# Chapter 4

# Ideal Gas and the Kinetic Theory of Gases

In the following we will consider a sample of gas molecules, we will call it system, which has the following properties (see Figure 1):

| • | $N_A, \quad M = mN_A$ | $N, \quad M_{\text{sample}} = Nm = nM, \quad n = \dfrac{N}{N_A}$ |
|---|---|---|
| $m$ | 1 mole | Sample |

Figure (1)

**For one particle (molecule)**:
$m$ = molecular mass (mass of one molecule)
**For one mole**, we have:
$N_A$ = the Avogadro's number.
$M = m\,N_A$ = molar mass (mass of one mole).
**For the sample**, we have:
$N$ = number of molecules in the sample.
$M_{\text{sample}}$ (in kg) = mass of the sample $= N\,m$

**Examples**: For $O_2$; $M$ = 32 g/mol, and for $H_2$ is $M$ = 2 g/mol.

**Ideal (perfect) Gas Law**: Consider a sample of $n$ moles (or $N$ molecules) of gas contained in a volume $V$ and at absolute temperature $T$ and pressure $P$. The empirical relation between $P$, $V$, and the absolute temperature $T$ is given by the relation:

$$P\,V = n\,R\,T = N\,k\,T,$$

where $R$ is the universal gas constant = 8.314 J/(mol.K), $[P]$ = N/m² = Pascal, $[V]$ = m³, and $[T]$ = K. If the sample volume contains $M_{sample}$ kg of gas that has a molar mass $M$ (= the mass of 1 mol of that substance), then

$$n = \frac{M_{sample}}{M} = \frac{M_{sample}}{mN_A} = \frac{mN}{mN_A} = \frac{N}{N_A}.$$

where $k = R/N_A = 1.38\times10^{-23}$ J/K is the Boltzmann's constant..
**Avogadro's number:** is the number of particles (or atoms or molecules) in 1 mole (kmol), and is the same for all substances. $N_A = 6.023\times10^{23}$ particles/mole,

Mass of an atom or molecule, $m = \dfrac{M}{N_A}$.

**Comments**:

I. The ideal gas is held at low density.

II. The temperature ($T$) always measured in K, not in $^oC$ or $^oF$.

III. $1 \text{ atm.L} = 1.013\times10^{5}\ \frac{\text{N}}{\text{m}^2}\times10^{-3}\ \text{m}^3 = 101.3\ \text{N.m} = 101.3\ \text{J}.$

IV. The real gas behaves more like an ideal gas at low pressure and high temperature.

➔ The molecular weight ($M$) of $O_2$ is 32 g/mol, therefore, 32 g of $O_2$ contains $N_A$ molecules and ½ mole has a mass of 16 g.

➔ Gold has a molar mass of 197 g/mole.

a) How many moles of gold in a 2.5 g sample of pure gold?

✓ $$n = \frac{M_{sample}}{M} = \frac{2.5\text{ g}}{197\text{ g/mole}} = 0.013\text{ mole}.$$

b) How many atoms are in the sample?

✓ $$N = nN_A = 7.6\times10^{21}\text{ atoms}.$$

➔ How many hydrogen atoms and how many oxygen atoms are in an ice cube of mass 8 grams?

✓ The molecular weight of $H_2O$ is (2×1+16) = 18. Thus 8 grams of $H_2O$ is 8/18 = 0.44 mole. One mole has $N_A$ molecules, so 0.44 mole has ($0.44\times N_A$) = $2.68\times10^{23}$ molecules. In each $H_2O$ there is one oxygen atom and two hydrogen atoms, so in 8 grams of ice cube there are $2.68\times10^{23}$ oxygens and $5.36\times10^{23}$ hydrogen.

**Absolute Zero:** is unique temperature at which $P$ and $V$ reach zero.

**Standard Condition (Standard Temperature and Pressure) S.T.P.** is defined to be:

$$T = 273.15\text{ K} = 0\ ^oC, \qquad P = 1.013\times10^{5}\ \frac{\text{N}}{\text{m}^2}\text{(or Pa)} = 1\text{ atm}$$

➔ Show that one mole of any gas at S.T.P. occupies a volume of 22.4 L.

