Chapter14
Fluids

Fluid is the name given to a substance which begins to flow when external force is applied on it, typically a liquid or a gas. Fluids do not have their own shape but take the shape of the containing vessel. The branch of physics which deals with the study of fluids at rest is called hydrostatics, such as the pressure of a fluid at a particular depth, or the *buoyant force* acting on an object in a fluid. The branch which deals with the study of fluids in motion is called hydrodynamics, such as a fluid flows through a pipe, the *flow rate* through the cross section is the same at any point in the pipe. *Bernoulli's equation* relates static pressure of a fluid to its dynamic (moving) pressure.

We begin our study with fluid statics, the study of fluids at rest in equilibrium. Then with fluid dynamics, the study of fluids in motion. It conforms to the shape of their container because it cannot withstand shearing stress. It can, however, exert a force perpendicular to its surface.

QUICK REFERENCE

Important Terms

Absolute (total) pressure	the total static pressure at a certain depth in a fluid, including the pressure at the surface of the fluid
Archimedes principle	the buoyant force acting on an object in a fluid is equal to the weight of the fluid displaced by the object.
Bernoulli's principle	the sum of the pressures exerted by a fluid in a closed system is constant.
density	the ratio of the mass to the volume of a substance
flow rate continuity	the volume or mass entering any point must also exit that point
fluid	any substance that flows, typically a liquid or a gas
gauge pressure	the difference between the static pressure at a certain depth in a fluid and the pressure at the surface of the fluid
hydrodynamics	the study of fluids in motion
hydrostatics	the study of fluids in rest
ideal fluid	a noncompressible, nonviscous fluid which exhibits steady flow, that is, the velocity of the fluid particles is constant
liquid	substance which has a fixed volume, but retains the shape of its container
Pascal's priciple	If gravity effect is neglected, the pressure applied to an enclosed fluid is transmitted equally (**undiminished**) to every portion of the fluid and the walls of the containing vessel.
pressure	force per unit area. the SI unit for pressure equal to one newton of force per square meter of area

Equations and Symbols

$P = \frac{F}{A}$ $\rho = \frac{m}{V}$ $P_{depth} = P_0 + \rho g h$ $F_B = W_{fluid} = \rho g V_{fluid}$ $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$ (mass flow rate) $A_1 v_1 = A_2 v_2$ (volume flow rate) $P_1 + \frac{1}{2}\rho {v_1}^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho {v_2}^2 + \rho g y_2$	where P = pressure F = force perpendicular to a surface A = area ρ = density m = mass V = volume F_B = buoyant force W = weight g = acceleration due to gravity v = speed or velocity y = height above some reference level

General information (for reading)

Forms of Matter

1. **SOLID**: Solids have the following properties:

a- Molecules in solids are arranged in a regular shape (crystalline form), with small intermolecular spaces, and held together by strong attractive forces (cohesive forces).
b- Solids have a definite shape and volume (as it is difficult to compress or extend a solid).
c- Molecules of solids can vibrate to-and-fro about a fixed position (zero resultant force position) alternately attracting and repelling one another.
d- The amplitude of molecular vibrations increases on heating.

2. **LIQUID**: Liquids have the following properties:

a. The attractive forces between molecules in liquids are not strong enough to give the liquid a definite shape, so it takes the shape of its container.
b. Intermolecular spaces are wider than in solids.
c. Molecules are vibrating to-and-fro attracting and repelling one another (or: sliding over each other). In the same time the liquid molecules can move freely among one another (i.e. exchanging their places) in all directions.
d. Rise of temperature causes the molecules to move faster, leading to expansion of liquid.

3. **GAS**: Gases have the following properties:

a- The gas has no regular shape.
b- The intermolecular forces are negligible; consequently, the gases have very large intermolecular spaces.
c- The molecules in gases move freely in all directions (i.e. randomly).
d- The speed of gas molecules are very high, colliding with one another or to the walls of the containing vessel, producing a pressure.
e- A gas is perfectly free to expand and completely fill the containing vessel.
f- Rise in temperature of the gas increases its kinetic energy. Also, the collision with the walls increases, so that the gas pressure increases.

Property	Solid	Liquid	Gas
Volume	Definite	Definite	Variable – expands or contracts to fill container
Shape	Definite	Takes up shape of bottom of container	Takes up the shape of the whole container
Density	High	Medium	Low
Expansion when heated	Low	Medium	High
Effect of applied pressure	Very Slight	Slight decrease in volume	Large decrease in volume
Movement of particles	Very slow	Medium	Fast

14-1 FLUIDS, DENSITY, AND PRESSURE

Density: In a fluid, the **density** is defined as the mass per unit volume. If a mass m of homogeneous material has volume V, the density ρ is

$$\text{density} = \frac{\text{mass}}{\text{volume}}, \quad \rho \equiv \frac{m}{V}, \qquad (\rho = \text{greek letter "rho"}) \qquad (14.1)$$

The SI unit of density is the kilogram per cubic meter (kg/m^3). The cgs unit, the gram per cubic centimeter (g/cm^3), is also widely used.

Example: Calculate the density of water at $4\ \text{C}^\circ$

Answer: $$\rho_{\text{water}}\ 4\ \text{C}^\circ = \frac{1\ \text{gram}}{\text{cm}^3} \times \frac{1\ \text{kg}}{10^3\ \text{gram}} \times \left(\frac{100\ \text{cm}}{1\ \text{m}}\right)^3 = 10^3\ \frac{\text{kg}}{\text{m}^3},$$

Material	water	air	Iron	Pb	Al	Cork
Density kg/m^3	10^3	1.21	7.9×10^3	11.9×10^3	2.7×10^3	?

Sometimes instead of density we use the term “relative density” or “specific gravity” which is defined as :

$$\text{relative density} = \text{RD} = \frac{\text{Density of body}}{\text{Density of water } 4\ \text{C}^\circ} = \frac{\rho}{\rho_{\text{water}}}$$

Material	Iron	Pb	Al	Cork
relative density	7.9	11.9	2.7	?

- Specific gravity is a poor term, since it has nothing to do with gravity; relative density would have been better.

Example: A living room has floor dimensions of 3.5 m and 4.2 m and a height of 2.4 m. Calculate the weight of the air with pressure is 1.0 atm.

Answer:

$$\begin{aligned} mg &= (\rho V)g \\ &= (1.21\ \text{kg/m}^3)(3.5\ \text{m} \times 4.2\ \text{m} \times 2.4\ \text{m})(9.8\ \text{m/s}^2) \\ &= 418\ \text{N} \approx 420\ \text{N}. \qquad \text{(Answer)} \end{aligned}$$

Fluid Pressure:

Consider a small surface of area *dA* centred on a point in the fluid; the normal force exerted by the fluid is $dF_\perp$ (Fig 14.3). We define the pressure *p* at that point as the normal force per unit area

$$p = \frac{dF_\perp}{dA}$$

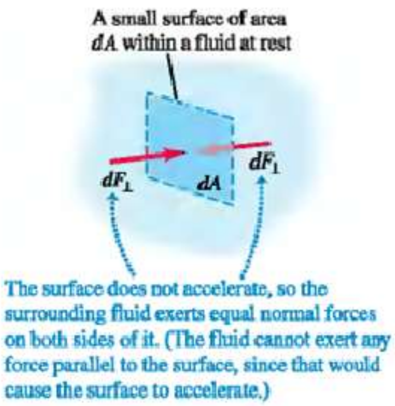
If the pressure is the same at all points of a finite plane surface with area *A*, then

$$p = \frac{F_\perp}{A}$$

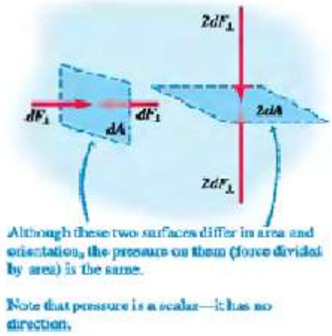
where $F_\perp$ is the net normal force on one side of the surface. The SI unit of pressure is the Pascal, where

$$1\text{ Pascal} = 1\text{ Pa} = \text{N/m}^2$$

14.3 Forces acting on a small surface within a fluid at rest.



14.4 The pressure on either side of a surface is force divided by area. Pressure is a scalar with units of newtons per square meter. By contrast, force is a vector with units of newtons.



Sample Problem 14.01 Atmospheric pressure and force

A living room has floor dimensions of 3.5 m and 4.2 m and a height of 2.4 m.

(a) What does the air in the room weigh when the air pressure is 1.0 atm?

KEY IDEAS

(1) The air's weight is equal to *mg*, where *m* is its mass. (2) Mass *m* is related to the air density ρ and the air volume *V* by Eq. 14-2 ($\rho = m/V$).

Calculation: Putting the two ideas together and taking the density of air at 1.0 atm from Table 14-1, we find

$$\begin{aligned} mg &= (\rho V)g \\ &= (1.21\text{ kg/m}^3)(3.5\text{ m} \times 4.2\text{ m} \times 2.4\text{ m})(9.8\text{ m/s}^2) \\ &= 418\text{ N} \approx 420\text{ N}. \quad \text{(Answer)} \end{aligned}$$

This is the weight of about 110 cans of Pepsi.

(b) What is the magnitude of the atmosphere's downward force on the top of your head, which we take to have an area of 0.040 m^2?

KEY IDEA

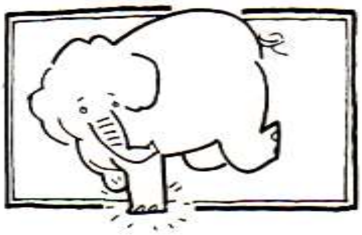
When the fluid pressure *p* on a surface of area *A* is uniform, the fluid force on the surface can be obtained from Eq. 14-4 ($p = F/A$).

Calculation: Although air pressure varies daily, we can approximate that $p = 1.0$ atm. Then Eq. 14-4 gives

$$\begin{aligned} F = pA &= (1.0\text{ atm})\left(\frac{1.01 \times 10^5\text{ N/m}^2}{1.0\text{ atm}}\right)(0.040\text{ m}^2) \\ &= 4.0 \times 10^3\text{ N}. \quad \text{(Answer)} \end{aligned}$$

This large force is equal to the weight of the air column from the top of your head to the top of the atmosphere.

