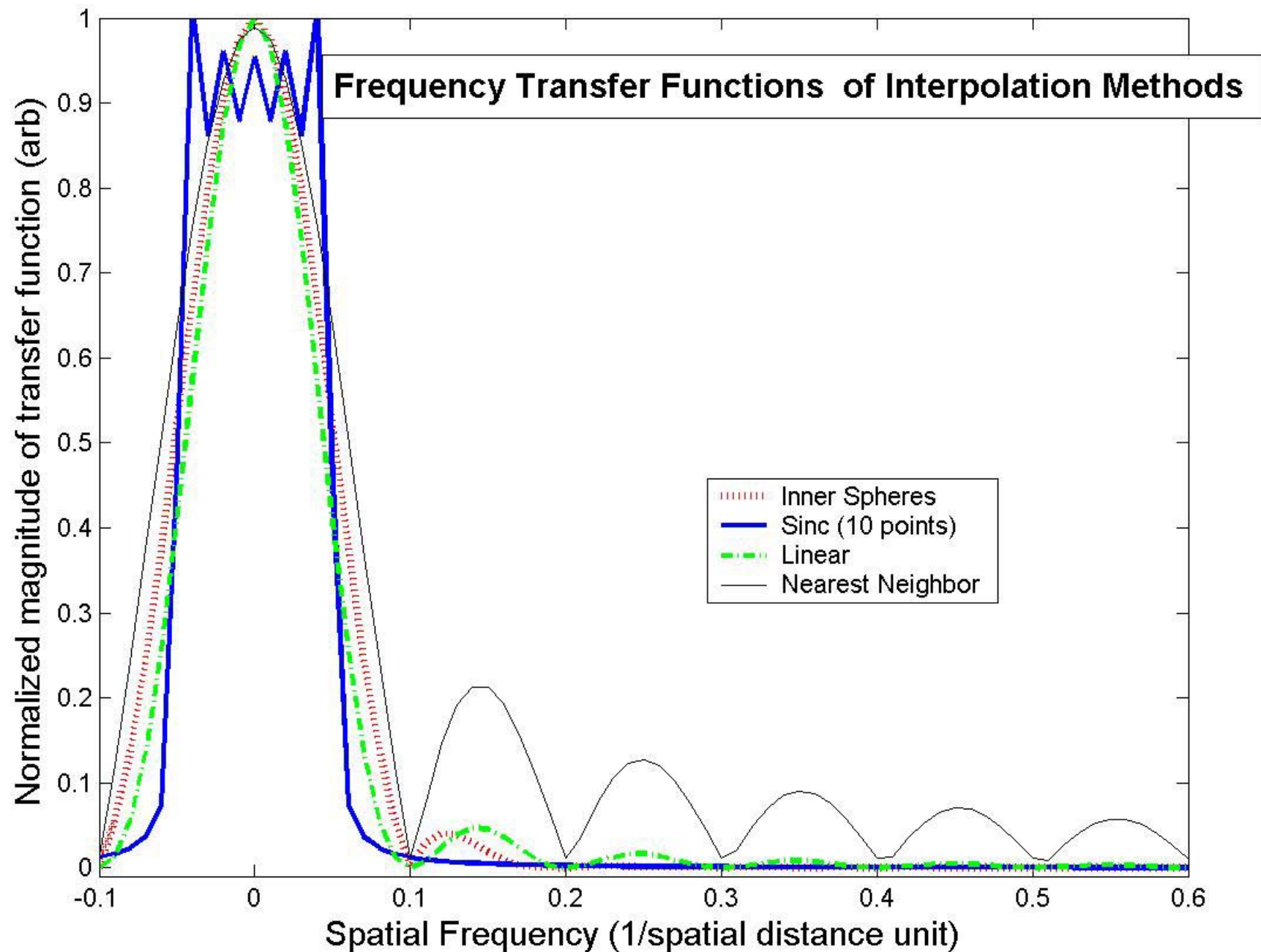


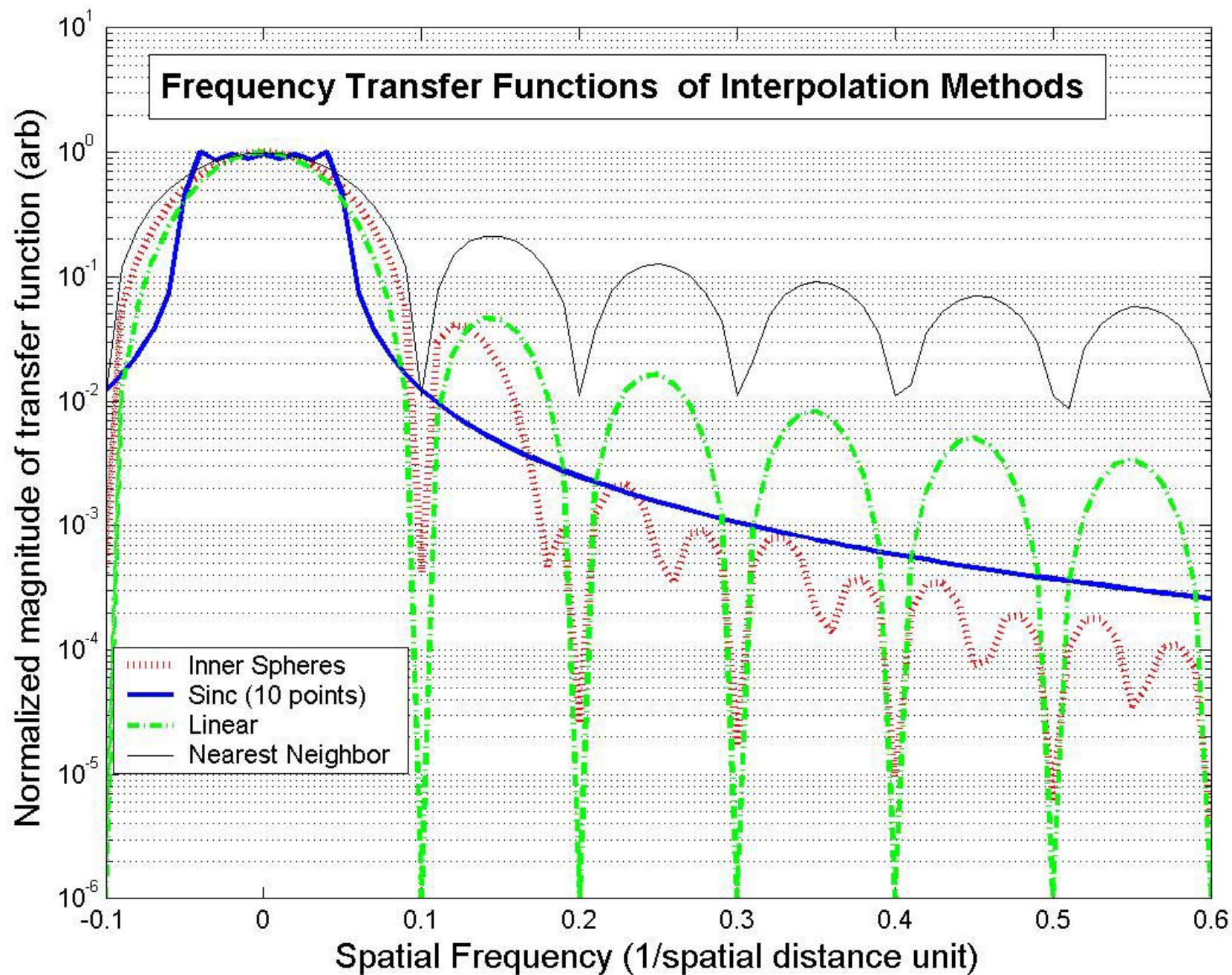
Voxel Inner Spheres Interpolation (not normalized):



Voxel Inner Spheres Interpolation (normalized):