✓ $$V = \frac{n\,R\,T}{P} = \frac{(1\text{ mol})(8.314\frac{\text{J}}{\text{mol.K}})(273.15\text{ K})}{(1.013\times10^{5}\frac{\text{N}}{\text{m}^2})} = 22.4\times10^{-3}\ \text{m}^3 = 22.4\text{ L}$$

➔ Find the density of ammonia, $NH_3$,

✓ At STP one mole of any gas occupies 22.4 L. The molecular mass of $NH_3$ is:

$$1\text{N} = 1\times14.01\text{ u} = 14.01\text{ u}$$

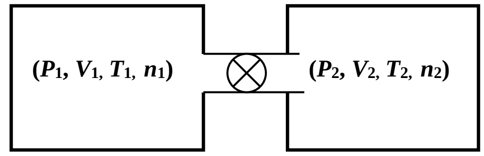
$$3\text{H} = 3\times1.008\text{ u} = \underline{3.02\text{ u}}$$

$$17.03\text{ u} = 17.03\text{ g/mole}$$

One mole of $NH_3$, therefore, has a mass of 17.03 g at STP, and its density is:

$$\rho = \frac{m}{V} = \frac{17.03\text{ g}}{22.4\text{ L}} = 0.76\text{ g/L} = 0.76\text{ kg/m}^3.$$

➔ Consider two thermally insulated vessels connected by a short tube with a valve. Both the vessel are filled with ideal gas having volume $(P_1, V_1, T_1, n_1)$ and $(P_2, V_2, T_2, n_2)$, as shown in the figure. If the valve is opened, find the final pressure and temperature of gas.




✓ Before opening the valve the ideal gas law gives,

$$P_1V_1 = n_1RT_1, \quad P_2V_2 = n_2RT_2,$$

After connection and at equilibrium, the final pressure "$P$" will be the same, so the ideal gas law gives,

$$P(V_1 + V_2) = R(n_1T_1 + n_2T_2)$$
$$= P_1V_1 + P_2V_2$$

This means: $P = \dfrac{P_1V_1 + P_2V_2}{(V_1 + V_2)}$

➔ The density of air at 0 °C and 1 atm pressure is 1.293 kg/m³. Find its density at 100 °C and 2 atm pressure?

✓ The given data are:

$$P_i = 1.01\times10^5 \text{ Pa}, \quad P_f = 2.02\times10^5 \text{ Pa}, \quad T_i = 273 \text{ K}, \quad T_f = 373 \text{ K}.$$

Use the ideal gas law as a ratio: $\dfrac{P_iV_i}{T_i} = \dfrac{P_fV_f}{T_f}$, $\qquad V_j = \dfrac{\text{m}}{\rho_\text{j}}$., one can find

$$\rho_f = \rho_i \frac{P_fT_i}{P_iT_f} = 1.293 \frac{2(273)}{1(373)} = 1.893 \text{ kg/m}^3.$$

➔ a- A spherical balloon of radius 0.80 m is filled with an ideal gas and located in a region where the pressure is $3.0\times10^5$ Pa and the temperature is 12 °C. Assume the balloon has the same pressure and temperature as its surroundings. Find the number of moles of ideal gas in the balloon. [V=(4/3)πr³=2.14 m³]

✓ $$n = \frac{PV}{RT} = \frac{(3.0\times10^5 \text{ N/m}^2)(2.14 \text{ m}^3)}{(8.314 \text{ J/K})(285 \text{ K})} = 272 \text{ moles}.$$

b- The balloon in part (a) is moved to another region where the pressure is $1.5\times10^5$ Pa. When the balloon reaches the same pressure and temperature as this new region, its radius is found to increase by 0.22 m. What must the temperature of this region be in °C?

✓ $$T_f = \frac{P_fV_f}{nR} = \frac{(1.5\times10^5 \text{ N/m}^2)\dfrac{4}{3\pi}(0.80{+}0.22)^3}{(8.314 \text{ J/K})(285 \text{ K})} = 295 \text{ K} = 22 \text{ °C}.$$

**Kinetic theory of gases**: is a sub-branch of statistical mechanics in which no attempt is made to derive the thermodynamic laws, but rather their meaning in terms of averages of microscopic variables is made clearer. Basic assumptions:

1- the system (gas) consists of identical particles, of negligible volume (compared with the volume of the gas), called molecules.
2- the molecules are perfectly elastic, i.e. no energy lost in collision,
3- the time of collision is negligible compared with the time between collisions,
4- the attraction between the molecules is negligible. In these conditions, the gas is ideal. The molecules in the gas obey Newton's law and move randomly with a wide distribution of speeds ranging from zero to very large values.