Example: An elephant weighing 40000 N stands on one foot of area $1000\,\text{cm}^2$. What is the pressure exerted on the ground? [Hint: $1\,\text{cm}^2 = \frac{1}{10000}\,\text{m}^2$]



Answer:

$$p = \frac{F}{A} = \frac{40000}{0.1} = 400000\ \text{pa}$$

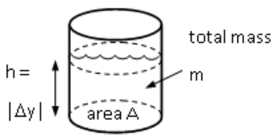
Example: What is the pressure exerted by a girl weighing 400 N standing on one stiletto heel of area $1.0\,\text{cm}^2$?

Answer:

$$p = \frac{F}{A} = \frac{400}{0.0001} = 4000000\ \text{pa}$$

What is your comments?

Example: Consider a bucket of water. What is the pressure at the bottom of the bucket *due to the weight of the water*?

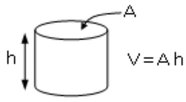


Answer:

$$p = \frac{F_{\perp}}{A} = \frac{\text{weight}}{A} = \frac{mg}{A}$$

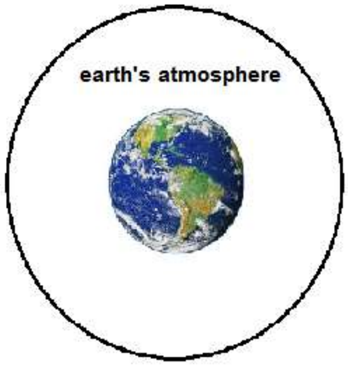
Will now show that $p = \rho g h$

$$\rho = \frac{m}{V} \Rightarrow m = \rho V, \quad p = \frac{mg}{A} = \frac{\rho V g}{A} = \frac{\rho (A h) g}{A} = \rho g h$$



More generally, $\Delta p = \rho g \Delta h$, where Δh is the change in the depth below the surface of the water. This derivation assumes that density ρ = constant, which it is for the case of water, because water is incompressible.

14-2 FLUIDS AT REST



Atmospheric pressure p_0**,** the gaseous envelope surrounding the earth is called the earth's atmosphere and the pressure exerted by the atmosphere is called atmospheric pressure. This pressure varies with weather changes and elevation. Normal atmospheric pressure at sea level is 1 atmosphere (atm), defined as follows:

$$(p_0)_{av} = 1\ \text{atm} = 1.01\times10^5\ \text{Pa} = 760\ \text{torr} = 14.70\ \text{lb/in}^2$$

Example: Calculate the magnitude of the atmosphere's downward force on the top of your head, which we take to have an area of 0.040 m2.

Answer:

$$F = pA = (1.0\ \text{atm})\left(\frac{1.01\times10^5\ \text{N/m}^2}{1.0\ \text{atm}}\right)(0.040\ \text{m}^2)$$
$$= 4.0\times10^3\ \text{N}. \qquad \text{(Answer)}$$

This large force is equal to the weight of the air column from the top of your head to the top of the atmosphere.

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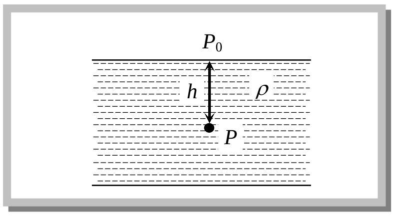
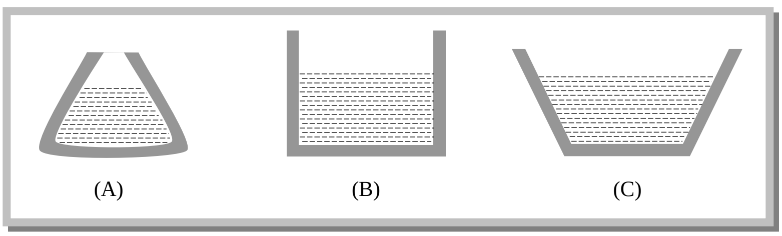
[1] If p_0 is the atmospheric pressure then for a point at depth h below the surface of a liquid of density ρ, hydrostatic pressure p is given by:

$$p = p_0 + \rho g h \qquad \text{(pressure at depth } h\text{)} \qquad (14\text{-}8).$$

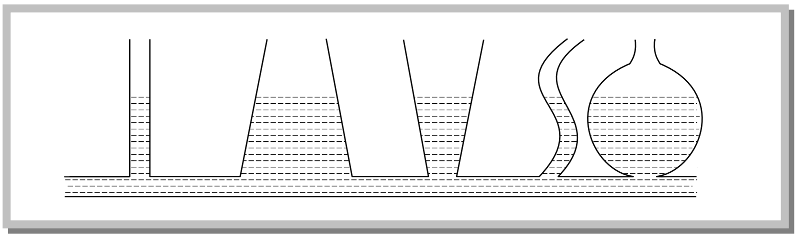


[2] Hydrostatic pressure depends on the depth of the point below the surface (h), nature of liquid (ρ) and acceleration due to gravity (g) while it is independent of the amount of liquid, shape of the container or cross-sectional area considered. So if a given liquid is filled in vessels of different shapes to same height, the pressure at the base in each vessel's will be the same, though the volume or weight of the liquid in different vessels will be different.

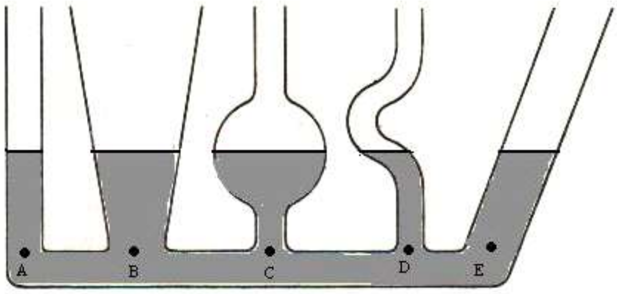


$$P_A = P_B = P_C \quad \text{but} \quad W_A < W_B < W_C$$

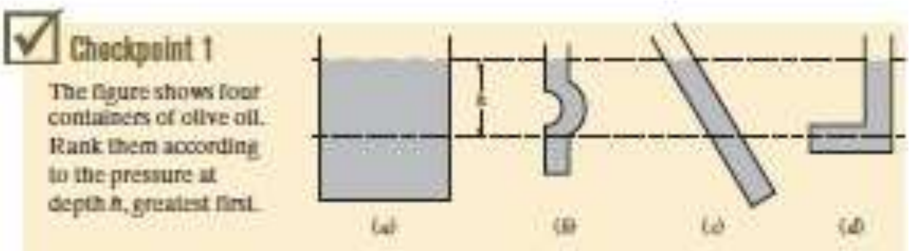
[3] In a liquid at same level, the pressure will be same at all points, if not, due to pressure difference the liquid cannot be at rest. This is why the height of liquid is the same in vessels of different shapes containing different amounts of the same liquid at rest when they are in communication with each other.



- **Remember:** The pressure at a given depth is the same regardless of the shape of the container.



The pressure at A, B, C, D and E are the same



[4] Gauge pressure: The excess pressure above atmospheric pressure is called **gauge pressure**. In other words, the pressure difference between hydrostatic (total) pressure p and atmospheric pressure p_0 is called gauge pressure.

$$\text{gauge pressure } = p - p_0 = \rho g h$$

***Example*.** If pressure at half the depth of a lake is equal to 2/3 pressure at the bottom of the lake then what is the depth of the lake?

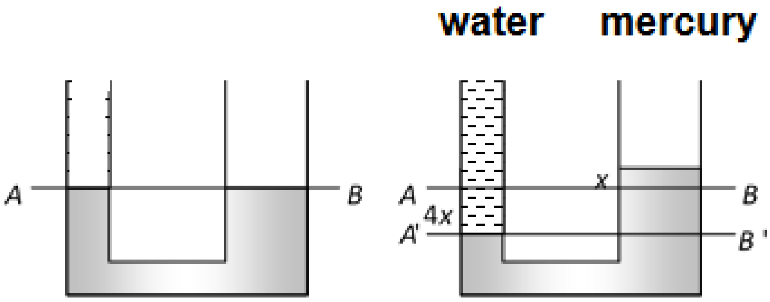
Solution : Pressure at bottom of the lake = $P_0 + h\rho g$ and pressure at half the depth of a lake $= P_0 + \frac{h}{2}\rho g$

According to given condition

$$P_0 + \frac{1}{2}h\rho g = \frac{2}{3}(P_0 + h\rho g) \Rightarrow \frac{1}{3}P_0 = \frac{1}{6}h\rho g$$

$$\Rightarrow h = \frac{2P_0}{\rho g} = \frac{2\times 10^5}{10^3 \times 10} = 20m.$$

***Example*.** A U-tube in which the cross-sectional area of the limb on the left is one quarter, the limb on the right contains mercury (density 13.6 g/cm^3). The level of mercury in the narrow limb is at a distance of 36 *cm* from the upper end of the tube. What will be the rise in the level of mercury in the right limb if the left limb is filled to the top with water?

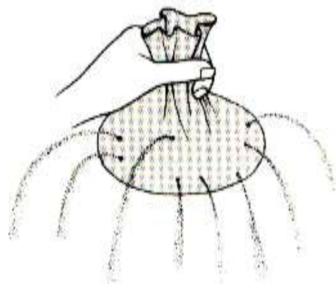


Solution : If the rise of level in the right limb be *x cm.* the fall of level of mercury in left limb be 4*x cm* because the area of cross section of right limb 4 times as that of left limb.
∴ Level of water in left limb is (36 + 4*x*) *cm.*
Now equating pressure at interface of mercury and water (at *A′ B′*)

$$(36 + 4x)\times 1 \times g = 5x \times 13.6 \times g$$

By solving we get $x = 0.56$ *cm.*

Notes and summary

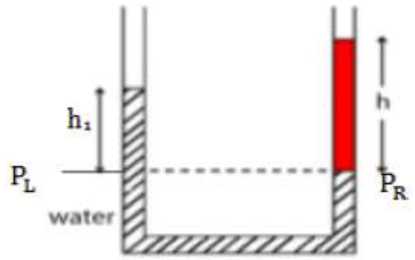


1- Pressure acts in all directions.
2- Pressure increases with depth. The deeper into a liquid, the greater the weight of liquid above and the higher the pressure.
3- Pressure depends on the density of the liquid.
4- Pressure does not depend on the shape, or the area, of the container holding the liquid.
5- The pressure exerted on us by the atmosphere is about 10^5 (N/m²). Fortunately, the pressure inside our bodies is equal to this value, so we are not crushed.
6- When you travel up a hill quickly in a car, the outside air pressure drops as you rise up through the atmosphere and you experience a popping sensation in your ears.
7- At high altitude, for example in an airplane, nose-bleeding may occur due to the greater excess pressure of the blood.
8- It should be noted that the pressure, for example, within a lake is not just the pressure due to water. We must add the pressure exerted by the atmosphere on the lake's surface.

$$p = p_a + \rho g h, \quad p_a = \text{ the atmospheric pressure}$$

9- Dams are designed so that their thickness increases with depth because they have to withstand a much higher pressure from the water at the bottom of a lake.

