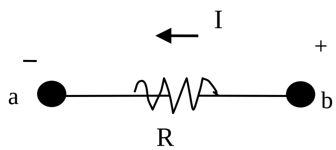
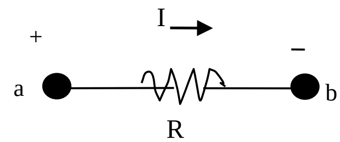
Kirchhoff's Laws

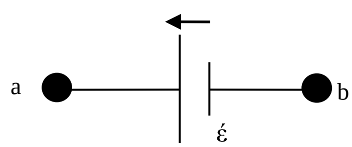
Junction rule: "The sum of the currents entering any junction must be equal to the sum of the current leaving that junction." In symbols $\sum I_{in} = \sum I_{out}$. It is a statement of conservation of charge.

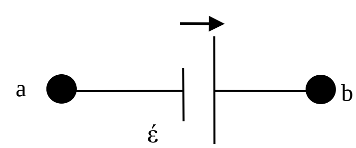
Loop rule: "The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero." In symbols $\sum V_i = 0$. It is a statement of conservation of energy.





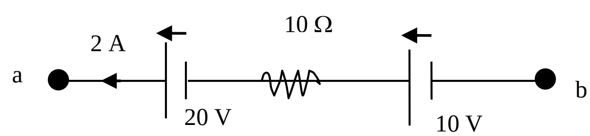
$$\Delta V = V_b - V_a = +iR \quad\quad \Delta V = V_b - V_a = -iR$$
$$\Rightarrow \; V_b - iR = V_a \quad\quad \Rightarrow \; V_b + iR = V_a$$





$$\Delta V = V_b - V_a = -\varepsilon \quad\quad \Delta V = V_b - V_a = +\varepsilon$$
$$\Rightarrow \; V_b + \varepsilon = V_a \quad\quad \Rightarrow \; V_b - \varepsilon = V_a$$

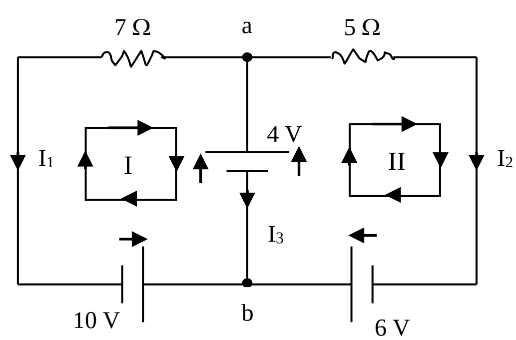
1- Find $V_a - V_b$.



Start with point a and move to the right, one can have

$$V_a - 20 + 2\times 10 - 10 = V_b \quad \Rightarrow \quad V_a - V_b = 10 \text{ volts}$$

2- A- Find the values of the currents in figure (3).



Ans: At the junction b

$I_1 + I_2 + I_3 = 0$ (1)

Loop I implies

$+7I_1 - 10 - 4 = 0 \quad \Rightarrow \quad I_1 = 2A$ (2)

Loop II implies

$4 + 6 - 5I_2 = 0 \quad \Rightarrow \quad I_2 = 2A$ (3)

from (2) and (3) in (1), one can find

$I_3 = -I_1 - I_2 = -4A$

So, the current I_3 will be in the opposite direction.

B- With three different methods, prove that $V_a - V_b = +4V$.

Through loop I, one finds $V_a - I_1(7) + 10 = V_b \Rightarrow V_a - V_b = 14 - 10 = 4$ Volts

Through loop II, one finds $V_a - I_2(5) + 6 = V_b \Rightarrow V_a - V_b = 10 - 6 = 4$ Volts

Through the middle connection, one finds $V_a - 4 = V_b \Rightarrow V_a - V_b = 4$ Volts

3- The two branch currents in the circuit shown in figure are $I_1 = \frac{1}{3}$A and $I_2 = \frac{1}{2}$A .

A- Determine the emfs, E_1 and E_2.

B- With three different methods, prove that $V_a - V_b = 3$ volts.

