WAVE MOTION

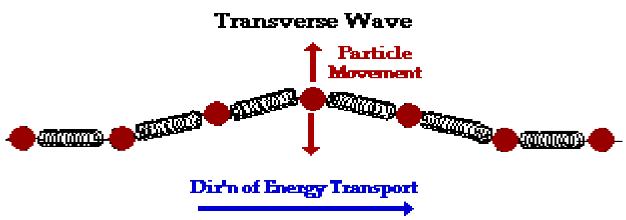
Introduction: Wave is defined as "*A disturbance which propagates* (*transfers energy*) *from one point to other points without giving the medium as a whole any permanent displacement*". A wave carries energy and momentum, but there is no transport of matter. It could be a short pulse or ongoing disturbances. If they occur in a repetitive fashion, they are said to be **periodic**. The main types of waves are:

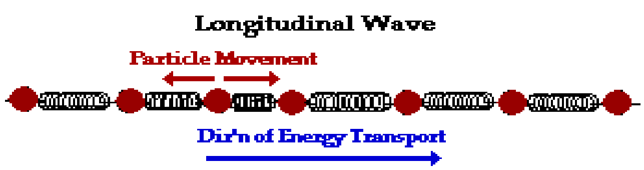
- ***mechanical waves*:** e.g. sound, earthquake, etc., travel only in a medium (solid, liquid, or gas),
- ***electromagnetic waves*:** (material not needed, e.g. light, X-rays, etc.) or
- ***matter waves*:** (e.g. electron, proton, etc.)

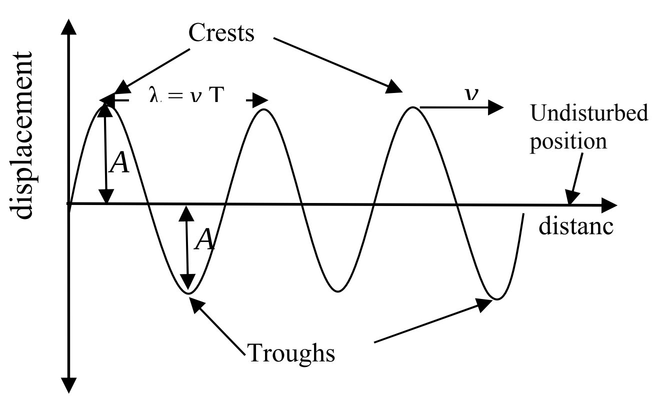
Mechanical waves require: vibrating source (e.g. a pendulum, tuning fork, vibrator,….); medium that can be disturbed; and some physical connection or mechanism through which adjacent positions of the medium can influence each other.

Motion of waves are:

- **Longitudinal:** the oscillating particles of the medium are displaced parallel to the direction of motion (direction of energy transfer). (e.g. sound waves),
- **Transverse:** the oscillating particles of the medium are displaced in a direction perpendicular to the motion of the wave. (e.g. wave on vibrating string, light).
- **Combination** (e.g. earthquake wave, water wave).







Periodic wave are characterized by:

1- **Amplitude** (A, $[A] = \text{m}$): "*the maximum displacement of a particle in a medium on either side of its undisturbed position*". This is the **size** of the wave measured from the middle position.
2- **Wavelength** (λ-the Greek letter lambda, $[\lambda] = m$) "the distance between two successive crests or two successive troughs" (or the distance between two successive points moving at the same phase, i.e. corresponding points on the wave). In term of the wavelength, we can define the wave number "k" as $k = 2\pi/\lambda$, $[k] = \text{rad/m}$.
3- **Periodic Time** (T, $[T]$ = s) "time taken to produce one complete oscillation".
4- **Frequency** ($f = 1/T$, $[[f] = \text{s}^{-1} \equiv \text{Hz}$ (Hertz)) "It is the number of complete waves passing a given point per second". Also, the **angular frequency**, ω, is defined by $\omega = 2\pi f$, and has a units of rad/s.
5- **Wave (Phase) Speed** (v, $[v]$ = m/s) is the rate at which the outline of the wave is traveling in the direction of the wave. It is defined as the product of frequency times the wavelength, or in symbols $v = f\lambda = \omega/k$.

It is important to note that the variables f, λ, ω, and k are all positive quantities.

➔ A frequency vibrator generates a harmonic wave traveling along a rope. It is observed that the wave completes 60 vibrations in 30 seconds. Also, a given crest travels 400 cm along the rope in 10 seconds. Calculate the frequency, the speed of the wave and the wavelength.

✓

$$\text{Frequency: } f = \frac{\text{total number of vibrations}}{\text{total time}} = \frac{60}{30} = \underline{2.0 \text{ Hz}}$$

$$\text{Wave speed: } v = \frac{\text{total length of the rope}}{\text{travelling time}} = \frac{4.0 \text{ m}}{10 \text{ s}} = \underline{0.4 \text{ m/s}}$$

$$\text{Wavelength: } \lambda = \frac{v}{f} = \frac{0.4}{2.0} = \underline{0.2 \text{ m}}$$

➔ In a vibrating string, waves travel a distance l = 45 cm in time t = 3.0 s. If the distance between two successive crests is 3.0 cm, what is the frequency of the vibrator causing the waves?

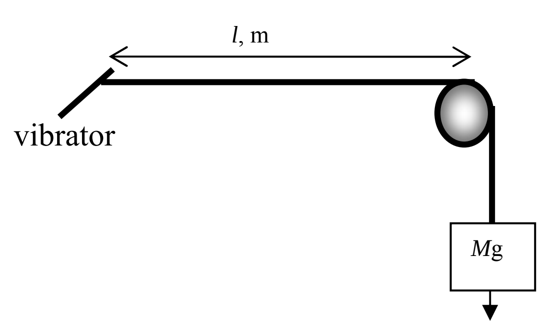
✓

$$\text{Wave length: } \lambda = 3.0 \text{ cm} = \underline{0.03 \text{ m}};$$

$$\text{Wave speed: } v = \frac{l}{t} = \frac{0.45}{3.0} = \underline{0.15 \text{ m/s}}$$

$$\text{Frequency: } f = \frac{v}{\lambda} = \frac{0.45/3.0}{0.03} = \underline{5.0 \text{ Hz}}$$

Wave velocity on string:
The speed of a transverse pulse traveling on a stretched string, of length l and mass m, is given by:



$$v = \sqrt{\frac{F}{\mu}}$$

where F is the tension in the string and $\mu = \frac{m}{l}$ is the linear density of the string (i.e. the mass per unit length). If there is a weight, of mass M, that generates tension, then $F = Mg$.

Note that the linear density could be also related to the density ρ and the cross-sectional area A of a wire by the relation:

$$\mu = \frac{\text{mass}}{\text{length}} = \frac{\text{mass}}{\text{Volume}} \text{Area} = \rho \text{ Area}$$

➔ Determine the speed of transverse waves on a stretched string that is under a tension of 80 N if the string has a length of 2 m and mass of 5 g.

✓ With the given data, one finds

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{80 \text{ N}}{\frac{5.0\times10^{-3} \text{ kg}}{2.0 \text{ m}}}} \approx 179 \text{ m/s.}$$

➔ A 20-m copper wire, with 1-mm diameter and $\rho = 8920 \text{ kg/m}^3$ is stretched to a tension of 150 N. How long will it take a transverse wave to travel the entire length of the wire?

✓ With the given data, one finds $\mu = \rho A = 8920[\pi(\frac{10^{-3}}{2})^2] = 7.0\times10^{-3} \text{ kg/m}$

and, $v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{150 \text{ N}}{7.0\times10^{-3} \text{ kg/m}}} = 146 \text{ m/s.}$

So,

$$t = l/v = 20/146 = 0.137 \text{ s}$$

H.W. A 30-m steel wire, with 1-mm diameter and $\rho = 7860 \text{ kg/m}^3$ is stretched to a tension of 150 N. How long will it take a transverse wave to travel the entire length of the wire? [Answer: 0.192 s.]