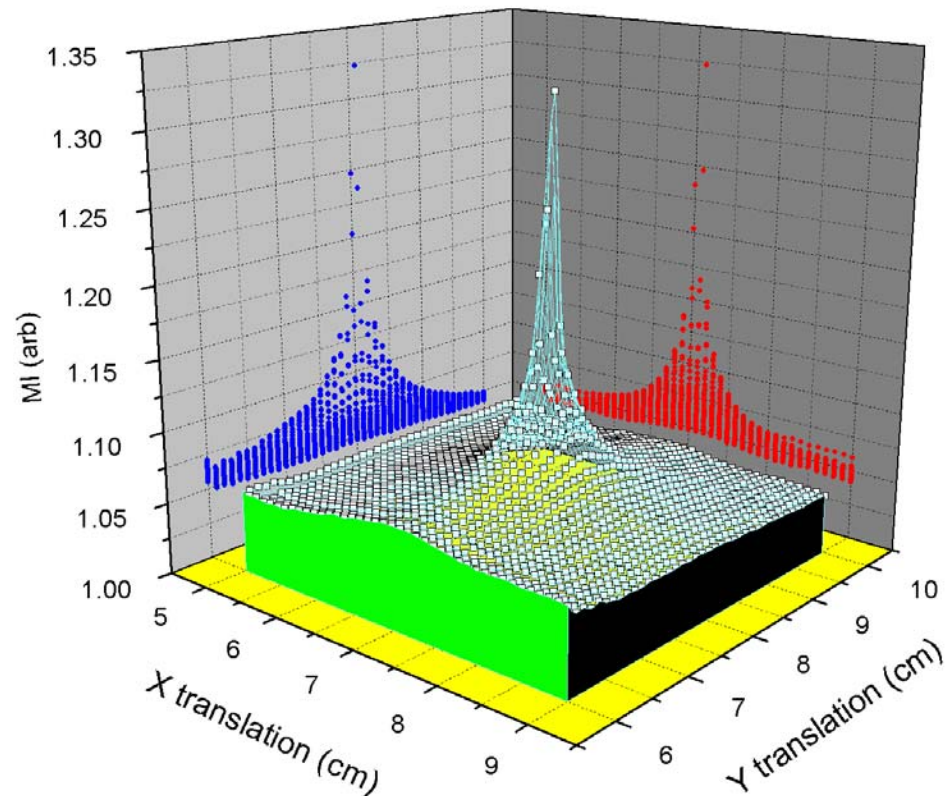


... Comments

- VinS
 - Side loops in FFT or high frequency artifacts are dampened a lot compared to linear
 - Similar to linear in the middle frequency band
 - It has the same spatial extent as linear (only 8 neighboring points are needed, if both radii are equal)

Effect of Translation from the correct position on MI measure using different interpolation methods

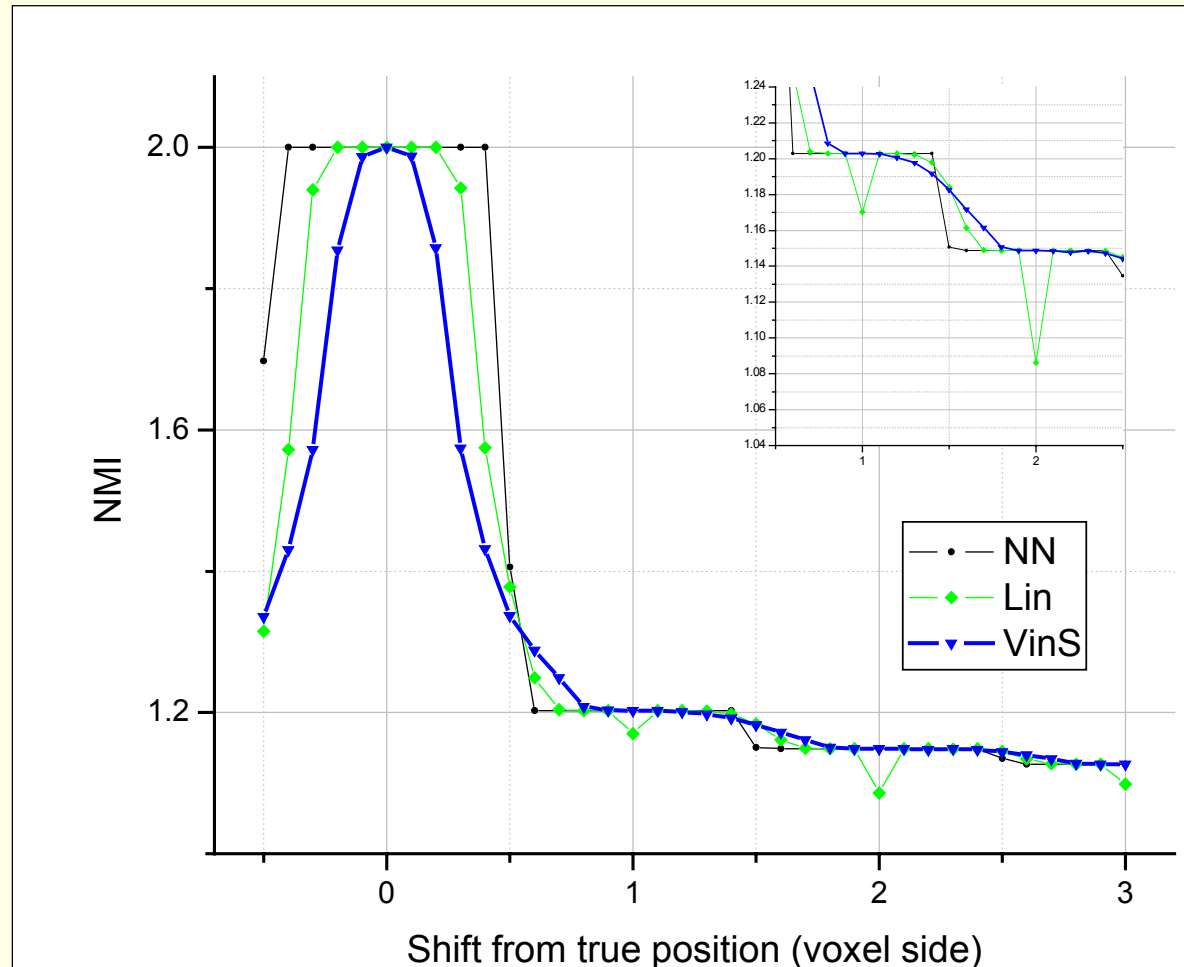
- This image was generated by taking whole voxel steps away from correct position and measuring the change in mutual information measure (MI).
- Notice the sharp increase in MI near the correct position
- Notice also how local minima can prevent simple search algorithms from approaching the right solution