**Equipartition of energy theorem**: states that "*The average kinetic energy per molecule per degree of freedom is (1/2) kT*". Consequently, the average translational kinetic energy per molecule is given by

$$\overline{K} = \frac{1}{2} m \overline{v^2} = \frac{3}{2} k T.$$

where $T$ is the absolute temperature. The pressure is

$$P = \frac{NkT}{V} = \frac{2}{3} \frac{N}{V} \overline{K}.$$

➔ 5-liter vessel contains nitrogen at a temperature of 27 $^o$C and a pressure of 3 atm. Find

a. the average kinetic energy per molecules.

✓ $$\overline{K} = \frac{1}{2} m \overline{v^2} = \frac{3}{2} k T = \frac{3}{2} \frac{RT}{N_A} = \frac{3}{2} \frac{(8.314)(300)}{6.02\times 10^{23}} = \underline{6.21\times 10^{21}\text{ J}}.$$

b. the total translational kinetic energy of the gas,

✓ $$K_{total} = N\,\overline{K} = \frac{3}{2} PV = \frac{3}{2}(3\times 1.01\times 10^{5})\times(5\times 10^{-3}) = \underline{2.27\times 10^{3}\text{ J}}.$$

c. the total number, $N$, of nitrogen atoms and the number of moles, $n$.

✓ $$N = \frac{K_{total}}{\overline{K}} = 3.66\times 10^{23}\text{ atoms.}$$

✓ $$n = \frac{N}{N_A} = \frac{3.66\times 10^{23}}{6.02\times 10^{23}} = 0.61\text{ mole.}$$

➔ A cylinder contains a mixture of helium and argon gas in equilibrium at a temperature of 150 $^o$C. What is the average kinetic energy of each gas molecule?

For helium:

✓ $$\overline{K} = \frac{3}{2}\, kT = \frac{3}{2}(1.38\times 10^{-23}\,\frac{\text{J}}{\text{K}})(423\text{K}) = 8.76\times 10^{-21}\text{ J.}$$

For Argon, Since the average kinetic energy depends only on the absolute temperature, so the average kinetic energy for argon will be the same as for helium, i.e. $8.76\times 10^{-21}$ J.

# True-False Questions

1- In using the Ideal gas law it is necessary that temperature be expressed in $^{o}C$. F
2- One mole of gas contains $6\times10^{23}$ molecules only if the gas is a monatomic gas. F
3- When making calculations with the ideal gas law, pressure must be expressed in atmospheres. F
4- When heat is added to a gas its temperature will raise by an amount independent of whether or not the pressure or volume is held constant. F (latent heat)
5- A real gas behaves more like an ideal gas at high temperatures than at low temperatures. T
6- For the same increase in temperature, solids generally expand less than liquids. T
7- The number of molecules (N), the universal gas constant (R) and the absolute temperature (T) are all thermodynamic variables. F
8- As the temperature increases from zero $^{o}C$ to 4 $^{o}C$, the water's density increases. T
9- Water, ice and water vapor can coexist in equilibrium. T
10- If the average kinetic energy of the molecules in a solid increases, this means the temperature increases. T

# Supplementary Problems

- A sample of an ideal gas exerts a pressure of 60 Pa when its temperature is 400 K and the number of molecules present per unit volume is n. A second sample of the same gas exerts a pressure of 30 Pa when its temperature is 300 K. How many molecules are present per unit volume of the second sample?

(a) n/3
(b) 5*n/3
(c)@ 2*n/3
(d) n/2
(e) 3*n/2

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- Which one of the graphs in Figure (1) best represents the variation of pressure with the volume of an ideal gas at constant temperature?