Ans: At the junction c

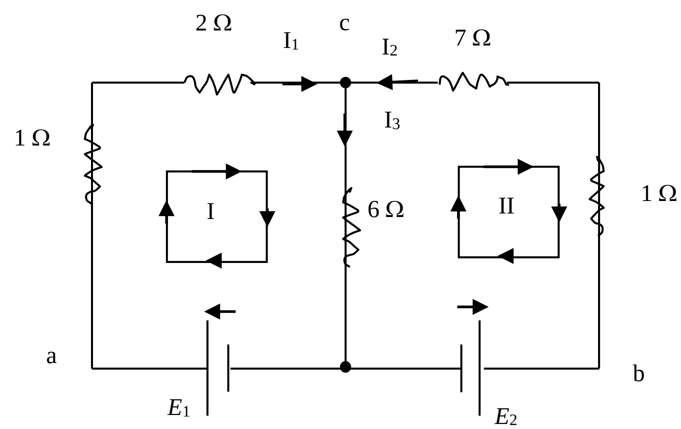
$$I_1 + I_2 = I_3 \quad \Rightarrow \quad I_3 = \frac{5}{6}\text{A} \qquad (1)$$

Loop I implies

$$E_1 - I_1(1+2) - 6I_3 = 0 \quad \Rightarrow \quad E_1 = 6 \text{ volts} \qquad (2)$$

Loop II implies

$$-E_2 + I_2(1+7) + 6I_3 = 0 \quad \Rightarrow \quad E_2 = 9 \text{ volts} \qquad (3)$$



- If the current I in figure (5) is equal to 4.0 A, then the potential difference between point 1 and 2, i.e. (V2 - V1), is:

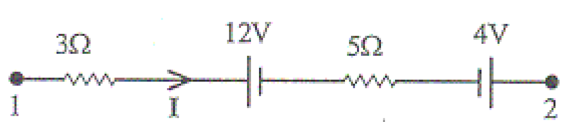


Figure (5)

$$V_1 - 3I - 12 - 5I + 4 = V_2$$

$$\Rightarrow V_2 - V_1 = -40 \text{ Volts}$$

- How long will it take a charged capacitor of 50.0×10^{-6} F to loss 30% of its initial energy if allowed to discharge through a 40 Ohm resistor?

$$\because U_i = \frac{1}{2}\frac{q_i^2}{C}, \quad \therefore U_f = \frac{1}{2}\frac{q_f^2}{C} = 0.7U_i = 0.7\frac{1}{2}\frac{q_i^2}{C}$$
$$\Rightarrow Q_f = \sqrt{0.7}Q_i$$
$$\therefore Q_f = Q_i e^{-t/RC} \Rightarrow \sqrt{0.7}Q_i = Q_i e^{-t/RC}$$
$$\Rightarrow t = -RC \ln\sqrt{0.7} = \underline{0.36\times10^{-3}\ \text{s}}$$

- The capacitor in figure (1) is initially charged to 50 V and then the switch is closed. What charge flows out of the capacitor during the first minute after the switch was closed?

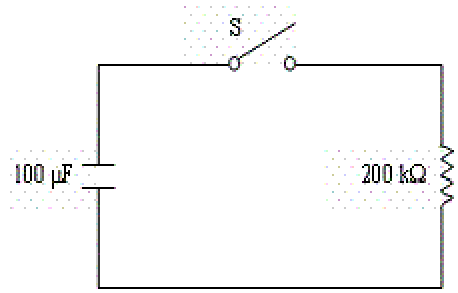


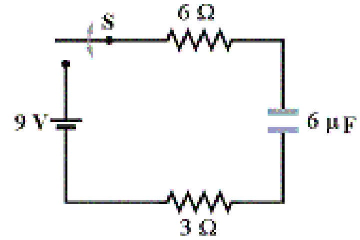
Figure 1

$$\because Q = Q_o(1-e^{-t/RC}),$$
$$Q_o = \varepsilon C = 50\times1.0\times10^{-4} = 5.0\ \text{mC},$$
$$\tau = RC = 2.0\times10^5\times1.0\times10^{-4} = 20.0\ \text{s}$$
$$\therefore Q = 5\ \text{mC}(1-e^{-60/20}) = \underline{4.75\ \text{mC}}$$

- At t=0, a 2.0×10^{-6} Farad capacitor is connected in series to a 20-V battery and a 2.0×10^{6} Ohm resistor. How long does it take for the potential difference across the capacitor to be 12 V?

$$\because V = V_{\max}(1-e^{-t/RC})$$
$$\therefore 12 = 20(1-e^{-t/4})$$
$$\Rightarrow t = RC = 2.0\times10^5\times1.0\times10^{-4} = \underline{3.66\ \text{s}}$$

- In the circuit shown in the following figure, the capacitor was initially uncharged. At time t = 0, switch S is closed. If τ denotes the time constant, the current through the 3-ohm resistor at $t = \tau/10$ is



$$I = \frac{\varepsilon}{R}e^{-t/\tau} = \frac{9}{9}e^{-1/10} = 0.90\ \text{A}$$

- A capacitor, initially uncharged in a single-loop RC circuit, is charged to 85% of its final potential difference in 2.4 s. What is its time constant in seconds?