Near the correct position (1/10 voxel side steps)

Observations:

- VinS show smooth drop off with minimal interpolation artifacts
- linear interpolation shows dip in whole voxel shifts



Far from correct position

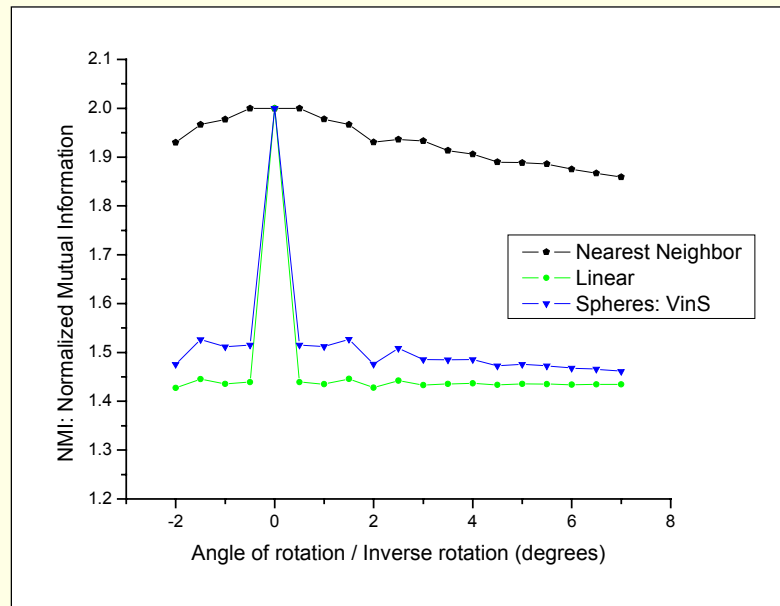


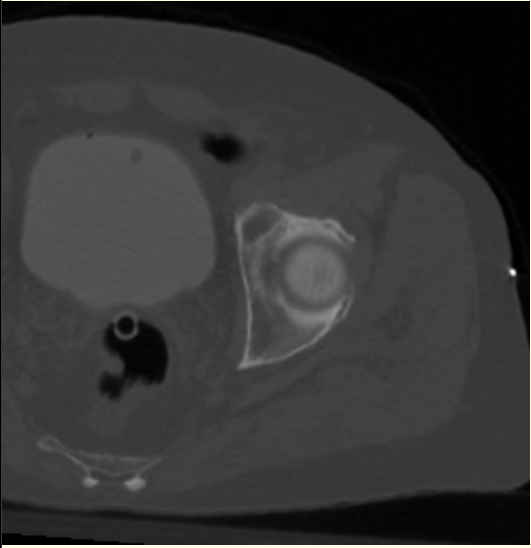
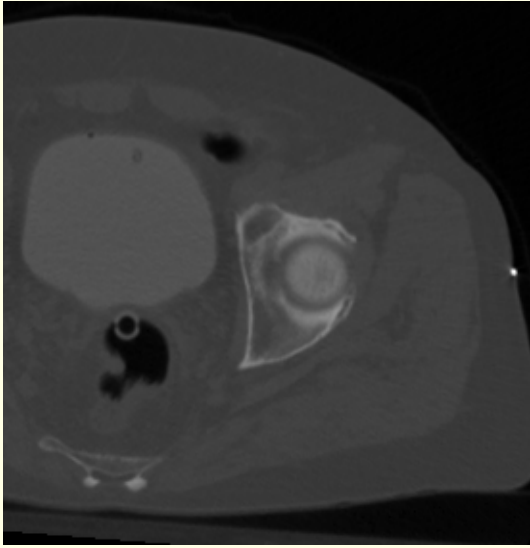
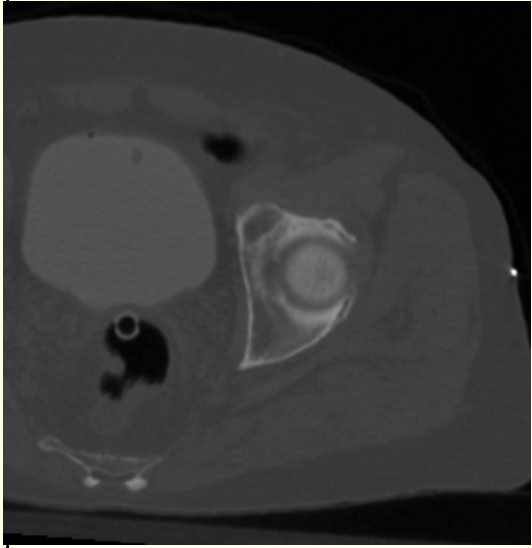