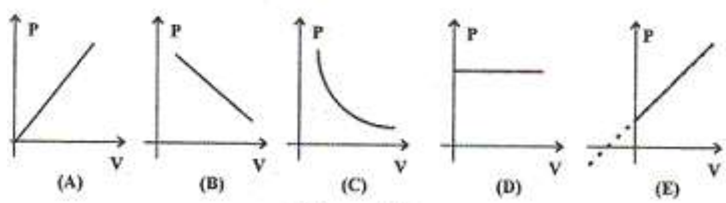


Figure # 1

(a) A.
(b) B.
(c) E.
(d)@ C.
(e) D.

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- Compute the number of molecules in 1.00 $cm^3$ of an ideal gas at a pressure of 100 Pa and temperature of 20 $^oC$.

(a) $4.34\times10^{16}$ molecules
(b)@ $2.47\times10^{16}$ molecules
(c) $43.0\times10^{21}$ molecules
(d) $6.02\times10^{23}$ molecules
(e) $3.62\times10^{17}$ molecules

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- A steel vessel contains 5 moles of an ideal gas at 0 $^oC$ and a pressure of 1 atm. It is heated at constant volume until its temperature is 100 $^oC$. How many moles of gas should be removed from the container to keep the pressure of the gas constant at 1 atm?

(a)@ 1.34 moles
(b) 0 moles
(c) 3.66 moles
(d) 4.32 moles

(e) 2.45 moles

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- A helium-filled balloon has a volume of 2 $m^3$. As it rises in the earth's atmosphere, its volume expands. What will its new volume be if its original temperature and pressure are 20 $^oC$ and 1 atm., and its final temperature and pressure are -40 $^oC$ and 0.1 atm.?

(a) 25 $m^3$
(b) 8 $m^3$
(c) 4 $m^3$
(d) 10 $m^3$
(e)@ 16 $m^3$

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- One mole of an ideal gas has a temperature of 25 $^oC$. If the volume is held constant and the pressure is doubled, the final temperature will be:

(a) 50 $^oC$.
(b) 174 $^oC$.
(c) 596 $^oC$.
(d) 25 $^oC$.
(e)@ 323 $^oC$.

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- In a constant-volume gas thermometer, the pressure is 0.019 atm at 100 $^oC$. Find the temperature when the pressure is 0.027 atm.

(a) 132 $^oC$
(b) 340 $^oC$
(c)@ 257 $^oC$
(d) 531 $^oC$
(e) 321 $^oC$

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- An ideal gas occupies a volume $V_1$ at a temperature of 100 $^oC$. If the pressure of the gas is held constant, by what factor does the volume change when the Celsius temperature is tripled?

(a)@ 1.54.
(b) 0.33.
(c) 3.55.
(d) 3.00.
(e) 6.00.

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- An ideal gas undergoes an isothermal process starting with a pressure of $2\times10^5$ Pa and a volume of 6 $cm^3$. Which of the following might be the pressure and volume of the final state?

(a) $4\times10^5$ Pa and 4 $cm^3$
(b)@ $6\times10^5$ Pa and 2 $cm^3$
(c) $3\times10^5$ Pa and 6 $cm^3$
(d) $1\times10^5$ Pa and 10 $cm^3$
(e) $8\times10^5$ Pa and 2 $cm^3$

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➢ An ideal gas occupies a volume of 12 L at 20 $^{o}C$ and a pressure of 1.0 atm. Its temperature is now raised to 100 $^{o}C$ and its pressure increases to 3.0 atm. The new volume is:

(a) 25 L.
(b)@ 5.1 L.
(c) 0.20 L.
(d) 21 L.
(e) 14 L.

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➢ The volume of an oxygen container is 50.0 L. As oxygen leaks from the container, the pressure inside the container drops from 21.0 to 9.00 atm, and its temperature drops from 303 to 283 K. The number of moles that leaks from the container is:

(a)@ 22.8 mol.
(b) 11.1 mol.
(c) 19.4 mol.
(d) 65.3 mol.
(e) 42.2 mol.

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➢ One mole of oxygen molecule (M = 32 g/mol) occupies a cubic vessel of side length 10 cm at a temperature of 27 $^{o}C$. Calculate the pressure of the gas on the walls.