One dimensional traveling wave: Mathematically, the displacement (or wave function) y, of a traveling wave with velocity v is represented by:

$$y = f(x \pm vt)$$

where (-) sign describes a wave traveling toward the positive x direction (i.e. to the right) and (+) sign describes a wave moving toward the negative x direction (i.e. to the left). A wave is said to be stationary if it is time independent. The quantity $u = x \pm vt$ is called the phase of the wave.

➔ Which of the following are descriptive of wave traveling in a medium?

$$y_1 = A(x+vt)^2, \quad y_2 = A(x+vt) + B(x-vt)^3,$$
$$y_3 = Ax(b+vt)^2, \quad y_4 = A\sin kx \cos kvt,$$
$$y_5 = Ae^{(x+vt)^2} + B\sin(x+vt)$$

✓ Here, one can see that y_1, y_2 and y_5 are of the form

$$y = f(x \pm vt) + g(x \pm vt)$$

and can describe traveling waves. The functions y_3 and y_4 are not of this form.

Harmonic wave: usually has a sinusoidal shape. The particle's displacement, $y(x, t)$, undergoes a harmonic motion and has the general form:

$$\underbrace{y(x,t)}_{\text{displacement}} = \underbrace{A}_{\text{amplitude}} \underbrace{\sin\underbrace{(\overbrace{k}^{\substack{\text{angular}\\ \text{wave number}\\ =2\pi/\lambda}} x \pm \overbrace{\omega}^{\substack{\text{angular}\\ \text{frequency}\\ =2\pi f}} t - \overbrace{\varphi}^{\substack{\text{phase}\\ \text{constant}}})}_{\text{Phase}}}_{\text{oscillating term}},$$

where the phase constant φ should be in **radians**. The phase velocity "v" is calculated by using the argument $kx \pm \omega t - \varphi =$ constant in which one finds $v = \dfrac{dx}{dt} = \dfrac{\omega}{k}$.

Homework: prove that:

$$y(x,t) = A\sin\left[\frac{2\pi}{\lambda}(x \pm vt) - \varphi\right] = A\sin\left[2\pi\left(\frac{x}{\lambda} \pm f\,t\right) - \varphi\right]$$

➔ A wave in a string, of linear density 0.13 g/m, is given by the equation:

$$y(x,t) = 0.018 \sin(3.0\,x - 240\,t)$$

In SI units. Calculate the tension in the string.

✓

$$v = \sqrt{\frac{F}{\mu}}$$

$$\Rightarrow \quad F = \mu v^2 = \mu\left(\frac{\omega}{k}\right)^2 = 0.13\times10^{-3}\times\left(\frac{240}{3}\right)^2 = \underline{0.83\text{ N}}$$

➔ A transverse harmonic wave in a string is described by:

$$y(x,t) = 3.0 \sin(0.3\,x - 8.0\,t - \varphi),$$

where x and y are in meters and t is in seconds. At $t = 0$ and $x = 0$, a point on the string has a positive displacement = 3.0 m. Calculate the phase constant φ.

✓
$$y(0,0)=3.0\sin(-\varphi)=3.0$$
$$\Rightarrow \quad \sin(-\varphi)=1$$
$$\Rightarrow \quad \varphi=\underline{-\pi/2}$$

Transverse particle's velocity

$$u(x,t)\equiv v_y(x,t)=\frac{dy(x,t)}{dt}$$
$$=\pm A\omega\cos(kx\pm\omega t-\Phi)$$
$$\Rightarrow \quad |u_{\max}|=\underline{A\omega}$$

where $u_{\max}$ is the maximum velocity of the particles in *y*-direction.

Transverse particle's acceleration

$$a_y(x,t)=\frac{du(x,t)}{dt}$$
$$=\mp A\omega^2\sin(kx\pm\omega t-\Phi)$$
$$\Rightarrow \quad |a_{y,\max}|=\underline{A\omega^2}=\underline{\omega|u_{\max}|}$$

where $a_{y,\max}$ is the maximum acceleration of the particles in *y*-direction.

➔ For a certain wave, the maximum transverse velocity is 6.0 m/s, and the maximum transverse acceleration is 12 m/s². Find the amplitude of this wave.

✓ $$\omega=\frac{|a_{y,\max}|}{|u_{\max}|}=\frac{12}{6.0}=2.0\,\frac{\text{rad}}{s}; \quad \Rightarrow \quad A=\frac{|u_{\max}|}{\omega}=\underline{3.0\text{ m}}$$

➔ For the wave

$$y(x,t)=0.2\ \sin(0.1\,x+120.0t+0.4),$$

where *x* and *y* are in cm and *t* is in seconds. Calculate the amplitude, wavelength, frequency, speed, maximum speed, and maximum acceleration.

✓ Compare with the general expression of the wave equation one can find:

Amplitude: $A=0.2\text{ cm }=\underline{0.2\times10^{-2}\text{ m}}$;

Wavelength: $\lambda=\frac{2\pi}{k}=\frac{2\pi}{0.1}=\underline{0.63\text{ m}}$;

Frequency: $f=\frac{\omega}{2\pi}=\frac{120}{2\pi}=\underline{19.10\text{ s}^{-1}}$;

speed: $v=\lambda f=\underline{12.00\text{ m/s}}$;

Maximum speed: $v_{y,\max}=A\omega=\underline{0.24\text{ m/s}}$;

Maximum acceleration: $a_{y,\max}=A\omega^2=\underline{28.8\text{ m/s}^2}$;

The energy (E , $[E]$ = J), associated with a segment of a string, of length Δl and mass Δm, moving with simple harmonic motion (SHM) is:

$$\Delta E = \frac{1}{2}kA^2 = \frac{1}{2}(\Delta m\omega^2)A^2 = \frac{1}{2}\mu\Delta l(\omega A)^2$$

Note that: Using Newton's law for SHM:

$$\because F = -kx,$$

$$\therefore ma = -kx \quad \Rightarrow \quad a = -\frac{k}{m}x = -\omega^2 x, \qquad \omega = \sqrt{\frac{k}{m}}.$$

Average Power ($P_{\text{ave}} = \dfrac{dE}{dt}$, $[P] = \text{W} = \dfrac{\text{J}}{\text{s}}$) transmitted by a harmonic wave on a string is given by:

$$P_{\text{ave}} = \frac{1}{2}\mu\,\omega^2 A^2 v$$

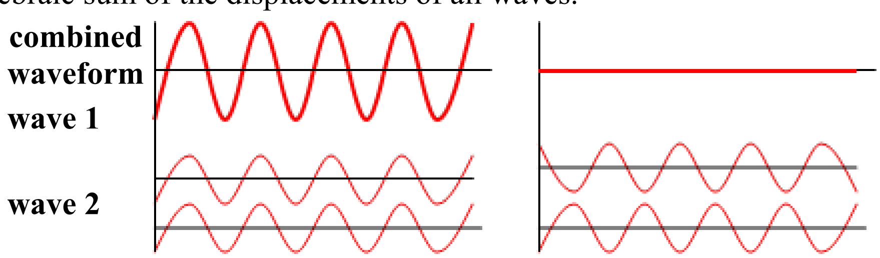
➔ A sinusoidal wave, given by the equation:

$$y(x,t) = 0.07\ \cos(6.0\,x\ -\ 30\,t\,)$$

where x and y are in meters and t is in seconds, is moving in a string of linear density□ = 1.2 g/m. At what rate is the energy transferred by the wave?

✓ $$P_{\text{ave}} = \frac{1}{2}\mu\,\omega^2 A^2 v = \frac{1}{2}(1.2\times10^{-3})(30)^2(0.07)^2(\frac{30}{6}) = \underline{1.32\times10^{-2}\ \text{W}}\,.$$

Superposition of harmonic waves: The net displacement (the resultant) of two or more waves equals the algebraic sum of the displacements of all waves.



