(*Prof. Dr. I. M. A. Nasser*)

ELECTRIC POTENTIAL

Introduction: For a conservative forces, such as gravitational force $\vec{F}_g = m\vec{g}$ and the electrostatic force $\vec{F}_E = q_o \vec{E}$, the total energy is conservative. In symbols:

$$K.E. + P.E. = \text{ constant } \Rightarrow \Delta K + \Delta U = 0$$

Where $\Delta K = (K_f - K_i)$ is the change in kinetic energy and $\Delta U = (U_f - U_i)$ is the change in potential energy. i and f represent the initial and final position respectively.

Suppose that a collection of electric charges create an electric field, then a test charge q_o placed in this field will experience a force. By moving the charge from point i to point f by an infinitesimal displacement $d\vec{s}$,

1- the work done by the electric field, W_E, is

$$dW_E = \vec{F}_E \cdot d\vec{s} = q_o \vec{E} \cdot d\vec{s}$$

As for the conservative field, dW_E, does not depend on the path we follow from i to f. This work decreases the potential energy of the electric field by an amount:

$$dU_E = -dW_E = -q_o \vec{E} \cdot d\vec{s}$$

2- For a finite displacement of the test charge between i and f, the change in **electrostatic potential energy, ΔU_E,** is

$$\Delta U_E = -q_o \int_i^f \vec{E} \cdot d\vec{s}$$

3- The **electrostatic potential difference**, ΔV_E, between points i and f is defined as the change in potential energy divided by the test charge q_o:

$$\Delta V_E = V_f - V_i = \frac{\Delta U_E}{q_o} = -\int_i^f \vec{E} \cdot d\vec{s}$$

The **electrostatic potential** has units of $\frac{\text{Work}}{\text{Charge}} = \frac{\text{J}}{\text{C}} = \text{ volt}$.

We can think of qV_A as the **electrostatic potential energy** of a charge q at a point *A* in an electric field *E*. The **electrostatic potential** is thus the potential energy per unit charge in the field *E*. Be careful not to confuse the different terms potential (in volts) and potential energy (in Joules).

Example 1: Consider two points in an electric field. The potential at point P_1 is V_1 = -30 V , and the potential at point P_2 is V_2 =150 V.

a- How much work is done by the field in moving a charge $q = -4.7\ \mu\text{C}$ from P_2 to P_1?

$$\begin{aligned} \Delta W_E &= -q\Delta V_E = -q(V_f - V_i) \\ &= -(-4.7\times 10^{-6}\text{ C})[(-30) - 150]\text{V} \\ &= -8.5\times 10^{-4}\text{ J} \end{aligned}$$

b- How much work is done by an external force in moving a charge $q = -4.7\ \mu\text{C}$ from P_2 to P_1?

$$\Delta W_{Ext} = -\Delta W_E = 8.5\times 10^{-4}\text{ J}$$

Example 2: What potential difference is needed to stop an electron with an initial speed of $v_i = 4.2\times10^6\ \frac{\text{m}}{\text{s}}$?

$$\Delta K + \Delta U = \frac{m_e}{2}(v_f^2 - v_i^2) + q_e\Delta V$$

$$\Rightarrow \Delta V = \frac{m_e v_i^2}{2q_e} \approx 50\ \text{V}$$

Potential difference in a uniform electric field

$$\Delta V_E = V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s} = -\vec{E}\cdot\int_i^f d\vec{s}$$

$$= -\vec{E}\cdot\Delta\vec{s} = -\vec{E}(\vec{s}_f - \vec{s}_i) = -Ed$$

If d is measured parallel to the field lines, $[E] = \frac{\text{volts}}{\text{meter}}$. The negative means that $V_f < V_i$.

Example 1: A 2 meter stick is parallel to a uniform $2000\frac{\text{N}}{\text{C}}$ electric field. Find the potential difference between its ends.

$$\Delta V = Er = 2000\times2 = 400\ \text{V}.$$

COMMENTS

1- Potential increases when moving "upstream" against *E*-lines.
2- Because work is a scalar quantity, so too is potential difference.
3- Since only differences in potential energy are important, the choice of a point at $V = 0$ is arbitrary. We usually choose $V = 0$ far away where $r_i \rightarrow \infty$.
4- The potential at distance r from an isolated charge Q is

$$V = k\frac{Q}{r}$$

5- the absolute potential at a point due to a number of point charges is

$$V = k\sum_i \frac{q_i}{r_i}$$

where the r_i are the distances of the charges q_i from the point in question. Positive charge q_i' s contribute positive terms to the potential, while negative q_i' s contribute negative terms.