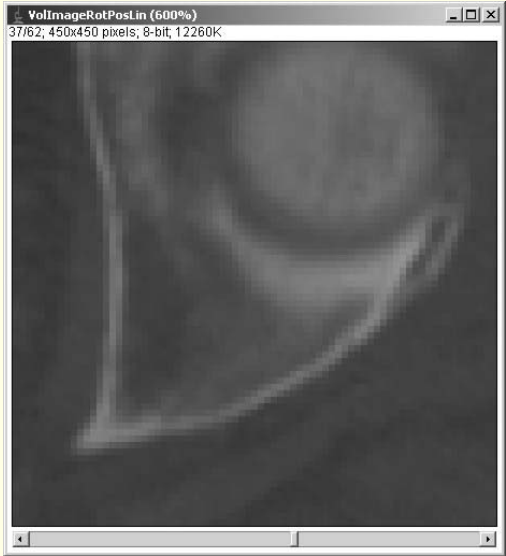
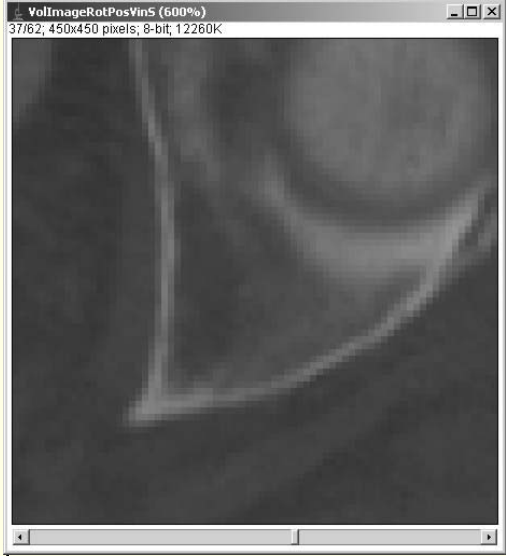
Observations:

- sub voxel accuracy is not achievable and should not be pursued
- VinS shows MI rise in mid-voxel shifts (as some other partial volume interpolators)
- linear interpolation shows dip in whole voxel shifts (also reported in literature)
- VinS interpolation artifacts are much less than linear

Rotation followed by Inverse Rotation study

- *Ideally*, a transformation followed by its inverse should be identity.
- *Practically*, it depends on the interpolator and the transformation
- MI deteriorates with any rotation/inv but keeps the same level of similarity



4.5° Rotation	Nearest Neighbor	Linear	VinS
An inter- polated Slice			
Zoom			

... Comments

- NN retains most similarity because it never generates new gray values. This by no means proves quality as seen in zoomed in images. Notice how NN method introduces steps or high frequency artifacts in other wise smooth CT image
- In medium zooming, linear and spheres interpolations looks quit similar visually.
- Loss of MI with Rot/InvRot is about the same for linear and spheres interpolation

Conclusions

- Reconstructing CT/MR slices that matches tracked US probes could be very useful in interventional surgery or as training tool. Interpolation plays important role in this process. The overlapping spheres method easily estimates the exact volume of overlap with much less computational cost. In addition, this method is better in view of the sampling theorem than linear method although it uses the same number of interpolation points. In translation study, it showed smooth loss of MI which could help search algorithms in registration process.

- **References:**

- T. M. Lehmann, C. Gönnér, and K. Spitzer, “Survey: Interpolation methods in medical image processing,” *IEEE Trans. Med. Imag.*, vol.18, pp. 1049–1075, Nov. 1999.
- J.Pluim et all, “Mutual Information matching and interpolation artifacts” SPIE Medical Imaging 1999, vol 3661
- Dr. J Holden, UW-Madison, Medical Physics; private correspondence.