Two waves in phase **Two waves 180° out of phase**

H.W. For the two identical traveling waves (except for phase differences φ):

$$y_1 = A\sin(kx-\omega t); \qquad y_2 = A\sin(kx-\omega t-\varphi)$$

use the identity $\sin a + \sin b = 2\sin(\frac{a+b}{2})\cos(\frac{a-b}{2})$) to prove that the resultant wave function is:

$$y = y_1 + y_2 = [\,\underbrace{2A\cos(\frac{\varphi}{2})}_{\text{resultant amplitude}}\,]\underbrace{\sin(kx-\omega t-\frac{\varphi}{2})}_{\text{oscillating term}}$$

➔ Two identical sinusoidal waves, are out of phase with each other, travel in the same direction. They interfere and produce a resultant wave given by the equation:

$$y(x,t) = 8.0\times10^{-4}\sin(4.0\,x - 8.0\,t + 1.57),$$

where x and y are in meters and t is in seconds. What is the amplitude of the two interfering waves?

✓ $$2A\cos(\frac{\varphi}{2}) = 8.0\times10^{-4} \quad\Rightarrow\quad A = \frac{8.0\times10^{-4}}{2\times\cos(1.57)} = \underline{0.5\text{ m}}.$$

Interference: *"superposition of two or more waves"* .

Resultant Amplitude

$$\overbrace{\qquad\qquad\qquad\qquad\qquad\qquad}^{(2A\cos(\frac{\varphi}{2}))}$$

fully constructive (amplitude is maximum)	fully destructive (amplitude is 0)
$\because \cos(\frac{\varphi}{2}) = \pm1$	$\because \cos(\frac{\varphi}{2}) = 0$
$\therefore \frac{\varphi}{2} = 0, \pm\pi, \pm2\pi, \ldots$	$\therefore \frac{\varphi}{2} = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$
$\Rightarrow \varphi = \pm n\pi, \quad n$ is even	$\Rightarrow \varphi = \pm n\pi, \quad n$ is odd

Standing Waves: *"superposition of two **identical waves*** (*same: frequency, amplitude and wavelength*) *moving in **opposite** directions*".

Standing wave is composed of nodes (i.e. amplitudes is zero) and antinodes (i.e. amplitude is maximum) with:

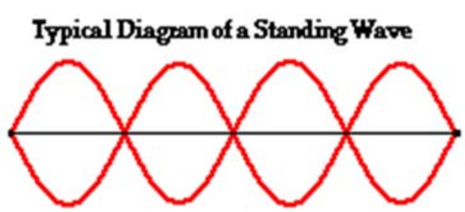
1- the distance between two successive nodes (or antinodes) = λ/2, and
2- the distance between two successive nodes and antinodes = λ/4.

Homework For the two opposite and equal waves:

$$y_1 = A\sin(kx - \omega t); \qquad y_2 = A\sin(kx + \omega t),$$

prove that the resultant wave is:

$$y = y_1 + y_2 = [\underbrace{2A\sin(kx)}_{\text{resultant amplitude}}]\ \underbrace{\cos(\omega t)}_{\text{oscillating term}}$$



$n = 4$

Resultant Amplitude

$$\overbrace{(2A\sin(kx)), \quad (k = \frac{2\pi}{\lambda})}$$

Anti-nodes

(maximum amplitude = $2A$)

$$\because \sin(kx) = \pm 1$$

$$\therefore kx = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \dots$$

$$= \pm n\frac{\pi}{2}, \quad n \text{ is odd}$$

$$\therefore x = \pm n\frac{\lambda}{4}, \quad n \text{ is odd}$$

or

$$x = \pm(n + \frac{1}{2})\frac{\lambda}{2}, \quad n = 0, 1, 2, \dots$$

nodes

(zero amplitude)

$$\because \sin(kx) = 0$$

$$\therefore kx = 0, \pm\pi, \pm 2\pi, \dots$$

$$= \pm n\frac{\pi}{2}, \quad n \text{ is even}$$

$$\therefore x = \pm n\frac{\lambda}{4}, \quad n \text{ is even}$$

or

$$x = \pm n\frac{\lambda}{2}, \quad n = 0, 1, 2, \dots$$

Resonances: For a string, with length L and fixed at both ends with a tension F, one can find

$$L = n\frac{\lambda}{2} \quad \Rightarrow \lambda = \frac{2L}{n}, \quad n = 1, 2, 3, \dots$$

then

$$f_n = \frac{v}{\lambda} = n\frac{v}{2L} = \frac{n}{2L}\sqrt{\frac{F}{\mu}}, \qquad n = 1, 2, 3, \dots$$

where n is the number of loops (segments). n takes the values: 1 for the fundamental (first harmonic) , 2 for the second harmonic, 3 for the third harmonic, etc.

Definition of the harmonic: Fundamental = first harmonic, and so on

Fundamental tone: the string vibrates as one segment

$$n=1 \qquad L=\frac{1}{2}\lambda \quad \therefore \lambda = 2L \qquad \Rightarrow \qquad f_1=\frac{1}{2L}v$$

Second harmonic: the string vibrates as two segments

$$n=2 \qquad L=\frac{2}{2}\lambda \quad \therefore \lambda = \frac{2}{2}L \qquad \Rightarrow \qquad f_2=\frac{2}{2L}v = 2f_1$$

Third harmonic: the string vibrates as three segments

$$n=3 \qquad L=\frac{3}{2}\lambda \quad \therefore \lambda = \frac{2}{3}L \qquad \Rightarrow \qquad f_3=\frac{3}{2L}v = 3f_1$$

Fourth harmonic: the string vibrates as three segments

$$n=4 \qquad L=\frac{4}{2}\lambda \quad \therefore \lambda = \frac{2}{4}L \qquad \Rightarrow \qquad f_4=\frac{4}{2L}v = 4f_1$$

In general:

$$f_1 : f_2 : f_3 : f_4 : \cdots f_n$$
$$1 : 2 : 3 : 4 : \cdots n$$

➔ A standing wave is established in a 15 cm long string fixed at both ends, see figure (1). If the string vibrates in three segments and the wave speed is 100 m/s, what is the frequency?

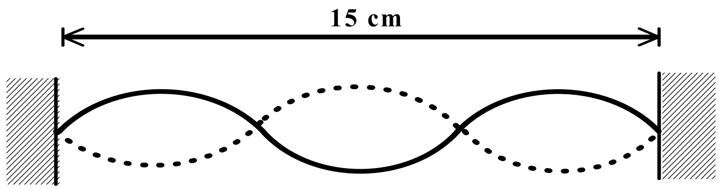


Fig. (1)

✓ First, from the figure, calculate the wavelength:

$$\frac{\lambda}{2} = \frac{15 \times 10^{-2}}{3} \quad \Rightarrow \quad \lambda = 0.1 \text{ m}.$$

So, the frequency will be

$$f = v/\lambda = 100/0.1 = \underline{1000 \text{ Hz}}.$$

➔ A string has linear density = 5.1 g/m and is under a tension of 120 N. If the vibrating length of the string is 60 cm, what is the lowest resonant frequency?

➔

✓

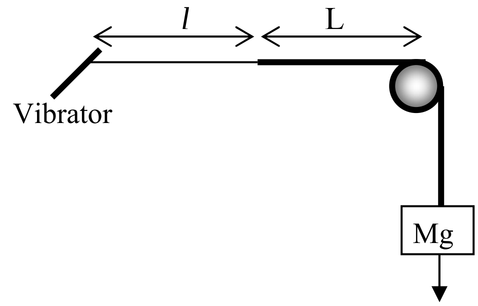
$$f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}} = \frac{1}{2 \times 0.6}\sqrt{\frac{120}{5.1 \times 10^{-3}}} = \underline{128 \text{ Hz}}$$

➔ The lowest resonant frequency in a certain string clamped at both ends is 50 Hz. When the string is clamped at its midpoint, the lowest resonant frequency is:

✓

$$f \, \alpha \frac{1}{L} \quad \Rightarrow \quad \frac{f_1}{f_2} = \frac{L_2}{L_1} = \frac{L/2}{L}$$

$$\Rightarrow \quad f_2 = 2f_1 = 2 \times 50 = \underline{100 \text{ Hz}}$$



➔ An aluminum wire of length l = 0.60 m and cross sectional area 1.0×10^{-6} m^2 is connected to a steel wire of the same cross-sectional area. The compound wire is loaded with a block M of mass 10 kg. The length of the steel wire L = 0.87 m. Transverse waves are set up in the compound wire by means of a vibrator of variable frequency. Find the lowest frequency of excitation for which standing waves are observed such that at the connection point is a node.
(Given: ρ(Aluminum) = 2.60 g/cm^3, ρ(Steel) = 7.80 g/cm^3).