Example: A uniform U-tube is partially filled with water. Oil, of density 0.75 g/cm^3, is poured into the right arm until the water level in the left arm rises h_1 = 3.0 cm (see Figure). The length of the oil column (h) is then:

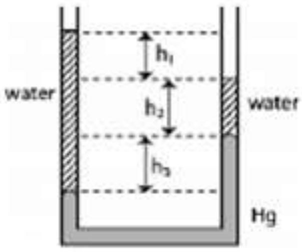


Answer:

$$P_L = P_R \Rightarrow \rho_{water} g h_1 = \rho_{oil} g h$$

$$h = \frac{\rho_{water}}{\rho_{oil}} h_1 = \frac{1.0\text{ g/cm}^3}{0.75\text{ g/cm}^3} \times 3 = 4.0\text{ cm.}$$

Example: A U-tube of constant cross sectional area, open to the atmosphere, is partially filled with Hg (ρ_{Hg}=13.6 g/cm^3). Water (ρ_w=1.00 g/cm^3) is then poured into both arms. If the equilibrium configuration of the tube is as shown in the Figure with h_3= 1.00 cm, determine the value of h_1. (Note that h_1, h_2 and h_3 are not drawn to scale).



Answer: At the bottom of h_3, the pressure at the left side will be equal to pressure at the right side, we can have:

$$p_o + \rho_w g(h_1 + h_2 + h_3) = p_o + \rho_w g h_2 + \rho_{Hg} g h_3$$

$$\Rightarrow \quad h_1 = \left(\frac{\rho_{Hg} - \rho_w}{\rho_w}\right) h_3 = \left(\frac{13.6-1}{1}\right) 1 = \underline{12.7\text{ cm}}.$$

Extra problems

Sample Problem 14.02 Gauge pressure on a scuba diver

A novice scuba diver practicing in a swimming pool takes enough air from his tank to fully expand his lungs before abandoning the tank at depth L and swimming to the surface, failing to exhale during his ascent. At the surface, the difference Δp between the external pressure on him and the air pressure in his lungs is 9.3 kPa. From what depth does he start? What potentially lethal danger does he face?

KEY IDEA

The pressure at depth h in a liquid of density ρ is given by Eq. 14-8 ($p = p_0 + \rho g h$), where the gauge pressure $\rho g h$ is added to the atmospheric pressure p_0.

Calculations: Here, when the diver fills his lungs at depth L, the external pressure on him (and thus the air pressure within his lungs) is greater than normal and given by Eq. 14-8 as

$$p = p_0 + \rho g L,$$

where ρ is the water's density (998 kg/m^3, Table 14-1). As he ascends, the external pressure on him decreases, until it is atmospheric pressure p_0 at the surface. His blood pressure also decreases, until it is normal. However, because he does not exhale, the air pressure in his lungs remains at the value it had at depth L. At the surface, the pressure difference Δp is

$$\Delta p = p - p_0 = \rho g L,$$

so

$$L = \frac{\Delta p}{\rho g} = \frac{9300 \text{ Pa}}{(998 \text{ kg/m}^3)(9.8 \text{ m/s}^2)}$$

$$= 0.95 \text{ m}. \qquad \text{(Answer)}$$

This is not deep! Yet, the pressure difference of 9.3 kPa (about 9% of atmospheric pressure) is sufficient to rupture the diver's lungs and force air from them into the depressurized blood, which then carries the air to the heart, killing the diver. If the diver follows instructions and gradually exhales as he ascends, he allows the pressure in his lungs to equalize with the external pressure, and then there is no danger.

Sample Problem 14.03 Balancing of pressure in a U-tube

The U-tube in Fig. 14-4 contains two liquids in static equilibrium: Water of density ρ_w (= 998 kg/m^3) is in the right arm, and oil of unknown density ρ_x is in the left. Measurement gives l = 135 mm and d = 12.3 mm. What is the density of the oil?

KEY IDEAS

(1) The pressure p_{int} at the level of the oil–water interface in the left arm depends on the density ρ_x and height of the oil above the interface. (2) The water in the right arm *at the same level* must be at the same pressure p_{int}. The reason is that, because the water is in static equilibrium, pressures at points in the water at the same level must be the same.

Calculations: In the right arm, the interface is a distance l below the free surface of the *water*, and we have, from Eq. 14-8,

$$p_{int} = p_0 + \rho_w g l \quad \text{(right arm)}.$$

In the left arm, the interface is a distance $l + d$ below the free surface of the *oil*, and we have, again from Eq. 14-8,

$$p_{int} = p_0 + \rho_x g(l + d) \quad \text{(left arm)}.$$

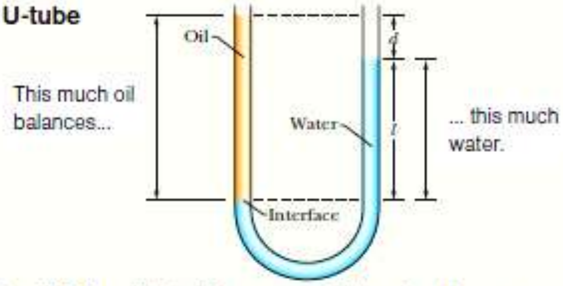


Figure 14-4 The oil in the left arm stands higher than the water.

Equating these two expressions and solving for the unknown density yield

$$\rho_x = \rho_w \frac{l}{l + d} = (998 \text{ kg/m}^3)\frac{135 \text{ mm}}{135 \text{ mm} + 12.3 \text{ mm}}$$

$$= 915 \text{ kg/m}^3. \qquad \text{(Answer)}$$

Note that the answer does not depend on the atmospheric pressure p_0 or the free-fall acceleration g.

Q15.

A cylindrical container has a layer of oil of thickness 0.120 m floating on water that is 0.250 m deep. The density of oil is 750 kg/m^3. What is the gauge pressure, in kPa, at the bottom of the container?

A) 3.33
B) 0.882
C) 2.45
D) 1.27
E) 6.35

Ans:

$\rho_x \rightarrow oil, h_x \rightarrow height\ of\ oil$

$\rho_w \rightarrow water, h_w \rightarrow height\ of\ water$

At the oil-water interface

$p_1 = p_0 + \rho_x g h_x\ (p_0 = atmospheric\ pressure)$

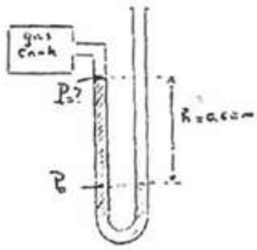
At the bottom:

$p_2 = p_1 + \rho_w g h_w = p_0 t + \rho_x g h_x + \rho_w g h_w$

Gauge Pressure:

$p = p_2 - p_0 = (\rho_x h_x + \rho_w h_w) g$

$= [(750)(0.120) + (1000)(0.250)](9.8) = 3.33\ \mathrm{kP_a}$



Example: An open-tube mercury manometer (see figure) is connected to a gas tank. What is the absolute pressure of the gas if h = 0.60 m and a nearby mercury barometer reads 76 cm-Hg? Density of mercury ρ_{Hg} =13.6×10^3 Kg/m^3.

Answer:

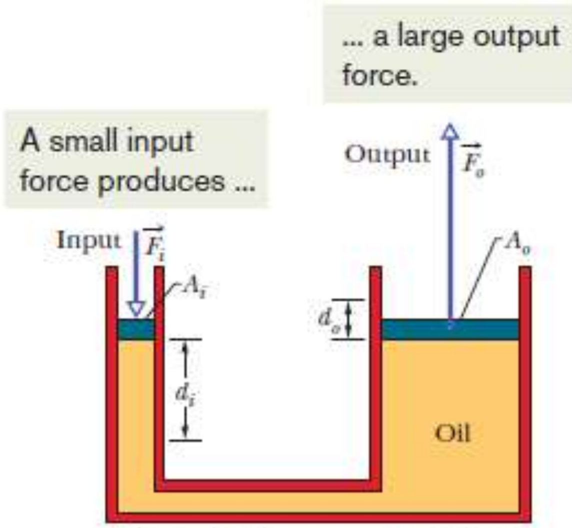
$$p_o = p + \rho_{Hg} g h \quad \Rightarrow \quad p = p_o - \rho_{Hg} g h = 1.01\times10^5 - 13.6\times10^3 \times 9.8 \times 0.6 = 2.1\times10^4\ \text{Pa}$$

14-4 PASCAL'S PRINCIPLE

Pascal's law: state that:

"If gravity effect is neglected, the pressure applied to an enclosed fluid is transmitted equally (**undiminished**) to every portion of the fluid and the walls of the containing vessel."

In other words, If gravity effect is neglected, the pressure at every point of liquid in equilibrium of rest is same.



A hydraulic arrangement that can be used to magnify a force $\vec{F}_i$. The work done is, however, not magnified and is the same for both the input and output forces.

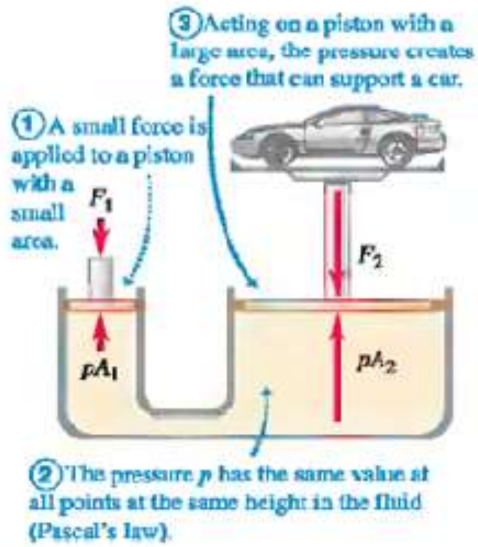


Figure 14-8 The hydraulic lift is an application of Pascal's law. The size of the fluid-filled container is exaggerated for clarity.