6- If two charges, q_1 and q_2, are separated by a distance r_{12}, the potential energy, U, of the pair of charges is given by:

$$U = k\frac{q_1 q_2}{r_{12}}$$

Note that: If the charges of the same sign, i.e. repel each other, a positive work must be done on the system (by external agent) to bring (assemble) the two charges near one another. If the charges are of opposite sign, the force is attractive and U is negative (bound system like H-atom). This means that positive work must be done to separate them.

7- The potential energy U for a number of point charges is given by:

$$U = k \sum_{\substack{i,j=1 \\ j>i}} \frac{q_i q_j}{r_{ij}}$$

where r_{ij} is the distance between the charge q_i and q_j .

8- If $\vec{E} = 0$ at a point P then V must be constant at P. For example, inside a hollow conducting sphere of radius *R* and charge *Q*, the electric field is zero but the electric potential, V, is constant ($V = k\dfrac{Q}{R}$).

9- If $V = 0$ at a point P then $\vec{E}$ may, or may not, be zero at P.

10- For a two points *A* and *B* with potential V_A and V_B respectively. If a positive charged particle accelerate from *A* to *B*, then $V_A > V_B$.

11- No work is done when a charge is moved along an **equipotential surface**. This means that electric field lines must be perpendicular to equipotential surfaces.

Extra Problems: (Answer is A1)

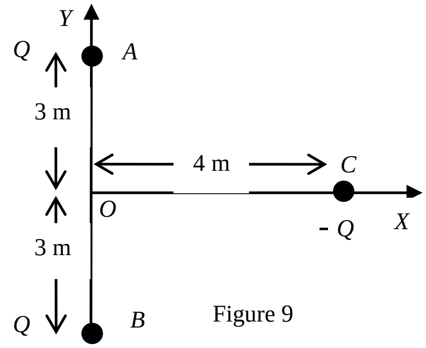

Figure 9

T-032

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Q0
18 Q0 Consider two concentric conducting shells of radii (a) and
25 Q0 (b), b > a. The smaller (inner) shell has a positive charge
   Q0 (q) and the larger (outer) shell has a charge (Q). If the
   Q0 potential on the inner shell is zero, what is the value of Q?
   Q0
   A1   Q = -b*q/a.
   A2   Q = -a*q/b.
   A3   Q =  b*q/a.
   A4   Q =  a*q/b.
   A5   Q = - q.   Q0
19 Q0 In figure 9, two equal positive charges, each of magnitude
25 Q0 5.0*10**(-5) C, are fixed at point A and B separated by a
   Q0 distance of 6 m. An equal and opposite charge moves towards
   Q0 them along the line CO. At point C, 4.0 m from O, the
   Q0 kinetic energy of the moving charge is 4.0 J. What is the
   Q0 kinetic energy of this charge when it passes point O?
   Q0
   A1   10.0  J.
   A2   4.3   J.
   A3   2.2   J.
   A4   12.5  J.
   A5   19.0  J.
   Q0
20 Q0 The potential of a charge distribution is given by:
25 Q0          V(x,y) = A [y*(x**2)- x*(y**2)],
   Q0 where A is in appropriate units. The electric field will
   Q0 be zero at the point:
   Q0
   A1   x = 0, and y = 0.
   A2   x = 1, and y = 0.
   A3   x = 0, and y = 1.
   A4   x = 1, and y = 1.
   A5   x = 1, and y = -1.