✓ Use the relation $v=\sqrt{\dfrac{F}{\mu}}=\sqrt{\dfrac{F}{\rho A}}$,

For aluminum: $f_{al}=\dfrac{n_{al}}{2\times 60}\sqrt{\dfrac{F}{A\times 2.60}}$;

For steel: $f_{st}=\dfrac{n_{st}}{2\times 87}\sqrt{\dfrac{F}{A\times 7.8}}$;

Equating the above frequencies (because they have a common vibrator) we can get:

$$\frac{n_{st}}{n_{al}}=\frac{5}{2}$$

Thus for standing waves, there are two loops (one node) in the aluminum wire and five loops (4 nodes) in the steel wire, with a node at the junction point.
Because the frequency in both wires is the same, the lowest frequency is:

$$f_{al}=\frac{2}{2\times 60}\sqrt{\frac{10\times 9.8}{A\times 2.60}}=323 \text{ Hz}$$

So, the total number of nodes formed at this frequency, excluding those at the ends, is 6.

➔ The wave function for a linearly polarized wave on a taut string is (in SI units)

$$y(x,t) = 0.35\,\sin(10\pi t - 3\pi x + \frac{\pi}{4})$$

(a) What is the velocity of the wave? (give the speed and the direction.),

✓ Compare the wave equation with the general one [use $\sin(-y) = -\sin(y)$],

$$y(x,t) = -0.35\,\sin(3\pi x - 10\pi t + \frac{\pi}{4})$$

we have $k = 3\pi \text{ m}^{-1}$, $\omega = 10\pi \text{ s}^{-1}$, $A = 0.35$ m, $\varphi = \pi/4$, consequently $v = \frac{\omega}{k} = 3.33 \text{ ms}^{-1}$ and the wave moves in the positive x-direction.

(b) What is the displacement at $t = 0$, $x = 0.1$ m?

✓ $$y\,(0.1, 0.0) = -0.35\,\sin(3\pi \times 0.1 + \frac{\pi}{4}) = \underline{0.06 \text{ m}}$$

(c) What is the wavelength and what is the frequency of the wave?

✓ $$\lambda = \frac{2\pi}{k} = \underline{0.67 \text{ m}}, \quad f = \frac{\omega}{2\pi} = \underline{5.0 \text{ Hz}}.$$

(d) What is the maximum magnitude of the transverse velocity of the string?

✓ $$v_{\max} = A\omega = \underline{11 \text{ m/s}}$$

True-False Questions

1. A wave transports both energy and mass. **F** (only energy)
2. Sound wave is a mechanical wave. **T**
3. A wave is a motion of a disturbance. **T**
4. Wavelength is the minimum distance between any two points on a wave that behave identically. **T**
5. A Wave on a stretched string is a longitudinal wave. **F**
6. The expression $y = A\cos(-kx + \omega t)$ represents a wave traveling in the negative x direction. **F**
7. The frequency of a wave is independent of its amplitude. **T**
8. When a wave travels from one medium to another its frequency does not change. **T**
9. If one doubles the amplitude of a wave, the energy carried by the wave will also be doubled. **F**
10. Changing the length of the string does not affect the power transmitted. **T**
11. In standing waves, the distance between two successive nodes = $\lambda/2$. **T**
12. In standing waves, the distance between two successive antinodes = $\lambda/2$. **T**
13. In traveling waves, the distance between two successive nodes = $\lambda/2$. **F**
14. If a wave on a string is reflected from a fixed end, then the reflected wave is 180^o out of phase with the original wave at the fixed end. **T**
15. The power transmitted by a sinusoidal wave on a string does not depend on the length of the string. **T**
16. The power transmitted by a sinusoidal wave on a string does not depend on the tension in the string. **F**

Supplementary Problems

- The velocity of a traveling wave on a string under fixed tension

(a) increases when the frequency increases
(b)@ does not change when the frequency increases
(c) decreases when the wave length increases
(d) decreases when the amplitude increases
(e) decreases when the frequency increases

- A sinusoidal transverse wave travels along a string. The time for a particle on the string to move from maximum displacement to zero displacement is 0.170 seconds. What is the frequency of the source?

(a) 8.96 Hz
(b) 5.88 Hz
(c) 2.94 Hz
(d)@ 1.47 Hz
(e) 23.3 Hz

- A sinusoidal wave is described as:

$$y(x,t) = 0.1\ \sin\left[10\pi(x/5 + t - 3/2)\right] \text{ (in SI units).}$$

What are the values of its frequency (f), and its velocity(v)?

(a) f=5 Hz, v = 1 m/s moving in +x-direction.
(b) f=5 Hz, v = 5 m/s moving in +x-direction.
(c) f=2 Hz, v = 1 m/s moving in -x-direction.
(d) f=2 Hz, v = 5 m/s moving in -x-direction.
(e)@ f=5 Hz, v = 5 m/s moving in -x-direction.

- A sinusoidal wave is described by the equation

$$y(x,t) = 2.0\sin(5x + 15t) \quad \text{(in SI units)}$$

How far does this wave move in 15 seconds?

(a) 15 m.
(b)@ 45 m.
(c) 5 m.
(d) 55 m.
(e) 90 m.

- A sinusoidal transverse wave is traveling on a string. Any point on the string:

(a) moves in the same direction as that of the wave.
(b) moves in a uniform circular motion with a different angular speed than that of the wave.
(c)@ moves in a simple harmonic motion with the same angular frequency as that of the wave.
(d) moves in a uniform circular motion with the same angular speed as the wave.
(e) moves in a simple harmonic motion with a different frequency than that of the wave.

- Consider a wave described by the equation:

$$y(x,t) = A\cos(kx - \omega t).$$

At t = 0, the displacement is zero at $x =$:

(a) $\frac{\lambda}{5}, \frac{2\lambda}{5}, \cdots.$

(b) $\frac{\lambda}{2}, \frac{3\lambda}{2}, \cdots$

(c)@ $\frac{\lambda}{4}, \frac{3\lambda}{4}, \cdots$

(d) $\frac{\lambda}{3}, \frac{5\lambda}{3}, \cdots$

(e) $\frac{\lambda}{2}, \lambda, \cdots$

$$\text{Hint}: \cos(kx) = 0 \quad \Rightarrow \quad kx = \frac{\pi}{2}, \frac{3\pi}{2}, \cdots$$

$$\because k = \frac{2\pi}{\lambda} \quad \therefore \quad x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \cdots$$

- A sinusoidal wave traveling in the positive x direction has amplitude of 10 cm, a wavelength of 20 cm, and a frequency of 5.0 Hz. A particle at x = 0 and t = 0 has a displacement of 10 cm. Write the equation of the displacement of the particles as a function of x and t.

(a)@ y = (0.1 m) sin[π(10x-10t-3/2)]
(b) y = (0.1 m) sin[π (10x+10t-3/2)]
(c) y = (0.1 m) sin(10x-10t+π/2)
(d) y = (0.2 m) sin(π (10x+10t-3/2)]
(e) y = (0.2 m) sin[π (10x-10t+3/2)]

- A harmonic wave is described by

$$y(x,t) = 0.2\sin(25x - 10t) \text{ (in SI units).}$$

How far does a wave crest move in 20 sec?

(a) 20 m
(b) 5 m
(c)@ 8 m
(d) 50 m
(e) 10 m

- A transverse wave of 0.5-m wavelength is moving in a stretched string with a speed of 5 m/s. If the maximum transverse acceleration of the particles in the string is 80 m/s^2, find the amplitude of the wave.

(a) 4 cm
(b) 6 cm
(c) 1 cm
(d) 3 cm
(e)@ 2 cm

- Ocean waves, with a wavelength of 12 m, are coming in at a rate of 20 crests per minute. What is their speed?