Working of hydraulic lift

The hydraulic lift shown schematically in Fig. (14.8) illustrates Pascal's law. A piston with small cross-sectional area A_1 exerts a force F_1 on the surface of a liquid such as oil. The applied pressure is transmitted through the connecting pipe to a larger piston of area A_2. The applied pressure is the same in both cylinders, so

$$p = \frac{F_1}{A_1} = \frac{F_2}{A_2} \qquad \text{so} \quad F_2 = \frac{A_2}{A_1} F_1 \qquad (14.13)$$

Examples: The hydraulic lift is a force- multiplying device with a multiplication factor equal to the ratio of the areas of the two pistons. Dentist's chairs, car lifts and jacks, many elevators, and hydraulic brakes all use this principle.

Comments:

i- Expressing the area as $A=\pi r^2=\pi(d/2)^2$, where r is the radius of the piston. One finds: $F_2=\left(\frac{r_2}{r_1}\right)^2 F_1=\left(\frac{d_2}{d_1}\right)^2 F_1$, d is the diameter of the piston.

ii- The volume of displaced liquid in both limbs is the same. So,

$$V_1=V_2 \quad\Rightarrow\quad A_1\ell_1=A_2\ell_2 \quad\Rightarrow\quad \frac{A_2}{A_1}=\frac{\ell_1}{\ell_2}$$

Consequently,

$$F_2=\frac{\ell_1}{\ell_2}F_1 \qquad (14.14)$$

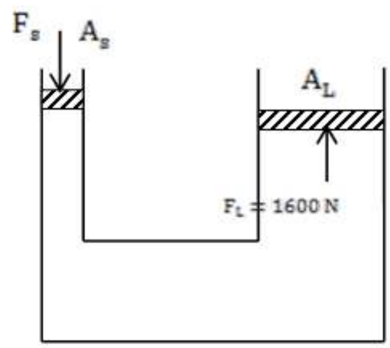
- "With a hydraulic lever, a given force applied over a given distance can be transformed to a greater force applied over a smaller distance."

Example: If you push on the smaller piston (A_1 = 0.01 m²) with a force of 12 N, how much force is on the larger piston (A_2 = 0.1 m²)?

Answer: Since the pressure is the same on both areas, then

$$\frac{F_1}{A_1}=\frac{F_2}{A_2} \quad\Rightarrow\frac{12}{0.01}=\frac{F_2}{0.1} \quad\Rightarrow F_2=120\text{ N}$$

Example: A hydraulic press has one piston of diameter 2.0 cm and the other piston of diameter 8.0 cm. What force must be applied to the smaller piston to obtain a force of 1600 N at the larger piston?



Answer: Use the ratio: $\frac{F_s}{A_s}=\frac{F_L}{A_L}$, then

$$F_s=\frac{F_L}{A_L}A_s=\frac{F_L}{\pi(d_L/2)^2}\pi(d_s/2)^2=F_L\left(\frac{d_s}{d_L}\right)^2=1600\left(\frac{2}{8}\right)^2=100\text{ N}$$

14-5 ARCHIMEDES' PRINCIPLE

Buoyancy is a familiar phenomenon: A body immersed in water seems to weigh less than when it is in air. When the body is less dense than the fluid, it floats. The human body usually floats in water, and a helium-filled balloon floats in air. *Archimedes's principle* states:

"***When a body is completely (or partially) immersed in a fluid, the fluid exerts an upward force on the body equal to the weight of the fluid displaced by the body***".

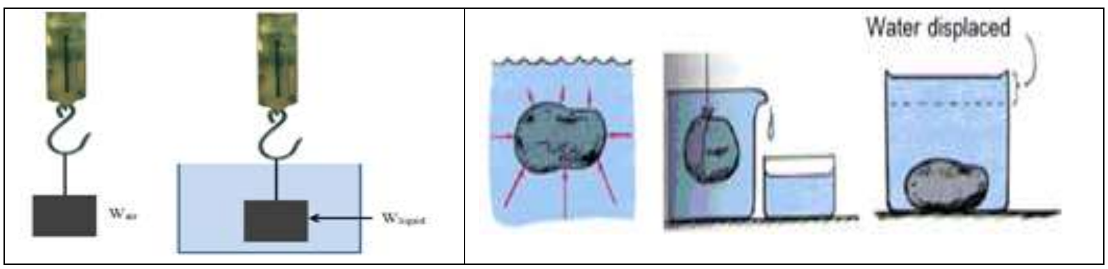
<u>To prove this principle experimentally</u>:

When a body is fully, or partially, submerged in a fluid a buoyant force F_b from the surrounding fluid acts on the body. The force is directed upward and has a magnitude equal to the weight of the fluid that has been displaced by the body.

m_f = the mass of the fluid that is displaced by the body $= \rho_f V_f = \rho_f V_{\text{object}}$.

W_{air} = Actual weight (in air) , W_{liquid} = Apparent weight (in liquid)

$$\overbrace{m_f \; g \qquad\qquad W_{\text{air}} - W_{\text{liquid}}}^{F_b \text{ (Buoyant force)}}$$



<u>Comments:</u>

i- The volume of the displaced liquid = volume of the submerged object $V_\ell = V_{\text{sub. object}}$.

ii- In case of floating $F_b = m_{object}\; g$, m_{object} is the mass of the object.

iii- If V is the volume of a block, of density ρ , and $V_i = xV$ is the submerged volume, then

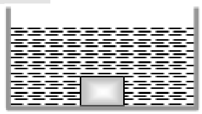
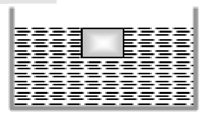
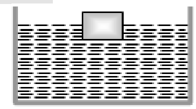
$$x = \frac{\rho}{\rho_\ell} = \frac{V_i}{V}$$

Floatation

Translatory equilibrium: When a body of density ρ and volume V is immersed in a liquid of density σ, the forces acting on the body are

Weight of body $W = mg = V\rho g,$ acting vertically downwards through centre of gravity of the body.

Upthrust force = $V\sigma g$ acting vertically upwards through the centre of gravity of the displaced liquid *i.e.*, centre of buoyancy.

If density of body is greater than that of liquid $\rho > \sigma$	If density of body is equal to that of liquid $\rho = \sigma$	If density of body is lesser than that of liquid $\rho < \sigma$
		
Weight will be more than upthrust so the body will sink	Weight will be equal to upthrust so the body will float fully submerged in neutral equilibrium anywhere in the liquid.	Weight will be less than upthrust so the body will move upwards and in equilibrium will float partially immersed in the liquid Such that, $W = V_{in}\sigma g \Rightarrow V\rho g = V_{in}\sigma g$ $V\rho = V_{in}\sigma$ Where V_{in} is the volume of body in the liquid

Important points

(i) A body will float in liquid only and only if $\rho \le \sigma$

(ii) In case of floating as weight of body = upthrust

So W_{App} = Actual weight – upthrust = 0

(iii) In case of floating $V\rho g = V_{in}\sigma g$

So the equilibrium of floating bodies is unaffected by variations in g though both thrust and weight depend on g.

Note:

1- When a balloon floats in equilibrium in air, its weight (including the gas inside it) must be the same as the weight of the air displaced by the balloon.
2- A fish's flesh is denser than water, yet a fish can float while submerged because it has a gas-filled cavity within its body. This makes the fish's average density the same as water, so its net weight is the same as the weight of the water it displaces.
3- When you swim in sea water (density is higher), your body floats higher than in fresh air (density is lower).

Example1: A solid sphere has actual weight of 10 N. When it is suspended from a spring scale and submerged in water, the scale reads 6.0 N. What is the radius of the solid sphere?

Answer: $F_b = \mathrm{W_{air}} - \mathrm{W_{liquid}} = 10 - 6 = 4\ \mathrm{N}$

But

$$F_b = \text{Weight of the displaced liquid} = \mathrm{m}_f\, g = \rho_f V_f\, g = \rho_f V_{\mathrm{sphere}}\, g = 4\ \mathrm{N}$$

$$\Rightarrow V_{\mathrm{sphere}} = \frac{4}{10^3 \times 9.8} = 4.1 \times 10^{-4}\ \mathrm{m}^3$$

Using

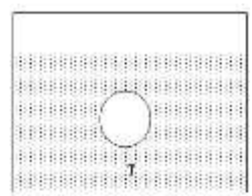
$$V_{\mathrm{sphere}} = \frac{4}{3}\pi r^3 \quad \Rightarrow \quad r = \underline{4.6\ \mathrm{cm}}$$

Example2: A block of wood of specific gravity 0.8 floats in water. What fraction of the volume of the block is submerged?

Answer: If V is the volume of the block and xV is the submerged volume, then

$$\mathrm{m}g = F_b \text{ or } \rho V g = \rho_w x V g \quad \text{so} \quad x = \frac{\rho}{\rho_w} = 0.8$$

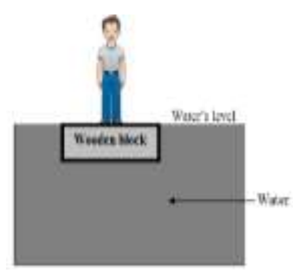
Example3: A 10 kg spherical object with a volume of 0.10 m^3 is held in static equilibrium under water by a cable fixed to the bottom of a water tank. What is the tension T in the cable? (See the figure)

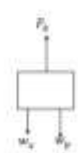


Answer: Since $T + mg = F_b = \rho_\ell V_\ell g$, then

$$T = -mg + \rho_\ell V_\ell g = -10 \times 9.8 + \rho_\ell V_b g = -98 + 1000 \times 0.1 \times 9.8 = \underline{882\ \mathrm{N}}$$

Example: A block of wood ($\rho = 850\ \mathrm{kg/m^3}$) floats on water with a 50.0 kg person standing on top of the block (see **Figure**). What minimum volume, in m^3, must the block have such that the top face of the block will be just in level with the water surface?





Answer:

$$b = bouyant, x = wood, p = person, f = water$$

$$F_b = W_x + W_p$$

$$\rho_f V \not{g} = m_x \not{g} + m_p \not{g}$$

$$\rho_f V = \rho_x V + m_p$$

$$\Rightarrow V = \frac{m_p}{\rho_f - \rho_x} = \frac{50.00}{1000 - 850} = 0.333\ m^3$$

Extra problems

Q: A block of wood floats in fresh water with two-thirds of its volume *V* submerged and in oil with 0.90 *V* submerged. Find the density of (a) the wood and (b) the oil.