Q0
Q13Q0 A 4.0 nano-C point charge is located at the origin, and a
25 Q0 second point charge (-5.0 nano-C) is placed on the y axis
   Q0 at y = 60 cm. If point A is at (45 cm, 0) and point B is at
   Q0 (80 cm, 0), what is the potential difference between points
   Q0 A and B (VA - VB)?
   Q0
```

A1 20 V
A2 30 V
A3 17 V
A4 40 V
A5 zero

T-031

Q0
13 Q0 The electric potential at points in the xy-plane is given by:
25 Q0 V = (x**3 - 2*x*y) Volts,
Q0 where x and y are in meters. The magnitude of the electric
Q0 field at the point with the coordinates x = 1 m and y = 2 m is:
Q0
A1 Sqrt(5) V/m.
A2 Sqrt(8) V/m.
A3 Sqrt(2) V/m.
A4 Sqrt(3) V/m.
A5 Zero.
Q0
14 Q0 In figure (7), what is the net potential at point P due to the
25 Q0 four point charges if V = 0 at infinity ? [take d = 2 cm,
Q0 q = 1.0 micro-C].

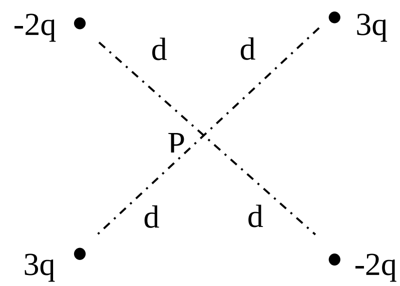


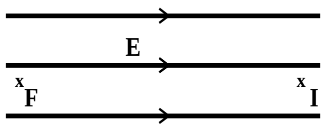
Figure (7)

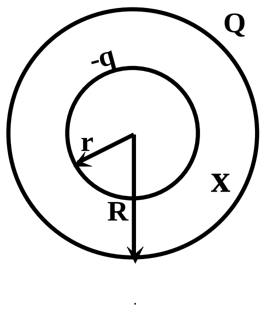
Q0
A1 9.0*10**5 V.
A2 - 9.0*10**5 V.
A3 4.6*10**7 V.
A4 - 4.6*10**7 V.
A5 Zero.
Q0

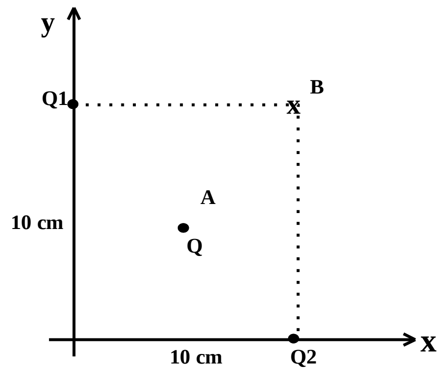
15 Q0 Which one of the following statements is true?
25Q0
Q0
A1 The electric field lines are perpendicular to the equipotential
A1 surfaces.
A2 We have to do work to move a charged particle along an
A2 equipotential surface.
A3 The electric field is a scalar quantity.
A4 The electric potential is a vector quantity.
A5 Any two equipotential surfaces are always parallel.
Q0
16 Q0 Two balls with charges 5.0 micro-C and 10 micro-C are at a
25Q0 distance of 1.0 m from each other. In order to reduce the
Q0 distance between them to 0.5 m the amount of work to be
Q0 performed is:
Q0
A1 0.45 J.
A2 45.0 J.
A3 1.2*10**(-4) J.
A4 4.5*10**(-4) J.
A5 0.23 J.

T-012

Fig 4 Fig 5 Fig 6





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Q0
Q11Q0 What is the external work required to bring four 2.0*10**(-9) C
012Q0 point charges from infinity and to place them at the corner of
25 Q0 a square of side 0.14 m
   Q0
   A1 1.4*10**(-6) Joule.
   A2 1.0*10**(-6) Joule.
   A3 0.3*10**(-6) Joule.
   A4 0.6*10**(-6) Joule.
   A5 1.8*10**(-6) Joule.
   Q0
Q12Q0 In figure (4), an electron moves from point 'I' to point 'F'
012Q0 in a uniform electric field directed as shown in the figure.
25 Q0
   A1 The electric field does positive work on the electron.
   A2 The electric field does negative work on the electron.
   A3 The electric potential energy of the electron increases.
   A4 The electron moves to a lower potential.
   A5 An external force is required to move the electron from I to F.
   Q0
Q13Q0 In figure (5), a hollow sphere, of radius r that carries a
25 Q0 negative charge -q, is put inside another hollow sphere, of
012Q0 radius R that carries a positive charge Q. At a distance x
   Q0 from the common center, such that r < x < R, the electric
   Q0 potential is:
   Q0
   A1  k*[(Q/R)-(q/x)].
   A2  k*[(Q/R)-(q/r)].
   A3  k*[(Q/R)+(q/x)].
   A4  k*[(Q/R)+(q/r)].
   A5  k*[(Q/x)-(q/R)].
   Q0
Q14Q0 In figure (6), Q1 = 2.0*10**(-6) C and Q2 = - 2.0*10**(-6) C.