(a) 30 m/s
(b) 8.0 m/s
(c) 16 m/s
(d) 24 m/s
(e)@ 4.0 m/s

- A transverse sinusoidal wave traveling in the negative x direction has an amplitude of 10.0 cm, a wavelength of 20.0 cm, and a frequency of 8.00 Hz. Write the expression for $y(x,t)$ in SI units if y(0,0) = 10.0 cm.

(a) $y(x,t)$ = (0.1 m) sin[20.0 x-8.00 t-(2π)]
(b) $y(x,t)$ = (0.1 m) sin[20.0 x+8.00 t+(2π)]
(c)@ $y(x,t)$ = (0.1 m) sin[31.4 x+50.3 t+($\pi/2$)]
(d) $y(x,t)$ = (0.1 m) sin[31.4 x+50.3 t+π]
(e) $y(x,t)$ = (0.1 m) sin[31.4 x-50.3 t-($\pi/2$)]

- The equation of a transverse wave traveling along a stretched string is given by:

  $y(x,t) = 2.0\times10^{-3}\ \sin(20x - 600t)$ (in SI units).

  What is the transverse speed at x = 1 m and t = 0.5 seconds?

(a)@ 1.1 m/s
(b) - 0.43 m/s
(c) zero
(d) 0.52 m/s
(e) - 0.21 m/s

- A stretched string has a mass per unit length of 0.500 kg/m and is under a tension of 10.0 N. A sinusoidal wave on this string has an amplitude of 0.120 mm, a frequency of 100 Hz, and is traveling in the negative x direction. What is the equation of the wave ?

(a) $y(x,t)$ = (0.120 mm) sin(444 x - 628 t)
(b) $y(x,t)$ = (0.120 mm) sin(444 x + 628 t)
(c)@ $y(x,t)$ = (0.120 mm) sin(140 x + 628 t)
(d) $y(x,t)$ = (0.120 mm) sin(140 x - 628 t)
(e) $y(x,t)$ = (0.060 mm) sin(140 x + 628 t)

- In a vibrating string waves travel a distance of 45 cm in 3.0 s. If the distance between two successive crests is 3.0 cm, what is the frequency of the vibrator causing the waves?

(a)@ 5.0 Hz.
(b) 20.0 Hz.
(c) 11.5 Hz.

(d) 15.0 Hz.
(e) 7.5 Hz.

A sinusoidal wave is given by the equation:

$y(x,t) = 7.0 \cos(-kx - \omega t + \varphi)$ (in SI units).

Which of the following statements is true about this wave:

(a) The wave is a standing wave.
(b) The wave is moving to the positive x-axis.
(c) The wave is moving with speed k/ω.
(d)@ The wave is moving to the negative x-axis.
(e) The wave is moving with speed $k\omega$.

- A water wave is described by the equation:

$y(x,t) = 0.40 \cos[0.10 \times (x + 3t)]$ (in SI units).

The maximum transverse speed of the water molecules is

(a)@ 0.12 m/s.
(b) 4.11 m/s.
(c) 0.04 m/s.
(d) 0.22 m/s.
(e) 1.20 m/s.

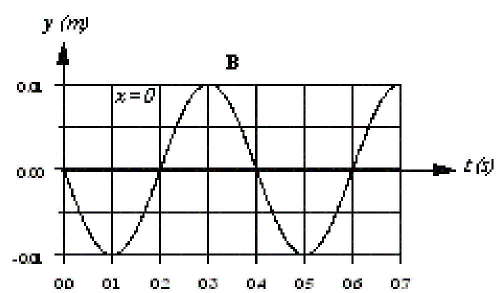
- Any point on a string carrying a sinusoidal traveling wave is moving with its maximum transverse speed when:

(a) the magnitude of its displacement is half the amplitude.
(b) the magnitude of its displacement is one fourth the amplitude.
(c)@ the magnitude of its displacement is zero.
(d) the magnitude of its acceleration is a maximum.
(e) the magnitude of its displacement is a maximum.

- A transverse sinusoidal wave of frequency 100 Hz is traveling along a stretched string with a speed of 20.0 m/s. What is the shortest distance between a crest and a point of zero transverse acceleration?

(a) 1.20 m.
(b) 0.15 m.
(c)@ 0.05 m.
(d) 0.10 m.
(e) 0.20 m.

- A sinusoidal traveling wave is generated on a string. The speed of the wave is 0.050 m/s, and it is traveling to the right along the x-axis. The figure shows the displacement of the particle of the string, at x=0, as a function of time. What is the equation of the wave? [y is given in meters]



(a)@ $y(x,t)$ = 0.01 sin[2π (50x - 2.5t)]
(b) $y(x,t)$ = 0.02 sin[2π (50x - 8.0t)]
(c) $y(x,t)$ = 0.01 sin[2π (50x + 2.5t)]
(d) $y(x,t)$ = 0.01 sin[2π (50x + 8.0t)]
(e) $y(x,t)$ = 0.01 sin[2π (50x - 8.0t)]

- A 100-Hz oscillator is used to generate a sinusoidal wave, on a string, of wavelength 10 cm. When the tension in the string is doubled, the oscillator produces a wave with a frequency and wavelength of:

(a) 50 Hz and 14 cm.
(b) 200 Hz and 20 cm.
(c) 100 Hz and 20 cm.
(d) 200 Hz and 14 cm.
(e)@ 100 Hz and 14 cm.

- Figure (1) shows two different wires, joined together end to end, and are driven by a vibrator of frequency 120 Hz. Wire (2) has a linear density four times that of wire (1). If a wave has a wavelength of 1 m in wire (1), what is the wavelength of the wave in wire (2)?

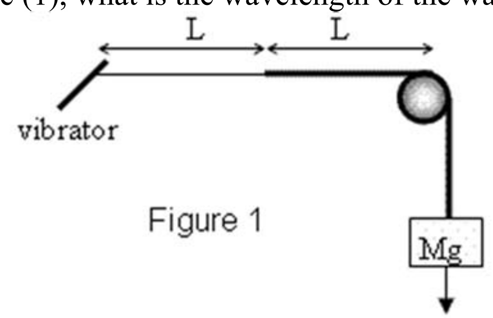


Figure 1

(a) 2.0 m.
(b)@ 0.5 m.
(c) 0.3 m.
(d) 1.5 m.
(e) 4.0 m.

- The equation for a transverse wave on a string is:
  $y(x,t) = 0.025\ \sin(25x - 500t)$ (in SI units).
  The tension in the string is 20 N. Find the linear density of this string.

(a) 0.02 kg/m
(b) 0.13 kg/m
(c)@ 0.05 kg/m
(d) 0.50 kg/m
(e) 0.25 kg/m

- A wave in a string, of linear density 0.13 g/m, is given by the equation:
  $y(x,t)$ = 0.018 sin(3.0 x - 24.0 t),

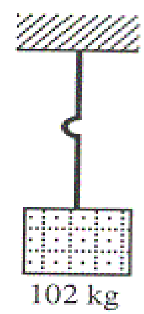
in SI units. The tension in the string is:

(a)@ 8.32×10^{-3} N.
(b) 3.90×10^{-3} N.
(c) 3.12×10^{-3} N.
(d) 2.34×10^{-3} N.
(e) 2.43×10^{-3} N.

➢ If a sinusoidal transverse wave is traveling on a string, then any point on the string

(a) moves in the same direction as the wave.
(b) moves in uniform circular motion with a different angular speed than that of the wave.
(c)@ moves in simple harmonic motion with the same frequency as that of the wave.
(d) moves in simple harmonic motion with a different frequency than that of the wave.
(e) moves in uniform circular motion with the same angular speed as that of the wave.

➢ A steel wire of mass 0.400 kg and length 0.640 m supports a 102-kg block (see the figure). The wire is struck exactly at the midpoint generating a pulse on the wire. How long does it take the peak of the pulse to reach the top of the wire?



