Glossary

The following terms ore commonly used in statistical mechanics. Therefore, it is convenient to know their definition.

- ***Accessible microstate:*** Any macrostate of a system which is its quantum state in which the system can be found without breaking any conditions imposed by the macroscopic information available, such as $N = \sum_i n_i, \quad E = \sum_i \varepsilon_i$, about the system.

- ***Configuration:*** Each way in which the molecules of a substance (particularly those of a gas inside a container) can be distributed is called a configuration. There are two types of distribution in statistical mechanics.
  - ✓ ***Ordered distribution***. In such a case the number of possible configurations of the molecules is the minimum. This is rare case and needs special conditions, for example: $T \to 0$
  - ✓ ***Disordered*** *or* ***random distribution***. In this case the number of possible configurations is the maximum.

- **Canonical Ensemble:** ('canonical' because it is used so often to model real systems): It is a statistical representative of a system in equilibrium (thermal contact) with a heat reservoir (bath) at some fixed temperature. It has a fixed N, V and T but variable *E*. The system can exchange energy with the heat bath, so that the states of the system will differ in total energy. This is a ***closed system***.

- **Chemical potential (μ):**
  - ✓ May be regarded as the Lagrangian multiplier introduced to satisfy the constraint of a fixed number of particles. Also,
  - ✓ It is the change in the energy of the system as a result of the change in its number of particles, when every other thermodynamical variable that describes the state of the system, such as entropy, volume, etc., is kept constant.

    It has the units of energy/molecule, or otherwise stated, for a single species of particles in a grand canonical ensemble. $-\infty < \mu < 0$ for bosons and $-\infty < \mu < \infty$ for fermions.

- **Degeneracy**: **T**his is the number of accessible microstates.
  - i- <u>Degenerate energy levels</u>, different arrangements of a physical system which have the same energy.
  - ii- <u>Degenerate matter</u>, a very highly compressed phase of matter which resists further compression because of quantum mechanical effects
  - iii- <u>Degenerate semiconductor</u>, a semiconductor with such a high doping-level that the material starts to act more like a metal than as a semiconductor

- **Density of states:** It is the number of microstates (i.e. the number of independent quantum states) of an *N* particle system per unit energy range. In other words, the density of states, denoted by $g(E)$, indicates how densely packed quantum states in a particular system.

Macroscopically, the density of states can be treated as a continuous function of the internal energy of the system.

- **Equal A-Priori Probabilities:** *a priori* from the Latin, meaning ``conceived beforehand". A system in a given macrostate is equally likely to be found in any one of the Ω microstates compatible with it.

  Discussion: To a given macrostate of the system there are in general correspond a large number of microstates, and it seems natural to assume that at any time t the system is equally likely to be in any one of these microstates. The quantity $w(N;V;E)$ will be defined as the actual number of possible microstate.

- **Ensemble:** (also **statistical ensemble**)
  - i- A set of replicas of our system. Each replica could be in a different quantum state.
  - ii- The ensemble is a “mixture” of different quantum states, each with a certain probability, or relative frequency p_i.
  - iii- is an idealization consisting of a large number of virtual copies (sometimes infinitely many) of a system, considered all at once, each of which represents a possible state that the real system might be in.
  - iv- a statistical ensemble is a probability distribution for the state of the system.

- **Energy level:** The collection of all the states with the same energy is often called an energy level. It is composed of all states having the same energy.

- **Equilibrium state of a system:** A system of many particles, such as a gas. Whose macroscopic (i.e. large scale) state does not tend to change in time is said to be in equilibrium.

- **Microstate of a system:** The microstate of a system at any time is given by specifying the maximum possible information about the system (say. a gas) molecules at this time, e.g. the position and velocity of each molecule. It is a particular quantum state of a system.

- **Principle of Equal Equilibrium Probabilities:** For an isolated system in equilibrium, all microstates compatible with the given constraints (N; V; E) have equal a priori probability.

- **Phase space:** To describe both the position and the state of motion of a point particle in space, it is customary to set up a six-dimensional space, called the **phase pace** in which the six co-ordinates x. y, z. p_x. p_y, p_z are marked out along six mutually perpendicular axes (p_x. p_y, and p_z are momenta).

- **Statistical mechanics:** It is a probabilistic approach to equilibrium macroscopic properties of large numbers of degrees of freedom. Or, it is the branch of physics that applies statistical principles (or probability) to the mechanical behavior of large numbers of **small** particles (such as molecules, atoms, or subatomic particles) in order to explain the overall properties of the matter composed of such particles.

The **method of Lagrange multipliers** is a strategy for finding the local maxima and minima of a function subject to equality constraints.
For instance, consider the optimization problem

maximize $f(x, y)$

subject to $g(x, y) = 0$.

We need both f and g to have continuous first partial derivatives. We introduce a new variable (λ) called a Lagrange multiplier and study the Lagrange function (or Lagrangian) defined by:

$$\mathcal{L}(x, y, \lambda) = f(x, y) + \lambda \cdot g(x, y),$$

where the λ term may be either added or subtracted.