Example 1
A large container, see figure 1, of water (ρ = 1000 kg/m^3) contains a thin, light plate at a depth of 3 m below the surface of the water. Neglect the mass and volume of the thin plate. The plate can be elevated by a jack without disturbing the water in the container.
(a) What is the gauge pressure at the depth of the plate?
(b) What is the absolute pressure at the depth of the plate?

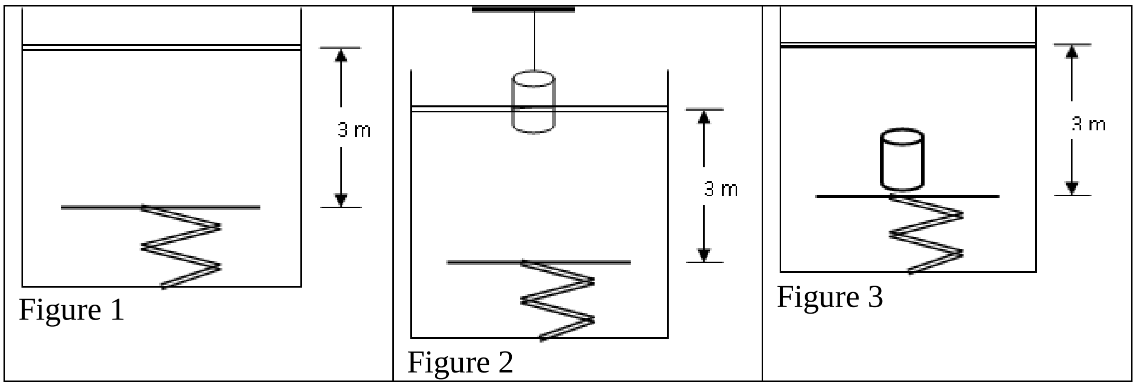


Figure 1 Figure 2 Figure 3

A solid aluminum cylinder (ρ = 2700 kg/m^3) of radius 0.25 m and height 1 m is lowered by a cable in the water until half the cylinder is beneath the surface of the water, see figure 2, where it remains at rest.
(c) What is the tension in the cable?
(d) The cylinder is then lowered onto the light plate, see figure 3, and the cable is removed. Find the force exerted by the plate on the cylinder if the jack lifts the plate upward at
i. a constant speed of 2 m/s
ii. an acceleration of 1 m/s^2.

Solution

(a) $p_{gauge} = \rho gh = (1000\,kg/m^3)(10\,m/s^2)(3m) = 3x10^4\ Pa$

(b) $p_2 = p_1 + \rho gh = 1.013x10^5\ Pa + (1000\,kg/m^3)(10\,m/s^2)(3m) = 1.313x10^5\ Pa$

(c) The tension in the cable is equal to the weight of the cylinder minus the buoyant force acting on the cylinder.

$$F_T = m_{Al}\,g - \rho g V_{displacedwater}$$

$$F_T = (\rho_{Al} V_{Al})g - \rho g V_{displacedwater}$$

The volume of the aluminum is

$$V_{Al} = \pi r^2 h = \pi(0.25m)^2(1m) = 0.20\,m^3$$

The volume of the displaced water is half of the volume of the aluminum, or 0.10 m^3 . Substituting the known values into the equation for the tension, we get

$$F_T = 4400\,N$$

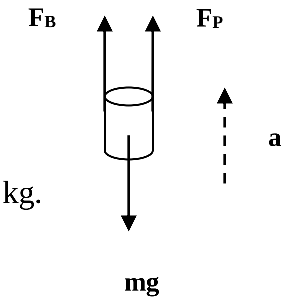
(d) i. For the jack to lift the aluminum cylinder it must apply a force equal to the apparent weight of the cylinder.

$$F = mg - F_{Buoyant} = (\rho_{Al} V_{Al})g - \rho g V_{displaced\ water} = gV(\rho_{Al} - \rho_{water})$$

$$F = (10\, m/s^2)(0.20\, m^3)(2700\, kg/m^3 - 1000\, kg/m^3)$$

$$F = 3400\, N$$

ii. Drawing the free-body diagram for the cylinder:



$$F_P + F_B - mg = ma$$

$$F_P = -F_B + mg + ma$$

where the mass of the aluminum cylinder is $\rho_{Al}\ V_{Al}$ = 540 kg.
Then

$$F_P = -\rho g V_{displacedwater} + mg + ma$$

Substituting, we get

$$F_P = 3940\, N$$

CHAPTER 14 REVIEW QUESTIONS
For each of the multiple choice questions below, choose the best answer.
Unless otherwise noted, use $g = 10$ m/s^2.