$$\because V = V_{\max}(1-e^{-t/RC})$$
$$\therefore 0.85 = (1-e^{-t/\tau}) \quad \Rightarrow \quad 0.85 = (1-e^{-t/\tau})$$
$$\therefore \ln 0.15 = -\frac{t}{\tau} \Rightarrow \tau = -\frac{2.4}{\ln 0.15} = 1.27 = \underline{1.3\text{ s}}$$

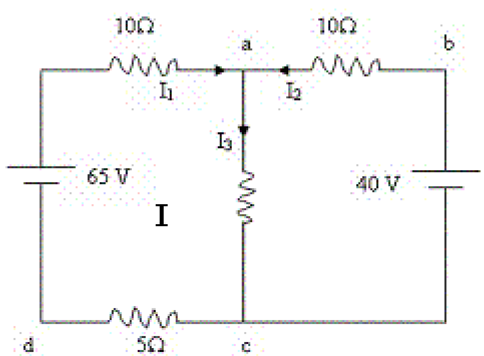
- A $1.0\ \mu\text{F}$ uncharged capacitor and a $3.0\ \text{k}\Omega$ resistor are connected in series, and then (at time t = 0) a 6-V potential difference is applied across them. Find the time at which the voltage on the capacitor is 3.8 V.

$$C = 1.0\ \mu\text{F}, R = 3.0\ \text{k}\Omega \quad \Rightarrow \quad \tau = 3.0\text{ ms}$$
$$\because V = \varepsilon(1-e^{-t/\tau}) \quad \Rightarrow \quad e^{-t/\tau} = 1-\frac{V}{\varepsilon} = 0.367$$
$$\therefore t = -\tau\ln(0.367) = \underline{3.0\text{ ms}}.$$

- Q16. A capacitor of capacitance C takes 2 s to reach 63 % of its maximum charge when connected in series to a resistance R and a battery of emf ε. How long does it take for this capacitor to reach 95 % of its maximum charge (from zero initial charge)? (Ans: 6s)

$$1-q(t) = q_0(1-e^{-t/\tau}) \quad \Rightarrow \quad e^{-t/\tau} = 1-0.63 = 0.37$$
$$\Rightarrow \quad \tau = -t/\ln(0.37) \approx 2\text{ s}$$
$$0.95q_0 = q_0(1-e^{-t/2}) \quad \Rightarrow \quad e^{-t/2} = 1-0.95 = 0.05$$
$$\Rightarrow \quad t = -2\ln(0.05) \approx \underline{6\text{ s}}$$

Example: For the circuit shown in figure (4), if $V_a - V_c = 20$ Volts, what is the potential difference $V_b - V_d$?



(a) -55 Volts.
(b) 25 Volts.
(c) -25 Volts.

In Loop I, $65-15I_1-20=0 \Rightarrow I_1 = 3\text{A}$

$$\therefore \quad V_b - V_d = V_b - V_c + V_c - V_d = 40+3\times 5 = \underline{55\text{ V}}$$

(d) 35 Volts.
(e) 55 Volts.

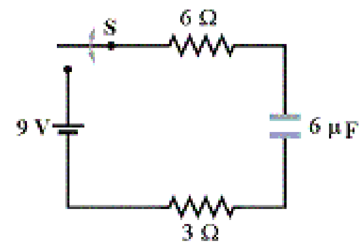
Example: A capacitor of capacitance C is discharging through a resistor of resistance R. In terms of RC , when will the energy stored in the capacitor reduces to one fifth of its initial value?

$$\because U = U_o e^{-2t/\tau}$$

$$\therefore \tfrac{1}{5} U_o = U_o e^{-2t/\tau} \Rightarrow t = \left(\frac{\ln 5}{2}\right)\tau = 0.80\ RC$$

Example: In the circuit shown in the following figure, the capacitor was initially uncharged. At time t = 0, switch S is closed. If τ denotes the time constant, the current through the 3-ohm resistor at $t = \tau/10$ is

$$I = \frac{\varepsilon}{R} e^{-t/\tau} = \frac{9}{9} e^{-1/10} = 0.90 \text{ A}$$



● A fully charged 6.0 μF capacitor is connected in series with a $1.5 \times 10^5\ \Omega$ resistor. What percentage of the original charge is left on the capacitor after 1.8 s of discharging?