012Q0 What is the external work needed to move a charge
25 Q0 Q = - 4.0*10**(-6) C at constant speed from point A at the
   Q0 center of the square to point B at the corner?
   Q0
   A1   Zero.
   A2   5.1*10**(-6) Joule.
   A3   7.2*10**(-6) Joule.
   A4 - 5.1*10**(-6) Joule.
   A5 - 7.2*10**(-6) Joule.
   Q0
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T-011

1. A particle [m = 8.0*10**(-9) kg, q = +6.0*10**(-9) C] has a speed of 80 m/s at point A and moves to point B where the potential is 2.0*10**3 V greater than at point A. What is the particle's kinetic energy at point B? (Assume that only electric forces act on the particle during its motion.)[14*10**(-6) J].
2. A 2 meters conducting rod is fixed perpendicularly to a uniform 200 N/C electric field. The potential difference between its ends is: Zero.
3. Two conducting spheres are very far apart. The smaller sphere carries a total charge of 6 micro-C. The larger sphere has a radius twice that of the smaller sphere and is neutral (Q = 0). After the two spheres are connected by a thin conducting wire, the charges on the smaller and the larger spheres, respectively are: [2 micro-C and   4 micro-C].

T-002

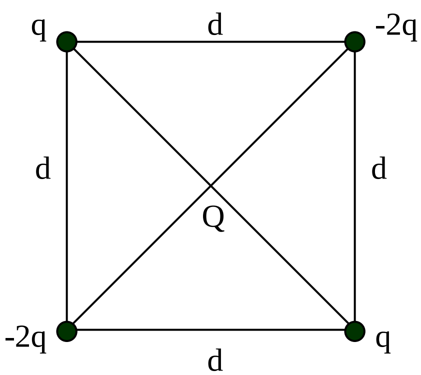

FIGURE 2

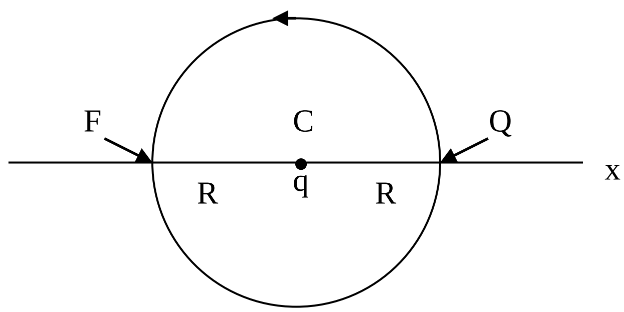

FIGURE 4

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Q0
12 Q0 A particle [m = 8.0*10**(-9) kg, q = +6.0*10**(-9) C] has
25 Q0 a speed of 80 m/s at point A and moves to point B where the
001Q0 potential is 2.0*10**3 V greater than at point A. What is the
002Q0 particle's kinetic energy at point B? (Assume that only
   Q0 electric forces act on the particle during its motion.)
   Q0
   A1 14*10**(-6) J.
   A2 38*10**(-6) J.
   A3 10*10**(-6) J.
   A4 28*10**(-6) J.
   A5 40*10**(-6) J.
   Q0
13 Q0 In figure 2, four charges are fixed at the corners of a square
25 Q0 whose sides are of length d. The work done by an external agent
002Q0 to bring a fifth charge, Q, from infinity to the center of
   Q0 the square is:
   Q0
   A1 - 2.8*k*q*Q/d.
   A2   1.4*k*q*Q/d.
   A3   2.8*k*q*Q/d.
   A4 - 1.4*k*q*Q/d.
   A5   3.4*k*q*Q/d.
   Q0
```

```
14 Q0 A charge q is located at the center of a circle with a large
25 Q0 radius R, see figure 4. Another charge Q is located on the
002Q0 circumference of the circle at the x-axis. What is the work,
   Q0 in Joules, needed to move Q from its location to point F, on
   Q0 the x-axis, along the circumference?
   Q0
   A1  Zero.
   A2  k*q*Q/(2*R).
   A3  k*q*Q/R.
   A4  2*k*q*Q/R.
   A5  k*q/(2R).
   Q0
15 Q0 Which of the following statements are CORRECT:
25 Q0  1. Electric charge is quantized.
002Q0  2. The potential at the center of a charged conductor is zero.
   Q0        ->
   Q0  3. If E  = 0 at a point P then V must be zero at P.
   Q0  4. The electric field inside a charged conductor is zero.
   Q0                                ->
   Q0  5. If V = 0 at a point P then E  must be zero at P.
   Q0
   A1  1 and 4.
   A2  2 and 4.
   A3  1, 2 and 3.
   A4  1, 2, and 5.
   A5  3 and 5.
   Q0
```