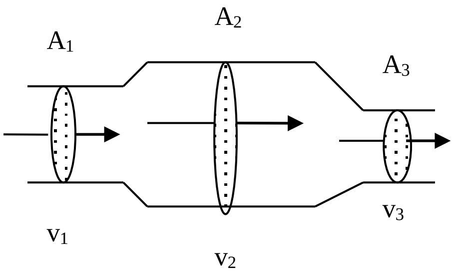
1. *Gauge pressure* at a certain depth below the surface of a fluid is equal to
   A. the pressure at the surface of the fluid
   B. the difference between the absolute pressure and the pressure at the surface of the fluid
   C. the sum of the absolute pressure and the pressure at the surface of the fluid
   D. the absolute pressure
   E. the density of the fluid

2. The pressure at the surface of the ocean is 1 atm (1×10^5 Pa). At what approximate depth in the ocean water ($\rho = 1025$ kg/m^3) would the absolute pressure be 2 atm?
(A) 1 m
(B) 5 m
(C) 10 m
(D) 100 m
(E) 1000 m

Questions 3 – 4: A ball weighing 6 N in air and having a volume of 5×10^{-4} m^3 is fully immersed in a beaker of water and rests on the bottom. The combined weight of the beaker and water without the ball is 10 N.

3. The buoyant force acting on the ball is most nearly
(A) 1 N
(B) 2 N
(C) 3 N
(D) 4 N
(E) 5 N

4. If the beaker, water, and the ball in the water are placed on a Newton scale, the scale will read
(A) 16 N
(B) 15 N
(C) 11 N
(D) 10 N
(E) 6 N

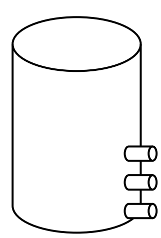


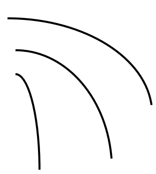
Questions 5-6: The three sections of the pipe shown above have areas A_1, A_2, and A_3. The speeds of the fluid passing through each section of the pipe are v_1, v_2, and v_3, respectively. The areas are related by $A_2 = 4A_1 = 8A_3$. Assume the fluid flows horizontally.

5. Which of the following is true of the speeds of the fluid in each section in the pipe?
(A) $v_3 = 2v_1$
(B) $v_3 = 8v_2$
(C) $v_2 = ½\ v_1$
(D) $v_2 = 16v_1$
(E) $v_3 = 64v_2$

6. Which of the following is true of the pressures in each section of the pipe?
(A) $p_1 > p_2 > p_3$
(B) $p_2 > p_1 > p_3$
(C) $p_3 > p_2 > p_1$
(D) $p_2 > p_3 > p_1$
(E) $p_1 > p_3 > p_2$

7. The large container above is filled with water. Three small spouts near the bottom of the container are of equal size and are initially corked. If the corks are removed from the spouts, which of the following best represents the path of the water stream from each spout?



(A)


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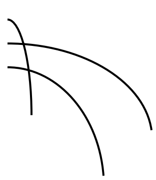

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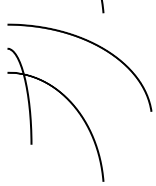
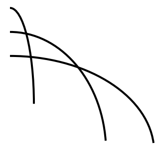