Answer:

a) Let *V* be the volume of the block. Then, the submerged volume is $V_s = 2V/3$. Since the block is floating, the weight of the displaced water is equal to the weight of the block, so

$\rho_{water} V_s = \rho_{block} V$, where ρ_{water} is the density of water, and ρ_{block} is the density of the block.

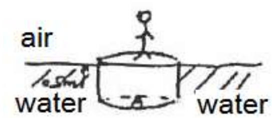
We substitute $V_s = 2V/3$ to obtain

$$\rho_{block} = 2\rho_{water}/3 = 2(1000\,\text{kg/m}^3)/3 \approx \underline{6.7\times10^2\ \text{kg/m}^3}$$

(b) If ρ_{oil} is the density of the oil, then Archimedes' principle yields $\rho_{oil} V_s = \rho_{block} V$. We substitute $V_s = 0.90\,V$ to obtain

$$\rho_{oil} = \rho_{block}/0.9 = \underline{7.4\times10^2\ \text{kg/m}^3}\ .$$

Q: What is the area of the smallest cylinder slab of ice, 0.5 m thick that will just support a man of mass 100 kg? The density of ice is 917 kg/m^3 and the density of water is 10^3 kg/m^3



Answer:

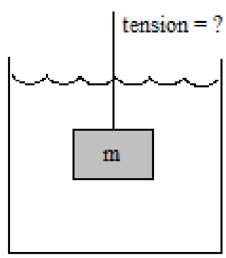
$$Mg + mg = F_b = \rho_{water} gV$$

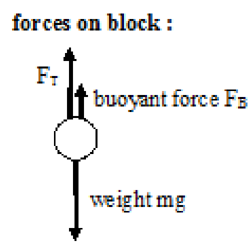
$$Mg + \rho_{ice} gV = F_b = \rho_{water} gV, \qquad V = AR$$

$$Mg = (\rho_{water} - \rho_{ice}) ghA$$

$$\therefore A = \frac{M}{(\rho_{water} - \rho_{ice})h} = \frac{100}{(1000-917)0.5} = 2.41\,\text{m}^2$$

Q: A block of copper (Cu) with mass m = 400 g and density $\rho_{Cu} = 8.9$ g/cm^3 is suspended by a string while under water. How does the tension in the string compare to the weight of the copper block?





Answer: Since the block is not moving, the net force on it is zero and we can write:

$F_T + F_B = mg$ (since | upward forces | = | downward forces |). So we have

$F_T = mg - F_B$. We must now compute the magnitude of the buoyant force F_B.

Archimedes says that F_B is the weight of the *displaced water* $= m_{water}\, g = \rho_{water} V g$ where V is the volume of the displaced water = volume of the copper block. We get V from

$$\rho_{Cu} = \frac{m}{V} \Rightarrow V = \frac{m}{\rho_{Cu}} \quad \text{so} \quad F_b = \rho_{water} V g = \rho_{water} \frac{m}{\rho_{Cu}} g = \frac{\rho_{water}}{\rho_{Cu}} m g$$

$$F_T = mg - F_b = mg - \left(\frac{\rho_{water}}{\rho_{Cu}} m g\right) = mg\left(1 - \frac{\rho_{water}}{\rho_{Cu}}\right) = mg\left(1 - \frac{1}{8.9}\right) = 0.89\, mg$$

So the tension in the string is only 89% of the weight of the copper block. The buoyant force is helping support the weight of the block, so the tension is less than the full weight of the block.

☺ This calculation can be turned around and used to compute the density of a block, given its mass and the tension in the string. Legend has it that Archimedes used this technique to determine the density of the king's crown (the king of Syracuse, a Greek colony in Sicily). The king was worried that the crown was not pure gold, and Archimedes was able to show that the density of his crown was considerably less than the density of gold (ρ_{Au} = 19.3 g/cm^3), confirming the king's suspicion.

Sample Problem 14.04 Floating, buoyancy, and density

In Fig. 14-11, a block of density ρ = 800 kg/m^3 floats face down in a fluid of density ρ_f = 1200 kg/m^3. The block has height H = 6.0 cm.

(a) By what depth h is the block submerged?

KEY IDEAS

(1) Floating requires that the upward buoyant force on the block match the downward gravitational force on the block. (2) The buoyant force is equal to the weight $m_f g$ of the fluid displaced by the submerged portion of the block.

Calculations: From Eq. 14-16, we know that the buoyant force has the magnitude $F_b = m_f g$, where m_f is the mass of the fluid displaced by the block's submerged volume V_f. From Eq. 14-2 ($\rho = m/V$), we know that the mass of the displaced fluid is $m_f = \rho_f V_f$. We don't know V_f but if we symbolize the block's face length as L and its width as W, then from Fig. 14-11 we see that the submerged volume must be $V_f = LWh$. If we now combine our three expressions, we find that the upward buoyant force has magnitude

$$F_b = m_f g = \rho_f V_f g = \rho_f LWhg. \qquad (14\text{-}20)$$

Similarly, we can write the magnitude F_g of the gravitational force on the block, first in terms of the block's mass m, then in terms of the block's density ρ and (full) volume V, and then in terms of the block's dimensions L, W, and H (the full height):

$$F_g = mg = \rho V g = \rho_f LWHg. \qquad (14\text{-}21)$$

The floating block is stationary. Thus, writing Newton's second law for components along a vertical y axis with the positive direction upward ($F_{net,y} = ma_y$), we have

$$F_b - F_g = m(0),$$

Floating means that the buoyant force matches the gravitational force.

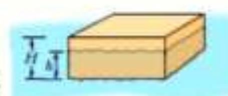


Figure 14-11 Block of height H floats in a fluid, to a depth of h.

or from Eqs. 14-20 and 14-21,

$$\rho_f LWhg - \rho LWHg = 0,$$

which gives us

$$h = \frac{\rho}{\rho_f} H = \frac{800\ \text{kg/m}^3}{1200\ \text{kg/m}^3}(6.0\ \text{cm})$$
$$= 4.0\ \text{cm}. \qquad \text{(Answer)}$$

(b) If the block is held fully submerged and then released, what is the magnitude of its acceleration?

Calculations: The gravitational force on the block is the same but now, with the block fully submerged, the volume of the displaced water is $V = LWH$. (The full height of the block is used.) This means that the value of F_b is now larger, and the block will no longer be stationary but will accelerate upward. Now Newton's second law yields

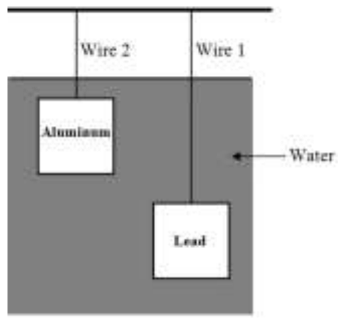
$$F_b - F_g = ma,$$

or

$$\rho_f LWHg - \rho LWHg = \rho LWHa,$$

where we inserted ρLWH for the mass m of the block. Solving for a leads to

$$a = \left(\frac{\rho_f}{\rho} - 1\right)g = \left(\frac{1200\ \text{kg/m}^3}{800\ \text{kg/m}^3} - 1\right)(9.8\ \text{m/s}^2)$$
$$= 4.9\ \text{m/s}^2. \qquad \text{(Answer)}$$



Q: Two cubes made of Lead and aluminum, having the same volume, are suspended at different depths in water by two wires, as shown in **Figure**. Wire 1 is used for lead and wire 2 is used for aluminum. If the densities are ρ(aluminum)= 2700 kg/m^3 and ρ(lead) = 11300 kg/m^3, which of the following statements is correct?

A) The tension is larger in wire 1.
B) The tension is larger in wire 2.
C) The buoyant force is larger on the lead cube.
D) The buoyant force is larger on the aluminum cube.
E) The tension and buoyant force are the same for both cubes.

Answer:

A

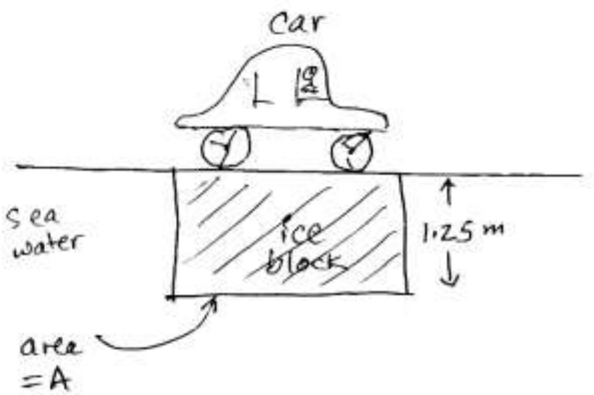
Q: What fraction of an iceberg of volume V_{tot} is submerged under sea water ($=V_{sub} / V_{tot}$)?
(Take: ρ_{ice} = 917 kg/m^3, ρ_{sea} = 1.03 × 10^3 kg/m^3.)

Answer:

$F_B = [M_{ice}]g \Rightarrow \rho_{sea}g(V_{sub}) = [\rho_{ice}V_{tot}]g$

$\Rightarrow V_{sub} / V_{tot} = \rho_{ice}/\rho_{sea\_water} = 917/1030 = 0.89$

Q: Determine the area "A" of a flat ice block 1.25 meter thick if it is to support a 2000-kg car above seawater. See the figure below. (take: ρ_{ice} = 920 kg/m^3, ρ_{sea} = 1020 kg/m^3.)



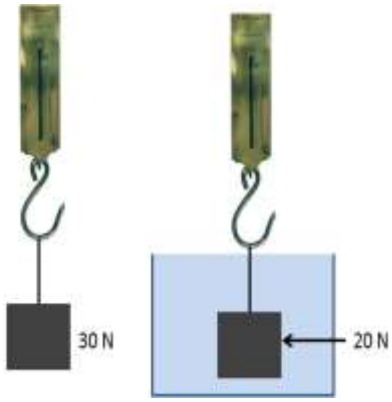
Ans:

$F_B = [M_{ice} + M_{car}]g \Rightarrow \rho_{sea}g(Ah) = [\rho_{ice} Ah + 2000]g$

$\Rightarrow A = 2000/[(\rho_{sea} - \rho_{ice})h] = 16\ m^2$

Example: An object hangs from a spring balance. The balance indicates 30N in air and 20N when the object is submerged in water. What does the balance indicate when the object is submersed in a liquid with a density that is half that of water?