T-001

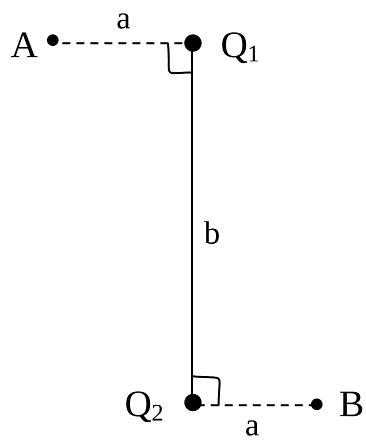

Figure 2

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Q0
11 Q0 Two point charges Q1 and Q2 are positioned as shown in
25 Q0 Figure(2). If Q1 = 2.0*10**(-9) C, Q2 = - 2.0*10**(-9) C,
   Q0 a = 3.0 m, and b = 4.0 m, what is the electric potential
   Q0 difference, VA - VB?
   Q0
   A1  4.8 V
   A2 -8.4 V
   A3  8.4 V
   A4 -6.0 V
   A5 -4.8 V
   Q0
12 Q0 A particle [m = 8.0*10**(-9) kg, q = +6.0*10**(-9) C] has
25 Q0 a speed of 80 m/s at point A and moves to point B where the
   Q0 potential is 2.0*10**3 V greater than at point A. What is the
   Q0 particle's kinetic energy at point B? (Assume that only
   Q0 electric forces act on the particle during its motion.)
```

```
   Q0
   A1 14*10**(-6) J.
   A2 38*10**(-6) J.
   A3 10*10**(-6) J.
   A4 28*10**(-6) J.
   A5 40*10**(-6) J.
   Q0
13 Q0 A 2 meters conducting rod is fixed perpendicularly to a
25 Q0 uniform 200 N/C electric field. The potential difference
   Q0 between its ends is:
   Q0
   A1  Zero.
   A2  400 Volts.
   A3 -400 Volts.
   A4  300 Volts.
   A5  150 Volts.
   Q0
14 Q0 Two conducting spheres are very far apart. The smaller sphere
25 Q0 carries a total charge of 6 micro-C. The larger sphere has a
991Q0 radius twice that of the smaller sphere and is neutral
   Q0 (Q = 0). After the two spheres are connected by a thin
   Q0 conducting wire, the charges on the smaller and the larger
   Q0 spheres, respectively are:
   Q0
   A1  2 micro-C  and   4  micro-C.
   A2 -4 micro-C  and  10  micro-C.
   A3  3 micro-C  and   3  micro-C.
   A4  0          and   6  micro-C.
   A5 -6 micro-C  and   12 micro-C.
   Q0
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Question 1
In figure (7), what is the net potential at point P due to the four point charges if V = 0 at infinity ? [take d = 2 cm, q = 1.0 micro-C].

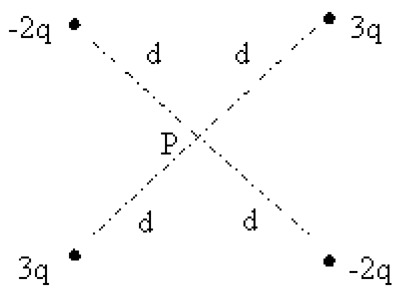

Figure (7)

a - 4.6*10**7 V.
b 4.6*10**7 V.
c 9.0*10**5 V.
d - 9.0*10**5 V.
e Zero.