(a) $3.33\times10^4$ Pa.
(b) $7.52\times10^6$ Pa.
(c)@ $2.49\times10^6$ Pa.
(d) $1.14\times10^4$ Pa.
(e) $5.01\times10^6$ Pa.

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➢ Oxygen gas at 20 $^{o}C$ is confined in a cube. What is the translational average kinetic energy per molecule?

(a) $9.1\times10^{-24}$ J
(b)@ $6.1\times10^{-21}$ J
(c) $4.1\times10^{-22}$ J
(d) $5.2\times10^{-21}$ J
(e) $2.1\times10^{-22}$ J

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➢ The mass of an oxygen molecule is 16 times that of a hydrogen molecule. At room temperature, the ratio of the rms speed of an oxygen molecule to that of a hydrogen molecule is:

(a)@ 1/4
(b) 16
(c) 1/16
(d) 1
(e) 4

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➢ The mass of a hydrogen molecule is $3.3\times10^{-27}$ kg. If $1.0\times10^{23}$ hydrogen molecules per second strike 2.0 cm$^2$ of wall at an angle of 55 degrees with the normal when moving with a speed of $1.0\times10^3$ m/s, what pressure do they exert on the wall?

(a) $0.9\times10^3$ Pa.
(b) $2.8\times10^3$ Pa.
(c) $8.6\times10^3$ Pa.
(d) $5.7\times10^3$ Pa.
(e)@ $1.9\times10^3$ Pa.

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➢ Find the RMS speed of nitrogen molecules (M=28 g/mole) at 0 $^oC$.

(a) $1.7\times10^2$ m/s.
(b)@ $4.9\times10^2$ m/s.
(c) $3.9\times10^2$ m/s.
(d) $3.2\times10^2$ m/s.
(e) zero.

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➢ The average translation kinetic energy of an ideal gas of helium atoms at room temperature (300 Kelvin) is $5.54\times10^{-21}$ J. The average translation kinetic energy of the ideal argon gas at room temperature is: [Atomic mass of helium = 2.0 Kg/kmole, Atomic mass of argon = 8.0 kg/kmole]

(a) $1.40\times10^{-21}$ J.
(b)@ $5.54\times10^{-21}$ J.
(c) $2.21\times10^{-21}$ J.
(d) $2.77\times10^{-21}$ J.
(e) $1.11\times10^{-20}$ J.

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➢ A closed tank, at room temperature, has a mixture of hydrogen molecules and helium atoms. The ratio of RMS speed of hydrogen molecules to that of helium is: [Note: The molar mass of the hydrogen molecule is 2.0 g/mol and the molar mass of the helium atom is 4.0 g/mol]

(a) 0.1
(b) 2.1
(c)@ 1.4
(d) 3.2
(e) 0.3

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➢ An ideal gas has an RMS speed of 254 m/s. If each gas particle has a mass of $6.62\times10^{-26}$ kg, what is the temperature of the gas?

(a) 310 K
(b) 611 K
(c)@ 103 K
(d) 425 K
(e) 79 K

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- Two moles of a monatomic ideal gas with an RMS speed of 254 m/s are contained in a tank that has a volume of 0.15 $m^3$. If the molar mass of the gas is 0.39 kg/mole, what is the pressure of the gas?

(a) $2.3\times10^5$ Pa.
(b)@ $1.1\times10^5$ Pa.
(c) $2.2\times10^4$ Pa.
(d) $3.2\times10^6$ Pa.
(e) $6.8\times10^4$ Pa.

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- Two identical containers, one has 2.0 moles of type 1 molecules, of mass m1, at 20 $^{o}$C . The other has 2.0 moles of type 2 molecules, of mass $m_2 = 2\ m_1$, at 20 $^{o}$C . The ratio between the average translational kinetic energy of type 2 to that of type 1 is:

(a) 16.
(b) 2.
(c) 8.
(d) 4.
(e)@ 1.

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- The average translational kinetic energy of the molecules of an ideal gas in a closed, rigid container is increased by a factor of 4. What happens to the pressure of the gas?

(a) it increases by a factor of 8.
(b)@ it increases by a factor of 4.
(c) it decreases by a factor of 8.
(d) it remains the same.
(e) it decreases by a factor of 4.

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