(**Ans:**



$$F_b = W - W_a$$
$$\rho_w V g = 30 - 20$$
$$V = \frac{10}{\rho_w g}$$

When in liquid

$$\frac{\rho_w}{2} V g = 30 - W_{ab}$$
$$\Rightarrow W_{ab} = 30 - \frac{\rho_w}{2} \cdot \frac{10 g}{\rho_w g} = 25\text{ N}$$

----

(48) Figure 14-46 shows an iron ball suspended by thread of negligible mass from an upright cylinder that floats partially submerged in water. The cylinder has a height of h = 6.00 cm, a face area of. 12.0 cm^2 on the top and bottom, and a density of 0.30 g/cm^3, and 2.00 cm of its height is above the water surface. What is the radius of the iron ball?

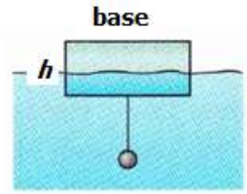


FIG. 14-46 Problem 48.

Ans:

Let ρ be the density of the cylinder (0.30 g/cm^3 or 300 kg/m^3) and ρ_{Fe} be the density of the iron (7.9 g/cm^3 or 7900 kg/m^3). The volume of the cylinder is

$$V_c = (6\times12)\text{ cm}^3 = 72\text{ cm}^3 = 0.000072\text{ m}^3,$$

and that of the ball is denoted V_b. The part of the cylinder that is submerged has volume

$$V_s = (4\times12)\text{ cm}^3 = 48\text{ cm}^3 = 0.000048\text{ m}^3.$$

Using the ideas of section 14-7, we write the equilibrium of forces as

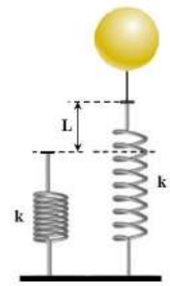
$$\rho g V_c + \rho_{Fe} g V_b = \rho_w g V_s + \rho_w g V_b \quad \Rightarrow \quad V_b = 3.8\text{ cm}^3$$

where we have used ρ_w = 998 kg/m^3 (for water, see Table 14-1). Using $V_b = \frac{4}{3}\pi r^3$ we find r = 9.7 mm.

Q24.

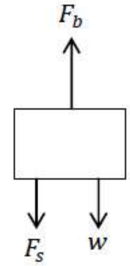
A spring of force constant k = 89 N/m is attached vertically to a table (**Figure 11**). A balloon, with negligible mass, is filled with helium to a volume of 5.0 m^3 and is connected with a light cord to the spring, causing it to stretch. Determine the extension L when the balloon is in equilibrium. [ρ (air) = 1.2 kg/m^3 and ρ (helium) = 0.18 kg/m^3].

Figure 11



A) 0.56 m
B) 0.23 m
C) 0.43 m
D) 0.15 m
E) 0.27 m

Ans:



$$F_b = bouyant, \quad W = weight, \quad S = Spring$$

$$A = air, \quad H = Helium, \quad B = baloon$$

$$F_b = F_s + W \Rightarrow F_s = F_b - W$$

$$W = (mg + m_H)g = (mg + \rho_H V_H)g$$

$$F_b = \rho_A V_H g \; ; \; F_s = kL$$

$$\therefore kL = (\rho_A - \rho_H)Vg - m_B g$$

$$\therefore L = \frac{g}{k}[(\rho_A - \rho_H)V - mg] = 0.56\, m$$

Chapter14
Fluids

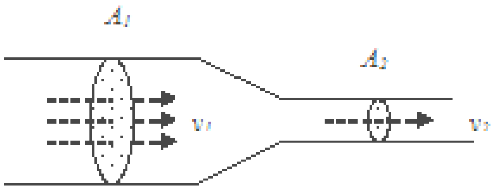
Summary of the given equations:

$\rho \equiv \frac{m}{V}$, $p = \frac{F_\perp}{A}$, $\Delta p = \rho g \Delta h$, $p = p_0 + \rho g h$ (pressure at depth h) ,

Pascal's Principle: $F_2 = \frac{A_2}{A_1} F_1 = \left(\frac{r_2}{r_1}\right)^2 F_1 = \left(\frac{d_2}{d_1}\right)^2 F_1 = \frac{\ell_1}{\ell_2} F_1$

14-6 THE EQUATION OF CONTINUITY

Consider a fluid flowing through a tapered pipe:



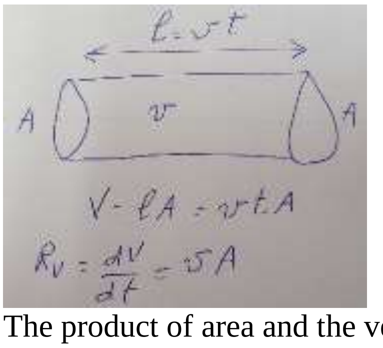
The area of the pipe on the left side is A_1, and the speed of the fluid passing through A_1 is v_1. As the pipe tapers to a smaller area A_2, the speed changes to v_2. Since **mass must be conserved**, the mass of the fluid passing through A_1 must be the same as the mass of the fluid passing through A_2. If the density of the fluid is ρ_1, and the density of the fluid at A_2 is ρ_2, the ***mass flow rate*** $\left(dm = \rho dV = \rho A dx \Rightarrow \frac{dm}{dt} = \rho \frac{dV}{dt} = \rho A v\right)$ through A_1 is $\rho_1 A_1 v_1$, and the mass flow rate through A_2 is $\rho_2 A_2 v_2$. Thus, by conservation of mass,

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

This relationship is called the ***equation of continuity***. If the density of the fluid is the same at all points in the pipe, the equation becomes

$$A_1 v_1 = A_2 v_2 \qquad \text{(Continuity equation)} \qquad (14.10)$$

Remember:

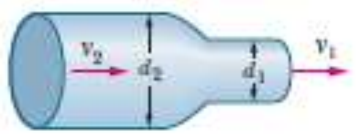
- 

- The product of area and the velocity of the fluid through the area is called the *volume flow rate* $R_V = \frac{dV}{dt} = Av = \text{constant}$.
- *mass flow rate* $R_\rho = \frac{dm}{dt} = \rho \frac{dV}{dt} = \rho R_V = \rho A v$
- Equation (14.10) shows that the volume flow rate has the same value at all points along any flow tube. When the cross section of a flow tube decreases, the speed increases, and vice versa.
- For circular pipe we have $A = \pi r^2$.

We can generalize Eq. (14.10) for the case in which the fluid is *not* incompressible. If ρ_1 and ρ_2 are the densities at sections 1 and 2, then

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \qquad \text{(continuity equation, compressible fluid)} \quad (14.11)$$

If $\rho_2 > \rho_1$, the volume flow rate at point 2 will be less than at point 1 ($A_2 v_2 < A_1 v_1$).



Example: In the figure, water flows through a horizontal pipe and then out into the atmosphere at a speed v_1 = 15 m/s. The diameters of the left and right sections of the pipe are 5.0 cm and 3.0 cm.

(a) In the left section of the pipe, what are the speed v_2?

(b) What volume of water flows into the atmosphere during a 10 min period?

Answer:

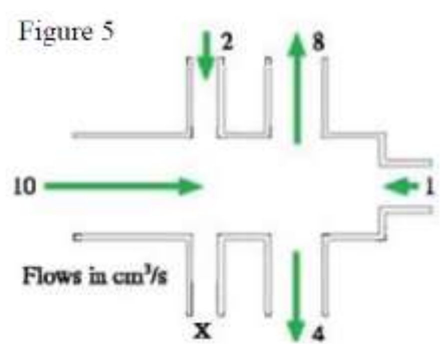
(a) The speed in the left section of pipe is

$$v_2 = v_1\left(\frac{A_1}{A_2}\right) = v_1\left(\frac{d_1}{d_2}\right)^2 = (15\,\text{m/s})\left(\frac{3.0\,\text{cm}}{5.0\,\text{cm}}\right)^2 = 5.4\,\text{m/s}.$$

(b) The volume of water (during 10 minutes) is

$$V = (v_1 t)A_1 = (15\,\text{m/s})(10\,\text{min})(60\,\text{s/min})\pi\left(\frac{0.03\,\text{m}}{2}\right)^2 = 6.4\,\text{m}^3.$$

Check if you have: $V = (v_1 t)A_1 = (v_2 t)A_2$



T132_Q20. Figure 5 shows volume flow rates (in cm^3/s) of a fluid from all but one tube. Assuming steady flow of the fluid, find the volume flow rate through the **X** tube and its direction.

Answer:

$$\text{in} \rightarrow +\text{ve}, \quad \text{out} \rightarrow -\text{ve}, \quad \text{net flow rate} = 0$$

$$\Rightarrow +2+10+1-8-4+X=0$$

$$\Rightarrow X = -1.$$

T12_Q12. A bucket with 0.0189-m^3 is to be filled through a pipe with 0.00780 m radius. If the water flows through the pipe end with a speed of 0.610 m/s, how long does it take to fill the bucket completely?

Answer

$$R_v = vA = \frac{dV}{dt} \quad \Rightarrow \quad dt = \frac{dV}{vA} = \frac{(0.0189)}{0.61\times\pi\times(0.0078)^2} = \underline{162}\text{ s.}$$

Example: Water is pumped out of a swimming pool at a speed of 5.0 m/s through a uniform hose of radius 1.0 cm. Find the mass of water pumped out of the pool in one minute. (Density of water = 1000 kg/m^3).

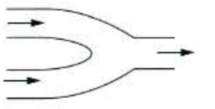
Answer:

$$R_\rho = \rho R_V = 1000\times v\times A = 1000\times 5.0\times\pi(0.01)^2 = 1.57\text{ kg/s}$$

$$\Rightarrow m = R_\rho t = 1.57\times 60 = \underline{94\text{ kg}}.$$

Notice that R_ρ **is the mass flow =** $\frac{dm}{dt} \quad \Rightarrow \quad m = R_\rho t$

Example: Two streams merge to form a river. One stream has a width of 8.0 m, depth of 4.0 m, and current speed of 2.0 m/s. The other stream is 7.0 m wide and 3.0 m deep, and flows at 4.0 m/s. If the river has a width of 10.0 m and a speed of 4.0 m/s, what is its depth?