===

Question 2
Two conducting spheres A and B are electrically isolated. The smaller sphere A has a total charge of 6*10**(-8) C and radius R. Sphere B has a radius (2*R) and is neutral. After connecting the two spheres by a conducting wire, find the charge on sphere B.

a - 6*10**(-8) Coulomb
b + 2*10**(-8) Coulomb
c + 4*10**(-8) Coulomb
d + 6*10**(-8) Coulomb
e + 3*10**(-8) Coulomb

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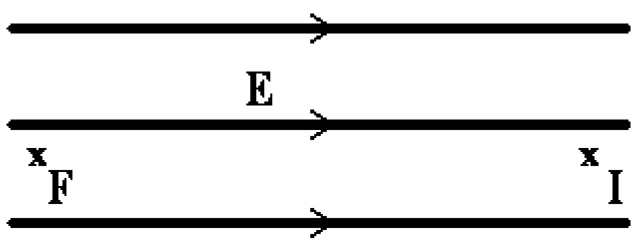
Question 3
In figure (4), an electron moves from point 'I' to point 'F' in a uniform electric field directed as shown in the figure.



a The electron moves to a lower potential.
b The electric field does positive work on the electron.
c The electric field does negative work on the electron.
d An external force is required to move the electron from I to F.
e The electric potential energy of the electron increases.

===

Question 4

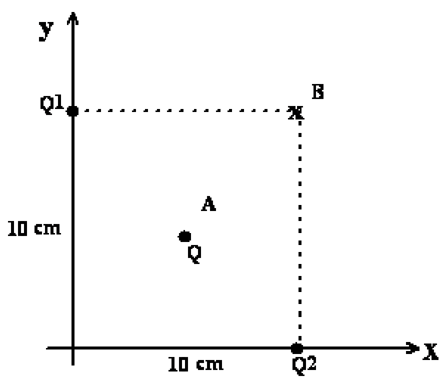
Which one of the following statements is true?

a We have to do work to move a charged particle along an equipotential surface.
b The electric field lines are perpendicular to the equipotential surfaces.
c The electric field is a scalar quantity.
d The electric potential is a vector quantity.
e Any two equipotential surfaces are always parallel.

===

Question 5

In figure (6), Q1 = 2.0*10**(-6) C and Q2 = - 2.0*10**(-6) C.
What is the external work needed to move a charge
Q = - 4.0*10**(-6) C at constant speed from point A at the center of the square to point B at the corner?



a - 7.2*10**(-6) Joule.
b 7.2*10**(-6) Joule.
c 5.1*10**(-6) Joule.
d Zero.
e - 5.1*10**(-6) Joule.

===

Question 6

Two large, parallel, conducting plates are 20 cm apart and have charges of equal magnitude but opposite signs on their facing surfaces. An electron, placed anywhere between the two plates experiences an electrostatic force of 1.6*10**(-15) N. Find the magnitude of the potential difference between the two plates.

a 4 kV
b 2 kV
c 50 kV
d 10 kV
e 30 kV

==

Question 7

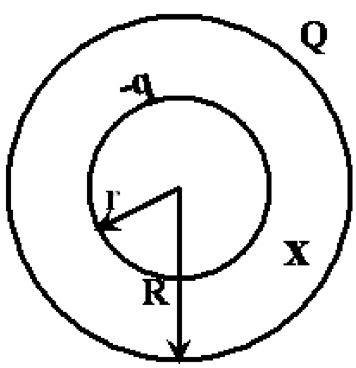
Which one of the following statements is CORRECT ?

a All points of a conductor in electrostatic equilibrium are at the same potential.
b Electric field lines are always parallel to equipotential surfaces.
c The electric field at the surface of a conductor in electrostatic equilibrium is parallel to the surface of the conductor.
d Electric field lines are always in the direction of increasing electric potential.
e If a conducting sphere carries a net charge, the charge will be uniformly distributed over its volume.

==

Question 8

In figure (5), a hollow sphere, of radius r that carries a negative charge -q, is put inside another hollow sphere, of radius R that carries a positive charge Q. At a distance x from the common center, such that r < x < R, the electric potential is:



a k*[(Q/R)-(q/x)].
b k*[(Q/R)-(q/r)].
c k*[(Q/R)+(q/x)].
d k*[(Q/x)-(q/R)].
e k*[(Q/R)+(q/r)].

==

Question 9

The electric potential at point A in an electric field is

15 V smaller than at point B. If a charge q = -2.0 C is moved from A to B, then the electric potential energy of this charge will:

a increase by 15 J.
b increase by 25 J.
c decrease by 30 J.
d decrease by 15 J.
e increase by 30 J.

==

Question 10

The electric potential at points in the xy–plane is given by:
V = (x**3 - 2*x*y) Volts,
where x and y are in meters. The magnitude of the electric field at the point with the coordinates x = 1 m and y = 2 m is:

a Sqrt(3) V/m.
b Zero.
c Sqrt(5) V/m.
d Sqrt(8) V/m.
e Sqrt(2) V/m.