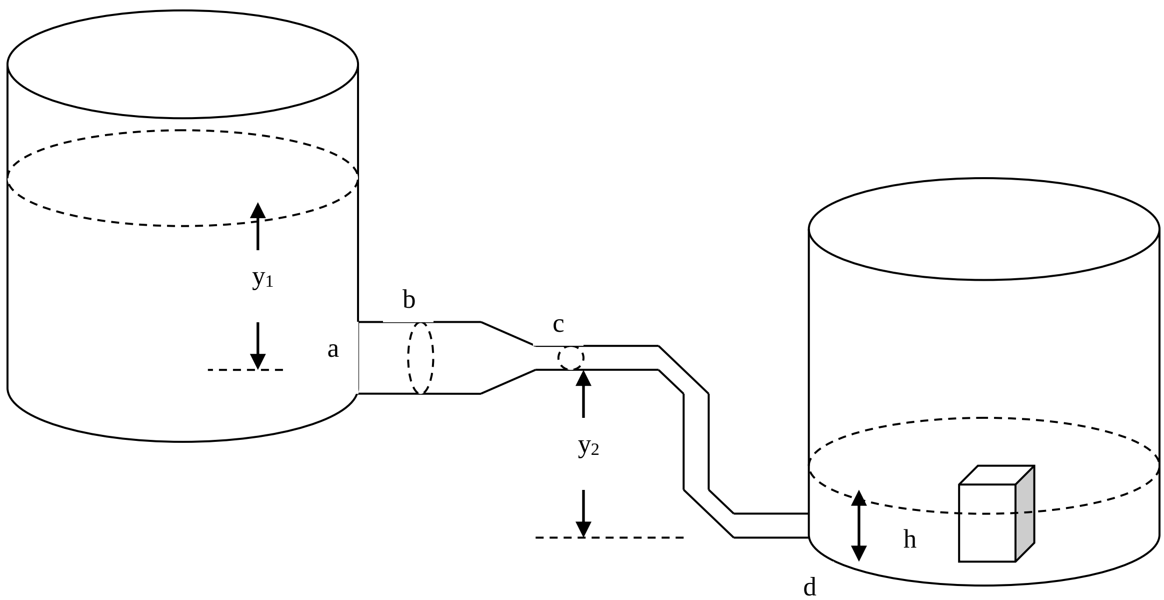
(D) 

(E) 

Free Response Question

Directions: Show all work in working the following question. The question is worth 15 points, and the suggested time for answering the question is about 15 minutes. The parts within a question may not have equal weight.

1. (15 points)



Note: Figure not drawn to scale.

A cylindrical-shaped pipe can carry water from a very large elevated container on the left to a lower container on the right. The area of the wider portion of the pipe containing the point *b* has a cross-sectional area A_b = 7.80 x 10^{-3} m^2 , and the narrower section of the pipe containing both points *c* and *d* has a cross-sectional area of A_c = 3.14 x 10^{-4} m^2. Point C is at a height of y_2 = 2 m above point *d*. A water valve closes the elevated container at point *a*, and thus there is initially only water in the upper container, and none in the pipe. The rectangular block in the lower container above has dimensions 10 cm x 3 cm x 3 cm and mass 0.075 kg, and it rests on the bottom of the lower container before any water enters the lower container.

(a) If the pressure at the surface of the water is 1 atm, what is the absolute pressure at point *a* which is at a depth of y_1 = 2 meters below the surface of the water in the tank?

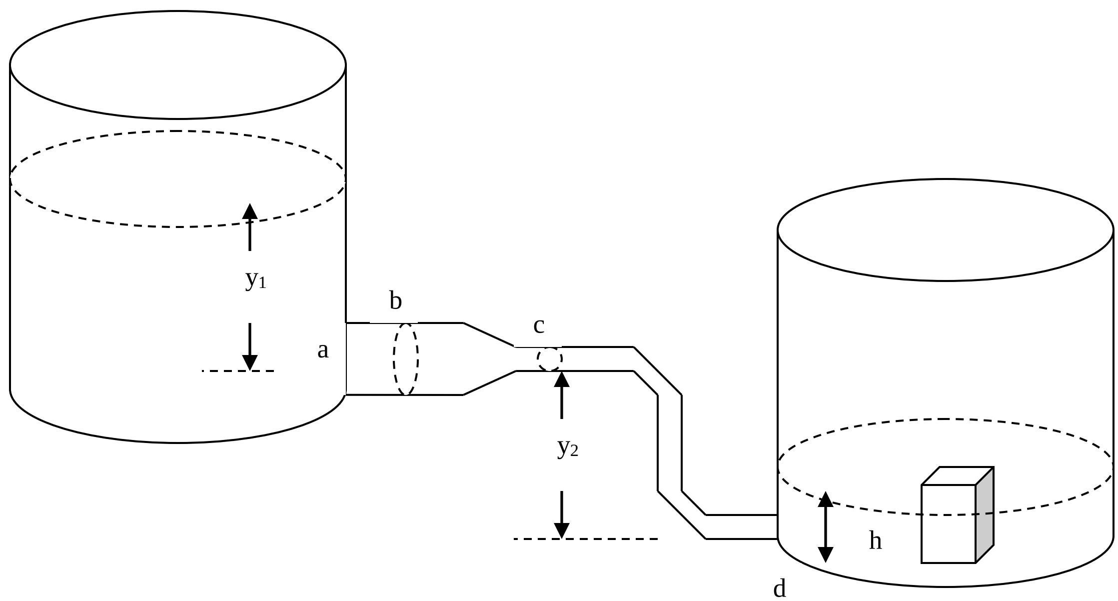
The valve at point *a* is opened to create an opening equal to the area of the pipe containing the point *b* so that water flows from the elevated container through the pipe, and into the lower container.