(a) 2.00×10^{-3} s
(b)@ 8.00×10^{-3} s
(c) 1.60×10^{-2} s
(d) 6.00×10^{-3} s
(e) 4.00×10^{-3} s

➢ In figure 2, two equivalent pulses, Pulse 1 and Pulse 2, are sent from points A and B at the same time, respectively. Which pulse reaches point C first?

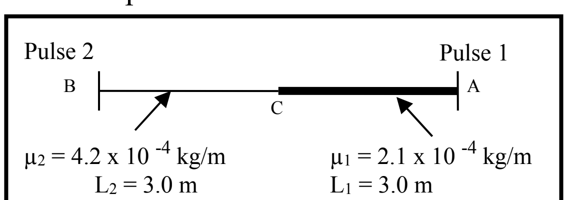


Figure 2

(a) Not enough information.
(b)@ Pulse 1.
(c) 312 Hz.
(d) Both at the same time.
(e) Pulse 2.

➢ The tension in a 60 m telephone wire is 800 N. A pulse initiated at one end of the wire is found to reach the other end in 1.5 s. What is the mass of the wire?

(a) 15 kg.

(b) 50 kg.
(c)@ 30 kg.
(d) 60 kg.
(e) 40 kg.

- The equation of a transverse wave on an 80 cm long stretched string is given by:
$$y(x, t)= 2.0 \sin [10 x-50 t] \qquad \text{(in SI units)}$$
If the mass of the string is 160 g, find the tension in the string.

(a)@ 5.0 N.
(b) 3.5 N.
(c) 25 N.
(d) 55 N.
(e) 15 N.

- A wave of wavelength 0.6 m is sent along a horizontal rope of linear density 13 g/cm, under a tension of 750 N. If the amplitude of the wave is 5.2 cm, then the average power transmitted along the rope is given by:

(a)@ 2.7 kW.
(b) 9.5 kW.
(c) 5.2 kW.
(d) 4.6 kW.
(e) 1.3 kW.

- A transverse wave in a 3.0 m long string is given by the harmonic wave equation:
$$y = 0.4 \cos[\pi\ (x/4 + 6t)] \qquad \text{(in SI units).}$$
If the string is kept under a constant tension of 70 N, find the power transmitted to the wave.

(a) 44 W
(b) 25 W
(c) 72 W
(d)@ 83 W
(e) 58 W

- Any point on a string carrying a sinusoidal wave will move with its maximum speed when the magnitude of its:

(a) displacement is maximum
(b) displacement is twice the amplitude
(c)@ displacement is minimum
(d) acceleration is maximum
(e) displacement is half the amplitude

- The speed of a transverse wave on a string is 170 m/s when the tension in the string is 120 N. What must be the tension to produce waves with a speed of 180 m/s if the amplitude is doubled?

(a) 240 N
(b) 540 N
(c) 127 N
(d) 270 N
(e)@ 135 N

- Transverse waves are being generated on a rope under constant tension. By what factor does the required power change if the length of the rope is doubled?

(a) 1/2.
(b)@ 1.
(c) 2.
(d) 4.
(e) 1/4.

- The power transmitted by a sinusoidal wave on a string does not depend on:

(a) the amplitude of the wave.
(b)@ the length of the string.
(c) the frequency of the wave.
(d) the tension in the string.
(e) the wavelength of the wave.

- A string under a tension of 15 N, is set into vibration to produce a wave of speed 20 m/s, and a maximum transverse speed of 8 m/s. For this wave, the average power is:

(a) 44 W.
(b) 30 W.
(c)@ 24 W.
(d) 15 W.
(e) 11 W.

- A string has a linear mass density of 0.10 kg/m and it is under tension of 10.0 N. What must be the frequency of traveling waves of amplitude 10.0 mm for the average power to be 0.5 W?

(a) 32 Hz
(b) 0.01 Hz
(c)@ 16 Hz
(d) 100 Hz
(e) 10 Hz

- Sinusoidal waves are generated on a string under constant tension by a source vibrating at a constant frequency. If the power delivered by the vibrating source is reduced to one half of the initial value, what is the ratio of the final amplitude to the initial amplitude?

(a) 1.0

(b) 2.0
(c) 0.50
(d)@ 0.71
(e) 1.4

➢ A sinusoidal wave, given by the equation:

$$y(x,t) = 0.07 \cos(6.0\ x - 30\ t) \text{ (in SI units)},$$

is moving in a string of linear density = 1.2 g/m At what rate is the energy transferred by the wave?

(a)@ 1.32×10^{-2} W.
(b) No enough information is given to solve this question.
(c) 3.02×10^{-2} W.
(d) 1.05×10^{-2} W.
(e) 2.21×10^{-2} W.

➢ Figure 1 shows the snap shot of part of a transverse wave traveling along a string. Which statement about the motion of elements of the string is correct? For the element at

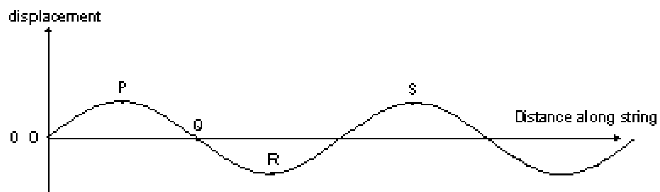


Figure 1

(a) Q, its displacement is a maximum.
(b) Q, its speed is zero.
(c)@ S, the magnitude of its acceleration is a maximum.
(d) P, its speed is a maximum.
(e) S, the magnitude of its acceleration is zero.

➢ Two harmonic waves are described by

$y_1 = 0.02 \sin[\pi(2x-120t)]$

$y_2 = 0.02 \sin[\pi(2x-120t-0.5)]$

in SI units. What is the amplitude of the resultant wave?

(a)@ 28 mm
(b) 0 mm
(c) 20 mm
(d) 10 mm
(e) 50 mm

➢ Fully DESTRUCTIVE interference between two sinusoidal waves of the same frequency and amplitude occurs only if they:

(a) travel in the same direction and are 90° out of phase
(b) travel in opposite directions and are in phase
(c)@ travel in the same direction and are 180° out of phase
(d) travel in the same direction and are in phase
(e) travel in opposite directions and are 90° out of phase

➢ The path difference between two waves is 5 m. If the wavelength of the waves emitted by the two sources is 4 m, what is the phase difference (in degrees)?

(a) 75
(b) 45
(c) 320
(d) 180
(e)@ 450

➢ A transverse harmonic wave in a string is described by:

$y(x,t) = 3.0 \sin(0.3\,x - 8.0\,t - \varphi)$ (in SI units).

At t = 0 and x = 0, a point on the string has a positive displacement and has velocity of zero. The phase constant φ (in degrees) is:

(a) 135.
(b)@ 270.
(c) 90.
(d) 45 .
(e) 180.

➢ Two harmonic waves traveling in the same medium are described by

$y_1(x,t) = 12\sin(3\pi x - 5\pi t),$

$y_2(x,t) = 12\sin(3\pi x - 5\pi t - 4),$

in SI units. What is the displacement of the resultant wave at x= 1.0 m and t= 1.0 s?

(a) -10 m

(b) 1.4 m
(c)@ 9.1 m
(d) -1.6 m
(e) -3.2 m

- Two waves are described as follows:

$$y_1(x,t) = 4\sin(x - v\,t),$$
$$y_2(x,t) = 4\sin(x + v\,t).$$

At what position and time do these two waves cancel?

(a) At x = 0 and at t = 0 only.
(b) They always cancel because v has opposite signs.
(c) They never cancel (they always add up).
(d) At t = 0 and at any position x.
(e)@ At x = 0 and at any time t.

- Two harmonic waves are described by:

$$y_1(x,t) = 4.0\sin(8.0x + 300t),$$
$$y_2(x,t) = 4.0\sin(8.0x + 300t - 2),$$

in SI units. What is the frequency of the resultant wave?

(a) 24 Hz.
(b) 38 Hz.
(c)@ 48 Hz.
(d) 33 Hz.
(e) 75 Hz.