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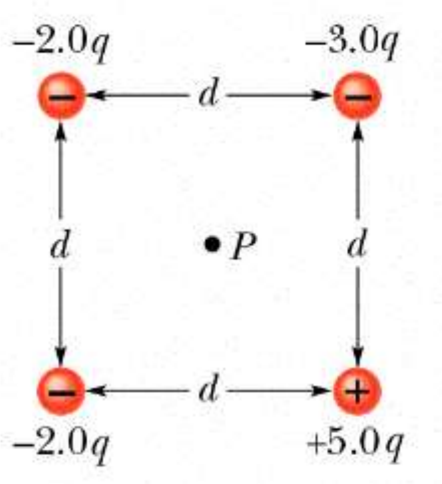
Question 11

In figure 6, point P is at the center of the square.
Find the net electric potential at point P.
Assume V = 0 at infinity.



a + 2.8 kq/d
b - 1.4 kq/d
c zero
d - 2.8 kq/d
e + 1.4 kq/d

==

Question 12

Two balls with charges 5.0 micro-C and 10 micro-C are at a distance of 1.0 m from each other. In order to reduce the distance between them to 0.5 m the amount of work to be performed is:

a 4.5*10**(-4) J.
b 1.2*10**(-4) J.
c 45.0 J.
d 0.23 J.
e 0.45 J.

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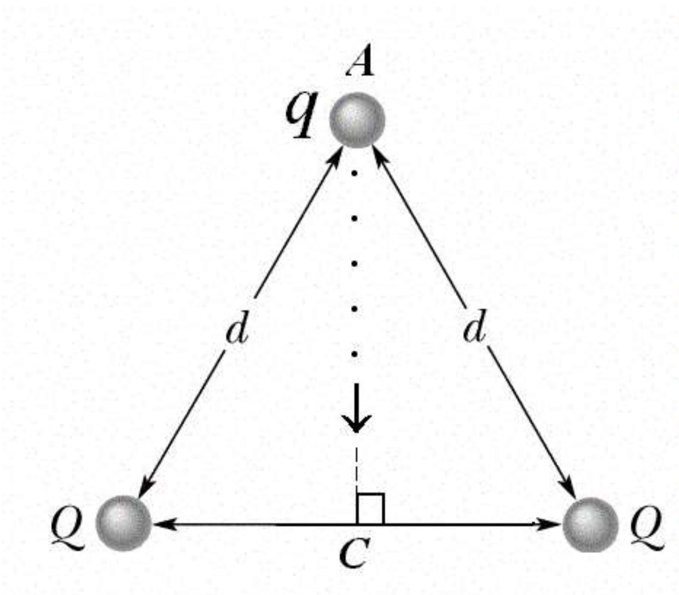
Question 13

Two positive charges, each of magnitude Q, are fixed at two corners of an equilateral triangle (see figure 5). The work required, by an external agent, to move a third positive charge q from A to C is:



a - 2kQq/d
b + kQq/d
c + 2kQq/d
d - kQq/d
e zero

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Question 14

Find the electric potential at the center of a charged metal sphere of radius 15 cm if the electric field at its surface is 1.2*10**4 N/C.

a 1.8 kV
b 2.7 kV
c zero

d 1.2 kV
e Infinity
==
Question 15
What is the external work required to bring four 2.0*10**(-9) C point charges from infinity and to place them at the corner of a square of side 0.14 m

a 1.8*10**(-6) Joule.
b 0.6*10**(-6) Joule.
c 0.3*10**(-6) Joule.
d 1.0*10**(-6) Joule.
e 1.4*10**(-6) Joule.
==
Question 16
Over a certain region, the electric potential is given as
V = (3.0*(x**2)*y)+(y**2)+(y*z) (volts).
Find the magnitude of the electric field at the point (1.0,1.0,1.0), where all distances are in meters.

a 3.6 N/C
b 2.4 N/C
c 8.5 N/C
d 1.0 N/C
e zero
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Question 17
Two electrons are fixed 2.0 cm apart. Another electron is shot from infinity with a speed v and comes to rest at a point midway between the two electrons. Find v.

a 742 m/s
b 318 m/s
c 255 m/s
d 612 m/s
e 963 m/s
==

Answers
1 c
2 c
3 b
4 b
5 d
6 b
7 a
8 a
9 c

10	c
11	d
12	e
13	c
14	a
15	e
16	c
17	b