Answer:

$$A_1 v_1 + A_2 v_2 = A_3 v_3$$
$$(8)(4)(2) + (7)(3)(4) = (10)(h)$$
$$\Rightarrow h = \underline{3.7\text{ m}}.$$

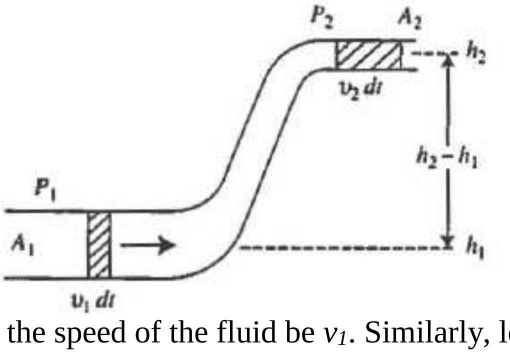
14-7 BERNOULLI'S EQUATION

According to the continuity equation, the speed of fluid flow can vary along the paths of the fluid. The pressure can also vary; it depends on the height and it also depends on the speed of flow. We can derive an important relationship called Bernoulli's equation that relates the pressure, flow speed, and height for flow of an ideal, incompressible fluid. Bernoulli's equation is an essential tool in analysing plumbing systems, hydroelectric generating stations, and the flight of airplanes.
Recall that in the absence of friction or other non-conservative forces, the total mechanical energy of a system remains constant, that is,

$$U_1 + K_1 = U_2 + K_2$$

$$mgh_1 + ½\, mv_1^2 = mgh_2 + ½\, mv_2^2$$

Bernoulli's principle states that "***the total pressure of a fluid along any tube of flow remains constant***". Consider a tube in which one end is at a height h_1 and the other end is at a height h_2:



Let the pressure at h_1 be p_1 and the speed of the fluid be v_1. Similarly, let the pressure at h_2 be p_2 and the speed of the fluid be v_2. If the density of the fluid is ρ, Bernoulli's equation is

$$p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 \qquad (14.17)$$

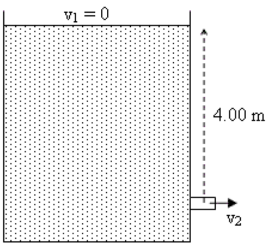
$$p + \rho g y + \frac{1}{2}\rho v^2 = \text{constant} \qquad (14.18)$$

- This equation states that: the sum of the pressure at the surface of the tube, the dynamic pressure caused by the flow of the fluid, and the static pressure of the fluid due to its height above a reference level remains constant.
- Note that if we multiply Bernoulli's equation by volume, it becomes a statement of conservation of energy.
- If a fluid moves through a horizontal pipe ($h_1 = h_2$), the equation becomes

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2 \qquad (14.19)$$

This equation implies that the higher the pressure at a point in a fluid, the slower the speed, and vice-versa. The equation of continuity and Bernoulli's principle are often used together to solve for the pressure and speed of a fluid, as the following review questions illustrate.

Example: A large tank open to atmosphere is filled with water. The figure shows this tank with a stream of water flowing through a hole (open to atmosphere) at a depth of 4.00 m. The speed of water, v_2, leaving the hole is:



Answer:

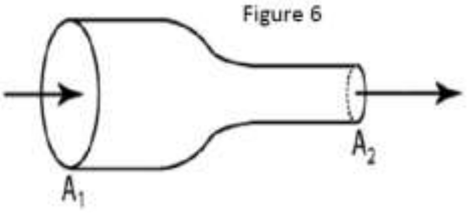
$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2;$$

where $p_1 = p_2 = p_{atm}$, $y_1 - y_2 = 4$ m, $v_1 \approx 0$

➔ $v = \sqrt{2g(y_1 - y_2)} = \sqrt{2 \times 9.8 \times 4.0} = \underline{8.85 \text{ m/s}}$.

T122_**Q25.** Water flows through a horizontal pipe of varying cross section, as shown in **Figure 6**, with A_1 = 10.0 cm^2 and A_2 = 5.00 cm^2. The pressure difference between the two sections is 300 Pa. What is the water speed at the left section of the pipe?

Answer:



Continuity Equation: $A_1 v_1 = A_2 v_2 \Rightarrow 10\, v_1 = 5\, v_2 \Rightarrow v_2 = 2v_1$

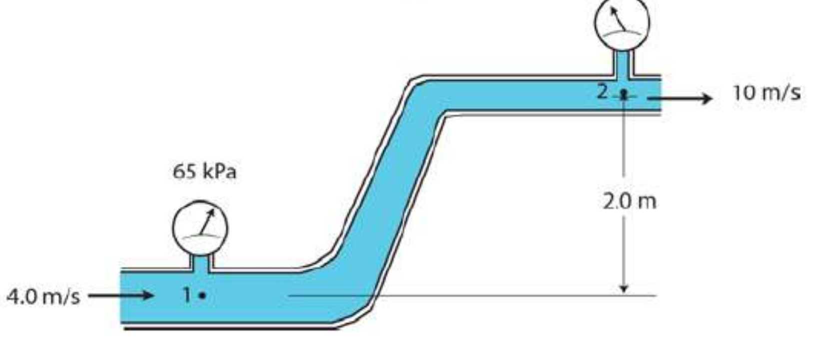
Bernuolli Equation: $p_1 + \frac{1}{2}\rho {v_1}^2 = p_2 + \frac{1}{2}\rho {v_2}^2$

$$p_1 - p_2 = \frac{1}{2}\rho\,({v_2}^2 - {v_1}^2)$$

$$300 = 500\left(4{v_1}^2 - {v_1}^2\right) \Rightarrow v_1 = 0.447 \text{ m/s}$$

T121_**Q24.** Water flows through a pipe as shown in **Figure 9**. The speed of water is 4.0 m/s at point 1 and 10 m/s at point 2. Find the gauge pressure at point 2 if the gauge pressure at point 1 is 65 kPa.

Figure 9



Answer:

$$P_1 + \frac{1}{2}\rho {v_1}^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho {v_2}^2 + \rho g y_2 \quad (\rho g y_1 \to 0)$$

$$65 \times 10^3 + \frac{1}{2} \times 10^3 \times 16 = P_2 + \frac{1}{2} \times 10^3 \times 100 + 10^3 \times 9.8 \times 2$$

$$P_2 = 3.4 \text{ kP}_\text{a}$$

Caution

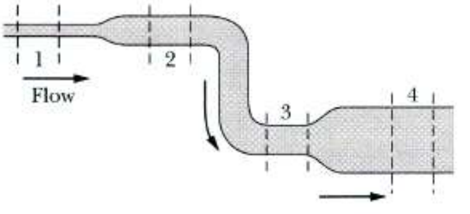
Bernoulli's principle applies only in certain situation. We stress again that Bernoulli's equation is valid for only incompressible, steady flow of a fluid with no viscosity. It's a simple equation that's easy to use.

T33_Q25. Air flows over the upper surface of an aircraft's wing at a speed of 135 m/s and under the lower surface at a speed of 120 m/s. The total wing area is 28 m^2. What is the lift on the wing? (Density of air = 1.20 kg/m^3)

Answer:

$$p_A + \frac{1}{2}\rho v_u^2 = p_l + \frac{1}{2}\rho v_d^2 \rightarrow F = (p_l - p_u) \times A = \frac{\rho}{2}(v_u^2 - v_l^2)A = 64.3\ kN$$

CHECKPOINT 4 Water flows smoothly through the pipe shown in the figure, descending in the process. Rank the four numbered sections of pipe according to (a) the volume flow rate R_V through them, (b) the flow speed v through them, and (c) the water pressure p within them, greatest first.



Note that: $v \propto 1/A.$

(a) all tie; (b) 1, then 2 and 3 tie, 4 (wider means slower); (c) 4, 3, 2, 1 (wider and lower mean more pressure)

Extra Problems

(54) The water flowing through a 1.9 cm (inside diameter d_0) pipe flows out through three 1.3 cm pipes ($d_1 = d_2 = d_3 = 1.3$ cm). (a) If the flow rates in the three smaller pipes (1, 2, 3) are: R_1=26 L/min, R_2=19 L/min, and R_3=11 L/min, what is the flow rate R_0 in the 1.9 cm pipe? (b) Find the speed of water in each of the pipes (with diameters: d_0, d_1, d_2, d_3).

Ans:

$$A_0 = \pi\left(\frac{1.9}{2\times 100}\right)^2 = 2.84\times10^{-4}\,\text{m}^2, A_1 = \pi\left(\frac{1.3}{2\times 100}\right)^2 = 1.33\times10^{-4}\,\text{m}^2$$

$$R_1 = \frac{26\times10^{-3}}{60} = 4.33\times10^{-4}\,\text{m}^3/\text{s}, R_2 = \frac{19\times10^{-3}}{60} = 3.17\times10^{-4}\,\text{m}^3/\text{s}, R_3 = \frac{11\times10^{-3}}{60} = 1.83\times10^{-4}\,\text{m}^3/\text{s}$$

$$(a) R_0 = R_1 + R_2 + R_3 = 9.33\times10^{-4}\,\text{m}^3/\text{s}$$

$$(b)\ v_0 = \frac{R_0}{A_0} = \frac{9.33\times10^{-4}}{2.84\times10^{-4}} = 3.29\,\text{m/s},$$

$$v_1 = \frac{R_1}{A_1} = \frac{4.33\times10^{-4}}{1.33\times10^{-4}} = 3.26\,\text{m/s},$$

$$v_2 = \frac{R_2}{A_1} = \frac{3.17\times10^{-4}}{1.33\times10^{-4}} = 2.39\,\text{m/s},$$

$$v_3 = \frac{R_3}{A_1} = \frac{1.83\times10^{-4}}{1.33\times10^{-4}} = 1.38\,\text{m/s}$$

Example: A garden hose has an inner diameter of 16 mm. The hose can fill a 10 liter bucket in 20 s. Find the speed of the water at the end of the hose (1 Liter = 10^{-3} m^3).

Answer:

$$R_V = vA = \frac{dV}{dt} \quad \Rightarrow \quad v = \frac{\frac{dV}{dt}}{A} = \frac{\left(\frac{10\times10^{-3}}{20}\right)}{\pi\times\left(8\times10^{-3}\right)^2} = \underline{2.5\ \text{m/s}}.$$

Example: In Fig. 14-49, water flows through a horizontal pipe and then out into the atmosphere at a speed $v1$ = 15 m/s. The diameters of the left and right sections of the pipe are 5.0 cm and 3.0 cm.