➢ The resultant wave, of two interfering waves, moving in the same direction is given by:
$y(x,t) = 10.0\cos(\pi/6)\sin(3.0x + 20\pi t + \pi/6)$
One of the two originally interfering waves could be:

(a)@ $y(x,t)$ = 5.0 sin(3.0 x + 20 π t + π/3).
(b) $y(x,t)$ = 10.0 sin(3.0 x + 20 π t + π/3).
(c) $y(x,t)$ = 5.0 sin(3.0 x + 20 π t + π/6).
(d) $y(x,t)$ = 10.0 sin(3.0 x - 20 π t).
(e) $y(x,t)$ = 10.0 sin(3.0 x + 20 π t).

➢ A sinusoidal wave of frequency 400 Hz has a speed of 330 m/s. How far apart are two points that differ in phase by π/2 ?

(a) 1.21 m
(b) 0.032 m
(c) 0.137 m
(d)@ 0.206 m
(e) 0.825 m

➢ Two identical sinusoidal traveling waves are moving in the same direction along a stretched string. The amplitude of the resultant wave is 1.80 times that of the common amplitude of the two combining waves. What is the phase difference (in degrees) between the two waves?

(a) 25.8
(b)@ 51.7
(c) 90.0
(d) 180
(e) 18.0

➢ The equation for a transverse wave on a string is:
$y(x,t) = 10.0\sin(0.157x - 50.3t)$ (in SI units).
At a given time, how far apart are two points that differ in phase by π/2?

(a)@ 10.0 cm
(b) 5.00 cm
(c) 0.120 cm
(d) 18.0 cm
(e) 40.0 cm

➢ Two sinusoidal waves having the same amplitude (A), frequency and wavelength, travel in the same direction and have a phase difference φ. Which one of the following statements is TRUE ?

(a)@ Their interference will be constructive if φ = 100 π.
(b) Their interference will be constructive if φ = π.
(c) For certain values of φ, the amplitude of the resultant reaches 4A.
(d) The frequency of the resultant wave is twice the original frequency.

(e) The resultant wave will be a standing wave for $\varphi = 0$.

- Two identical sinusoidal waves, are out of phase with each other, travel in the same direction. They interfere and produce a resultant wave given by the equation:

  $y(x,t) = 8.0\times10^{-4}\sin(4.0\,x-8.0\,t+1.57\text{ rad})$, (in SI units).

  What is the amplitude of the two interfering waves?

(a) 1.0 m.
(b) 4.0 m.
(c)@ 0.5 m.
(d) 2.5 m.
(e) 0.2 m.

- Two identical waves, moving in the same direction, have a phase difference of $\pi/2$. The amplitude of each of the two waves is 0.10 m. If they interfere, then the amplitude of the resultant wave is:

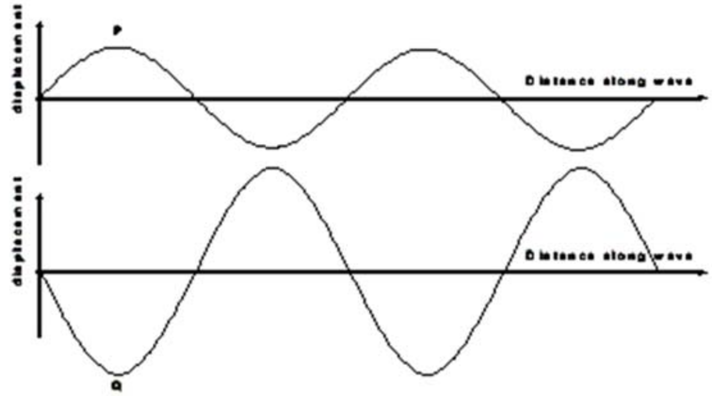
(a) 1.12 m.
(b) 0.05 m.
(c)@ 0.14 m.
(d) 0.21 m.
(e) Not enough information is given to solve this question.

$$\text{Hint}: A' = 2A\cos(\varphi/2)$$
$$= 2\times0.1\times\cos(180/2) = \underline{0.14\text{ m}}$$

- Two identical sinusoidal waves are traveling along a stretched string, both moving in the negative x direction. The two waves are out of phase by π radians. The amplitude of each wave is 1.00 cm and their frequency is 100 Hz. What is the displacement of the string at x=0 when t=10.0 s?

(a) 2.00 cm
(b) 1.41 cm
(c) 0.71 cm
(d) 1.00 cm
(e)@ zero

- Figure 2 shows the displacements at the same instant for two waves, P and Q, of equal frequency and having amplitude Y and 2×Y, respectively. If the two waves move along the positive x-direction, what is the amplitude of the resultant wave, and the phase difference between the resultant wave and the wave P?



The waves are superimposed to give a resultant wave.

Figure 2

(a)@ Resultant amplitude is Y, and the phase difference is π.
(b) Resultant amplitude is 2×Y, and the phase difference is π.
(c) Resultant amplitude is 3×Y, and the phase difference is π.
(d) Resultant amplitude is Y, and the phase difference is zero.
(e) Resultant amplitude is 2×Y, and the phase difference is zero.

- A transverse sinusoidal wave is traveling on a string with a speed of 300 m/s. If the wave has a frequency of 100 Hz, what is the phase difference between two particles on the string that are 85 cm apart?

(a) 0.6 radians.
(b) 4.1 radians.
(c)@ 1.8 radians.
(d) 3.4 radians.
(e) 5.6 radians.

$$\text{Hint}: \frac{\Delta\varphi}{\Delta \mathrm{r}} = \frac{2\pi}{\lambda} \Rightarrow \Delta\varphi = 2\pi\left(\frac{0.85}{v/f}\right) \approx 1.8 \text{ radians}$$

- Two harmonic waves traveling in opposite directions interfere to produce a standing wave described by:

$$y(x,t) = 0.3\sin(0.25x)\cos(120\pi t)$$

The speed of the two interfering waves is

(a) 1203 m/s
(b) 6527 m/s
(c)@ 1508 m/s
(d) 94.25 m/s
(e) 753.0 m/s

➢ A wave on a string is reflected from a fixed end. The reflected wave:

(a) is in phase with the original wave at the fixed end.
(b) has a larger amplitude than the original wave.
(c)@ is 180° out of phase with the original wave at the fixed end.
(d) cannot be transverse.
(e) has a larger speed than the original wave.

➢ The equation for a standing wave is given by:

$y(x,t) = 4.00\times10^{-3}\sin(2.09\text{ x})\cos(60.0\text{ t})$ (in SI units).

What is the distance between two consecutive antinodes?

(a) 0.56 m.
(b) 3.00 m.
(c) 2.20 m.
(d)@ 1.50 m.
(e) 5.00 m.

➢ The maximum amplitude of a standing wave on a string, with linear density = 3.00 g/m and tension of 15.0 N, is 0.20 cm. If the distance between adjacent nodes is 12.0 cm, what will be the wave function $y(x,t)$ of the standing wave? (Note that x is in cm and t is in seconds.)

(a) $y(x,t) = 0.20\sin(0.262\text{ x})\cos(2.20\times10^{3}\text{ t})$.
(b) $y(x,t) = 0.20\sin(0.421\text{ x})\cos(1.85\times10^{3}\text{ t})$.
(c)@ $y(x,t) = 0.20\sin(0.262\text{ x})\cos(1.85\times10^{3}\text{ t})$.
(d) $y(x,t) = 0.40\sin(0.262\text{ x})\cos(1.11\times10^{3}\text{ t})$.
(e) $y(x,t) = 0.40\sin(0.421\text{ x})\cos(1.85\times10^{3}\text{ t})$.

➢ A traveling wave is given by:

$$y(x,t) = 6.0\cos(0.63x - 25.1t)$$

in SI units. It interferes with a similar wave propagating in the opposite direction to produce a standing wave. The distance between the node and the consecutive antinode is:

(a) 1.0 cm.
(b)@ 2.5 cm.
(c) 0.5 cm.
(d) 5.0 cm.
(e) 7.9 cm.