(b) Consider the pressure at points *b*and *c*. At which of these points is the pressure the *least*? Justify your answer.

(c) If the speed of the water at point *b* is v_b = 6 m/s, what is the speed of the water at point *c*?

(d) Determine v_d, the speed at which the water initially enters the lower container.

(e) As the water level rises in the lower container, the block eventually begins to float. What is the height *h* of the water level at the instant the block is lifted off the bottom of the container, that is, the block just begins to float?



ANSWERS AND EXPLANATIONS TO CHAPTER 11 REVIEW QUESTIONS

Multiple Choice

1. B

$$p_{gauge} = p_{absolute} - p_{surface} = \rho gh$$

2. C

The gauge pressure is the difference between the absolute pressure and the pressure at the surface of the water:

$$p_{gauge} = p_{absolute} - p_{surface} = 2atm - 1atm = 1atm = 1x10^5\ Pa$$

$$p_{gauge} = \rho gh$$

$$h = \frac{p_{gauge}}{\rho g} = \frac{1x10^5\ Pa}{(1025\,kg/m^3)(10\,m/s^2)} \approx 10\,m$$

3. E

$$F_B = \rho g V_{disp\ fluid} = (1000\,kg/m^3)(10\,m/s^2)(5x10^{-4}\,m^3) = 5\,N$$

4. A

The scale will read the actual weight of the beaker, the water, and the ball, since the buoyant force is an internal force as far as the scale is concerned.
Weight on scale = 10 N + 6 N = 16 N

5. A

According to the equation of continuity, the speed of a fluid through a pipe is inversely proportional to the area of the pipe. Since $4A_1 = 8A_3$, $8v_1 = 4v_3$, or $v_3 = 2v_1$.

6. B

According to Bernoulli's principle, the higher the speed in a pipe, the lower the pressure of the fluid. Since $v_3 > v_1 > v_2$, then $p_2 > p_1 > p_3$.

9. D

If we neglect the small difference water level between the pipes, the Bernoulli equation becomes $p_1 + \frac{1}{2}\rho v_1{}^2 = p_2 + \frac{1}{2}\rho v_2{}^2$. Solving for the pressure difference, we get

$$p_1 - p_2 = \frac{1}{2}\rho v_2{}^2 - \frac{1}{2}\rho v_1{}^2 = 21Pa$$

Free Response Question Solution

(a) 3 points

$$p_a = p_{surface} + \rho gh = 1.013x10^5\ Pa + (1000\,kg/m^3)(10\,m/s^2)(2\,m) = 1.213x10^5\ Pa$$

(b) 3 points
The equation of continuity states that the speed in a pipe is inversely proportional to the area of the pipe:
$A_b v_b = A_c v_c$
Since the area at *b* is greater than the area at *c*, the speed at *c* is greater than the speed at *b*. According to the Bernoulli equation, a higher speed at a point indicates a lower pressure at that point. Thus, the pressure at point *c* is a lower than at point *b*.

(c) 3 points
$$A_b v_b = A_c v_c$$
$$\left(7.80\,x10^{-3}\,m^2\right)\left(6\,m/s\right) = \left(3.14\,x10^{-4}\,m^2\right)v_c$$
$$v_c = 149\,m/s$$

(d) 2 points
As the water enters the lower container at point *d* it must have the same speed as the water at point *c*. The water does not separate and is not compressed as it flows through the pipe from point *c* to point *d*, and thus keeps a constant speed between the two points.

(e) 4 points
As the lower container fills with water, there is a height h at which the water will cause the rectangular block to float. When the water reaches this height, the buoyant force acting on the block is just equal to the weight of the block:
$$F_B = mg$$
$$\rho g V_{disp\,water} = mg$$
$$\rho g\left(lwh\right) = mg$$
$$h = \frac{m}{\rho lw} = \frac{0.075\,kg}{\left(1000\,kg/m^3\right)\left(0.03\,m\right)\left(0.03\,m\right)} = 0.083\,m = 8.3\,cm$$