Solve: The time constant for this circuit is $\tau = RC = (1.5\times10^5\ \Omega)(6.0\times10^{-6}\ \text{F}) = 0.90$ s. At $t = 1.8$ s, which is 2τ, the fractional charge remaining is $\dfrac{q}{Q_0} = e^{-t/\tau} = e^{-(1.8\text{ s})/(0.90\text{ s})} = e^{-2} = 0.14$ or 14%.

●● When a capacitor is being charged up, (a) how many time constants are required for it to receive 95% of its maximum charge, and (b) what is the current in the circuit at that time?

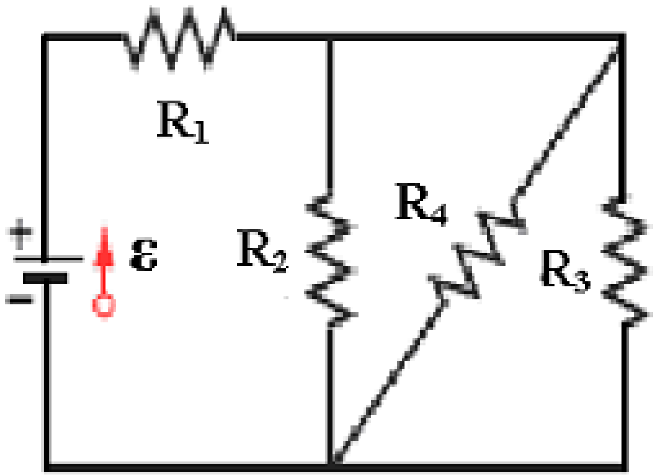
19.68. Set Up: The time constant is $\tau = RC$, so $q = Q_{\text{final}}(1 - e^{-t/RC})$ and $i = I_0 e^{-t/RC}$.

Solve: (a) $q = 0.95 Q_{\text{final}}$ gives $0.95 = 1 - e^{-t/RC}$. $e^{-t/\tau} = 0.05$. $-\dfrac{t}{\tau} = \ln(0.05)$. $\dfrac{t}{\tau} = 3.0$. Three time constants are required.

(b) $e^{-t/\tau} = 0.05$ so $i = 0.05 I_0$.

In **Figure 6**, $R_1 = 100\ \Omega$, $R_2 = R_3 = R_4 = 75\ \Omega$, and the ideal battery has emf $\boldsymbol{\varepsilon} = 6.0$ V. What is the current in R_1?

Figure 6



R_2, R_3 *and* R_4 *are in parallel*

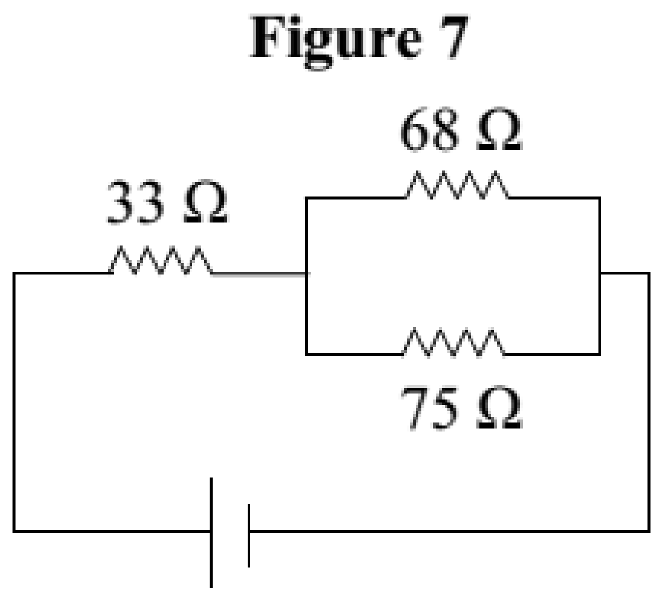
$$\therefore \mathrm{R_{eq}} = \frac{75}{3} = 25\ \Omega$$

R_1 *is in series with* R_{eq}

$$\therefore \mathrm{R'_{eq}} = \mathrm{R_{eq}} + \mathrm{R_1} = 125\ \Omega$$

$$\therefore \mathrm{i} = \frac{\varepsilon}{\mathrm{R'_{eq}}} = 48\ \mathrm{mA}$$

In the circuit shown in **Figure 7**, the 33Ω resistor dissipates 0.50 W. What is the emf of the ideal battery?



$$P = I^2R \rightarrow I = \sqrt{\frac{P_{33}}{R_{33}}} = 0.123A, R_{eq} = 33 + (\frac{1}{68} + \frac{1}{75})^{-1} = 68.88\Omega,$$

$$V = IR_{eq} = 8.5V$$