- A wave of speed 20 m/s on a string, fixed at both ends, has an equation for a standing wave given by:

$$y(x,t) = 0.05\ \sin(kx)\ \cos(30t),$$

in SI units. What is the distance between two consecutive nodes?

$$\text{Hint}: k = \omega/v = 30/20 = 1.5$$

$$d = \frac{\lambda}{2} = \frac{2\pi}{2k} \approx 2.1\,\text{m}$$

(a) 5.0 m.
(b) 0.1 m.
(c)@ 2.1 m.
(d) 3.2 m.
(e) 1.1 m.

- A 200-cm string is fixed at both ends. The mass per unit length of the string is 0.0150 g/cm. The tension in the string is 600 N. If the string vibrates in three equal segments, what is its fundamental frequency?

(a) 950 Hz
(b) 632 Hz
(c) 475 Hz
(d)@ 158 Hz
(e) 1330 Hz

- Consider a string fixed at both ends. It has consecutive standing wave modes with frequencies of 480 Hz and 600 Hz. The tension in the string is kept constant. Find the fundamental frequency.

(a) 480 Hz
(b) 300 Hz
(c) 150 Hz
(d)@ 120 Hz
(e) 600 Hz

- A string 180 cm long has a fundamental frequency of vibration of 300 Hz. What length of the same string, under the same tension, will have a fundamental frequency of 200 Hz?

(a) 147 cm.
(b) 900 cm.
(c)@ 270 cm.
(d) 220 cm.
(e) 120 cm.

- A stretched string, fixed at both ends, vibrates in its fundamental frequency. To double the fundamental frequency of the same string, one can change the tension in the string by a factor of:

(a) $\sqrt{2}$
(b) $1/\sqrt{2}$
(c) 2
(d) 1/2
(e)@ 4

- A standing wave is established in a 3.0-m-long string fixed at both ends. The string vibrates in three segments with amplitude of 1.0 cm. If the wave speed is 100 m/s, what is the frequency?

(a) 25 Hz
(b) 100 Hz
(c)@ 50 Hz
(d) 10 Hz
(e) 33 Hz

- A string of length L, mass per unit length μ, and tension F is vibrating at its fundamental frequency. What is the effect on the fundamental frequency if the length of the string is doubled and the tension is quadrupled?

(a) The fundamental frequency is doubled.
(b) The fundamental frequency is quadrupled.
(c) The fundamental frequency is halved
(d)@ The fundamental frequency does not change.
(e) The fundamental frequency is tripled.

- A stretched wire vibrates in its fundamental mode at 300 Hz. What would be the fundamental frequency if the wire were one third the original length, with twice the diameter and with four times the tension?

(a)@ 900 Hz.
(b) 150 Hz.
(c) 450 Hz.
(d) 300 Hz.
(e) 800 Hz.

- The lowest resonant frequency, in a certain string clamped at both ends, is 50 Hz. When the string is clamped at its midpoint, the lowest resonant frequency is:

(a) 50 Hz.
(b) 150 Hz.
(c)@ 100 Hz.
(d) 200 Hz.
(e) 250 Hz.

- Standing waves are produced in a string at the two consecutive resonant frequencies 155 and 195 Hz. If the mass of the string is 5.00 g and its length is 0.80 m, then the tension applied to the string should be:

(a) 17.2 N.
(b) 6.4 N.
(c) 28.5 N.
(d) 19.0 N.
(e)@ 25.6 N.

- A string that is stretched between two supports separated by 1.0 m has resonant frequencies of 500 Hz and 450 Hz, with no intermediate resonant frequencies, what is the wave speed in the string?

(a) 200 m/s
(b)@ 50 m/s
(c) 350 m/s
(d) 500 m/s
(e) 450 m/s

- A string having a mass per unit length of 10.0 g/m is under a tension of 160 N. This string resonates in the mode shown in figure 1. If the length of this string is 90.0 cm, find the frequency of oscillation.

Figure 1

(a) 60.0 Hz
(b)@ 211 Hz
(c) 120 Hz
(d) zero
(e) 310 Hz

- A string, 30.0 cm long, with a linear density of 0.65 g/m is set into vibration. It is found that normal modes of vibration are present ONLY at the frequencies of 880 Hz and 1320 Hz as the frequency of the source is varied over the range 500 Hz to 1500 Hz. What is the tension in the string?

(a) 56.4 N
(b) 50.4 N
(c)@ 45.3 N
(d) 36.5 N
(e) 78.2 N

- A string fixed at both ends vibrates in three loops. The string has a length of 1.0 m, a mass of 8.0 g and is under a tension of 15 N. What is the frequency?

(a) 150 Hz

(b) 130 Hz
(c) 29 Hz
(d)@ 65 Hz
(e) 86 Hz

➢ A 50 cm long string with a mass of 0.01 kg is stretched with a tension of 18 N between two fixed supports. What is the resonant frequency of the longest wavelength on this string?

(a) 50 Hz.
(b) 312 Hz.
(c)@ 30 Hz.
(d) 9.8 Hz.
(e) 150 Hz.

Hint : longest wavelength ≡ fundamental frequency

$$\mu = \frac{m}{L} = \frac{0.01}{50\times10^{-2}} = 0.02 \text{ kg/m}$$

$$v = \sqrt{\frac{\tau}{\mu}} = 30 \text{ m}, \qquad f = \frac{v}{2L} = 30 \text{ Hz}$$

➢ A string has linear density = 5.1 g/m and is under a tension of 120 N. If the vibrating length of the string is 60 cm, what is the lowest resonant frequency?

(a) 225 Hz.
(b)@ 128 Hz.
(c) 312 Hz.
(d) Not enough information.
(e) 158 Hz.

➢ A 40 cm string of linear mass density 8.0 g/m is fixed at both ends. The string is driven by a variable frequency audio oscillator ranged from 300 Hz to 800 Hz. It was found that the string is set in oscillation only at the frequencies 440 Hz and 660 Hz. What is the tension in the string?

(a) 125 N.
(b) 496 N.
(c) 500 N.
(d) 322 N.
(e)@ 248 N.

➢ A certain string, fixed at both ends, vibrates in seven segments at a frequency of 240 Hz. What frequency will cause it to vibrate in four segments?

(a) 411 Hz
(b) 89 Hz
(c)@ 137 Hz
(d) 274 Hz
(e) 420 Hz

➢ Vibrations with frequency 600 Hz are set up on a 1.33-m length of a string that is clamped at both ends. The speed of waves on the string is 400 m/s. How far from either end of the string does the first node occur?

(a)@ 0.33 m
(b) 0.49 m
(c) 0.75 m
(d) 0.17 m
(e) 0.66 m

- A point source emits sound waves which are reflected from a metal plate with air in between, as shown in figure 3. Standing waves are produced in between the source and the plate. If the points R, S and T are three successive nodes, what is the frequency of the wave? [Speed of sound in air is 342 m/s].

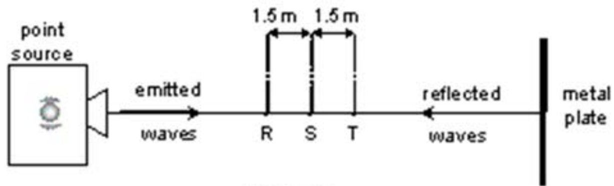


Figure 3

(a) 312 Hz.
(b) 158 Hz.
(c) Not enough information.
(d) 225 Hz.
(e)@ 114 Hz.

- When a certain string is clamped at both ends, the lowest four resonant frequencies are 50, 100, 150 and 200 Hz. When the string is also clamped at its midpoint (without changing the tension), then the lowest four resonant frequencies are:

(a) 50, 100, 150 and 200 Hz.
(b) 35, 50 , 75 and 100 Hz.
(c) 75, 150, 225 and 300 Hz.
(d) 50, 150, 250 and 300 Hz.
(e)@ 100, 200, 300 and 400 Hz.

- Transverse waves, with fixed amplitude, are being generated on a rope under constant tension. When the frequency of the wave is increased, which one of the following statements is correct:

(a) The wavelength increases and the transmitted power is the same
(b) Both the wavelength and the linear mass density decrease
(c) Both the wavelength and the maximum transverse speed increase
(d)@ The wavelength decreases and the transmitted power increases
(e) The maximum transverse speed is the same and the transmitted power increases

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