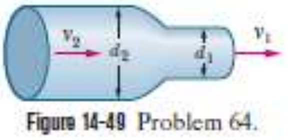


Figure 14-49 Problem 64.

(a) What volume of water flows into the atmosphere during a 10 min period?

(b) In the left section of the pipe, what are the speed v_2?

(c) In the left section of the pipe, what are the gauge pressure?

Answer:

(a) The volume of water (during 10 minutes) is

$$V = (v_1 t) A_1 = (15\,\text{m/s})(10\,\text{min})(60\,\text{s/min})\left(\frac{\pi}{4}\right)(0.03\,\text{m})^2 = 6.4\,\text{m}^3.$$

(b) The speed in the left section of pipe is

$$v_2 = v_1\left(\frac{A_1}{A_2}\right) = v_1\left(\frac{d_1}{d_2}\right)^2 = (15\,\text{m/s})\left(\frac{3.0\,\text{cm}}{5.0\,\text{cm}}\right)^2 = 5.4\,\text{m/s}.$$

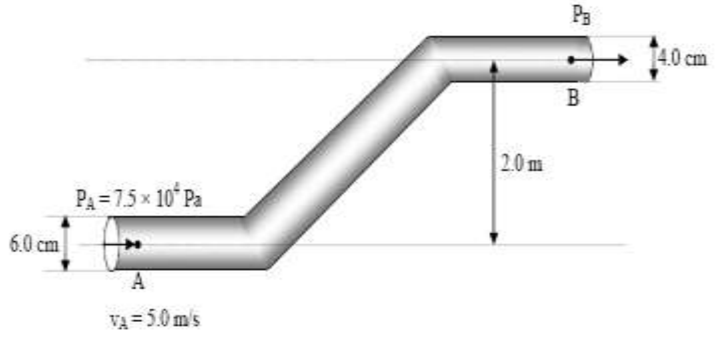
(c) Since $p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$ and $h_1 = h_2, p_1 = p_0$, which is the atmospheric pressure,

$$p_2 = p_0 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) = 1.01\times10^5\,\text{Pa} + \frac{1}{2}\left(1.0\times10^3\,\text{kg/m}^3\right)\left[\left(15\,\text{m/s}\right)^2 - \left(5.4\,\text{m/s}\right)^2\right]$$

$$= 1.99\times10^5\,\text{Pa} = 1.97\,\text{atm}.$$

Thus, the gauge pressure is (1.97 atm – 1.00 atm) = 0.97 atm = 9.8 x 10^4 Pa.

Example: Water flows through a pipe as shown in the figure. At the lower elevation, the water's speed (v_A) is 5.0 m/s and the gauge pressure (P_A) is 7.5 × 10^4 Pa. Find the gauge pressure at the higher elevation (P_B). (Diameter at A = 6.0 cm, diameter at B = 4.0 cm and the elevation of B relative to A is 2.0 m)



Answer:

Apply the continuity equation

$$\Rightarrow v_A A = v_B B \quad \Rightarrow \quad v_B = \left(\frac{A}{B}\right) v_A = \left(\frac{3}{2}\right)^2 5 = \underline{11.25\text{ m/s}}.$$

Apply the Bernoulli's equation

$$P_A + \frac{1}{2}\rho v_A^2 + \rho g\,(0) = P_B + \frac{1}{2}\rho v_B^2 + \rho g\,(2)$$

$$\Rightarrow \quad P_B = P_A + \frac{1}{2}\rho\left[5^2 - \left(11.25\right)^2\right] - \rho 9.8(2)$$

$$= 7.5\times10^4 + \underbrace{\frac{1}{2}\times1000(-101.6) - 1000\times9.8\times2}_{-1000\times(50.8+19.6)} = \underline{4.6\times10^3\text{ Pa}}.$$

Example:

Figure 14-56 shows a *siphon*, which is a device for removing liquid from a container. Tube *ABC* must initially be filled, but once this has been done, liquid will flow through the tube until the liquid surface in the container is level with the tube opening at *A*. The liquid has density 1000 kg/m^3 and negligible viscosity. The distances shown are h_1 = 25 cm, d = 12 cm, and h_2 = 40 cm.

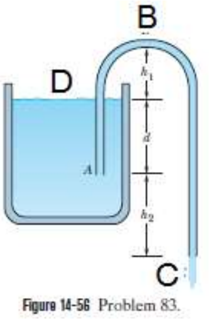


Figure 14-56 Problem 83.

(a) With what speed does the liquid emerge from the tube at *C*?

(b) If the atmospheric pressure is 1.0 x 10^5 Pa, what is the pressure in the liquid at the topmost point *B*?

Answer:

(a) We consider a point *D* on the surface of the liquid in the container, in the same tube of flow with points *A*, *B* and *C*. Applying Bernoulli's equation to points *D* and *C*, we obtain

$$p_D + \frac{1}{2}\rho v_D^2 + \rho g h_D = p_C + \frac{1}{2}\rho v_C^2 + \rho g h_C$$

which leads to

$$v_C = \sqrt{\frac{2(p_D - p_C)}{\rho} + 2g(h_D - h_C) + v_D^2} \approx \sqrt{2g\,(d + h_2)}$$

where in the last step we set $p_D = p_C = p_{\text{air}}$ and $v_D/v_C = 0$. Plugging in the values, we obtain

$$v_c = \sqrt{2(9.8\text{ m/s}^2)(0.40\text{ m} + 0.12\text{ m})} = 3.2\text{ m/s}.$$

(b) We now consider points B and C:

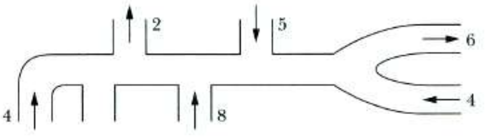
$$p_B + \frac{1}{2}\rho v_B^2 + \rho g h_B = p_C + \frac{1}{2}\rho v_C^2 + \rho g h_C\ .$$

Since $v_B = v_C$ by equation of continuity, and $p_C = p_{\text{air}}$, Bernoulli's equation becomes

$$\begin{aligned} p_B &= p_C + \rho g(h_C - h_B) = p_{\text{air}} - \rho g\,(h_1 + h_2 + d) \\ &= 1.0\times 10^5\text{ Pa} - (1.0\times 10^3\text{kg/m}^3)(9.8\text{ m/s}^2)(0.25\text{ m} + 0.40\text{ m} + 0.12\text{ m}) \\ &= 9.2\times 10^4\text{ Pa}. \end{aligned}$$

CHECKPOINT 3

The figure shows a pipe and gives the volume flow rate (in cm^3/s) and the direction of flow for all but one section. What are the volume flow rate and the direction of flow for that section?



$$\text{in} \rightarrow +\text{ve},\quad \text{out} \rightarrow -\text{ve},\quad \text{net flow rate} = 0$$

$$\Rightarrow +4 + 8 + 5 + 4 - 2 - 6 + X = 0$$

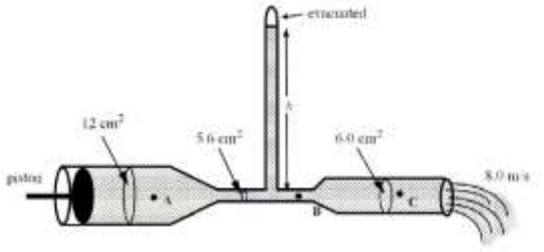
$$\Rightarrow X = -13.$$

T33_Q25. Air flows over the upper surface of an aircraft's wing at a speed of 135 m/s and under the lower surface at a speed of 120 m/s. The total wing area is 28 m2. What is the lift on the wing? (Density of air = 1.20 kg/m3)

Answer:

$$p_A + \frac{1}{2}\rho v_u^2 = p_l + \frac{1}{2}\rho v_d^2 \rightarrow F = (p_l - p_u) \times A = \frac{\rho}{2}(v_u^2 - v_l^2)A = 64.3\ kN$$

T112_ Q13. A glass tube has several different cross-sectional areas with the values indicated in the **Figure 6**. A piston at the left end of the tube exerts pressure so that mercury within the tube flows from the right end with a speed of 8.0 m/s. Three points within the tube are labelled A, B, and C. What is the total pressure at point A? Atmospheric pressure is 1.01 × 105 N/m2; and the density of mercury is 1.36 × 104 kg/m3.



Answer

First use the continuity equation

$$\Rightarrow v_A A_A = v_C A_C \quad \Rightarrow \quad v_A = \left(\frac{A_C}{A_A}\right) v_C = \left(\frac{8}{12}\right) 6 = \underline{4.0 \text{ m/s}}.$$

The pressures on either side of the junction must be equal. This requires:

$$p + \frac{1}{2}\rho v_A^2 = p_o + \frac{1}{2}\rho v_C^2$$

$$\Rightarrow p = p_o + \frac{1}{2}\rho\left(v_C^2 - v_A^2\right) = (1.01\times10^5) + \frac{1}{2}13600(8^2 - 4^2) = 4.27\times10^5 \text{ Pa}$$

T113_Q25. A liquid of density 1.00×10^3 kg/m^3 flows through a horizontal pipe that has a cross-sectional area of 2.00×10^{-2} m^2 in region A and a cross-sectional area of 10.0×10^{-2} m^2 in region B. The pressure difference between the two regions is 10.0×10^3 Pa. What is the mass flow rate between regions A and B?

Answer

$$A_1 v_1 = A_2 v_2$$

$$2 \times 10^{-2} v_1 = 10 \times 10^{-2} v_2 \Rightarrow v_1 = 5v_2$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2 \quad (\rho g y_1 \to 0,\ \rho g y_2 \to 0)$$

$$P_2 - P_1 = \frac{1}{2}\rho(v_1^2 - v_2^2) = \frac{1}{2} \times 10^3\left(25v_2^2 - v_2^2\right)$$

$$\therefore 10 \times 10^3 = \frac{1}{2} \times 10^3 \times 24v_2^2$$

$$v_2^2 = \frac{10}{12} \Rightarrow v_2 = \sqrt{\frac{10}{12}} = 0.9313 \text{ m/s}$$

$$\therefore \text{Mass flow rate: } = A_2 v_2 \rho = 91.3 \text{ kg/s}$$

Sample Problem 14.05 A water stream narrows as it falls

Figure 14-18 shows how the stream of water emerging from a faucet "necks down" as it falls. This change in the horizontal cross-sectional area is characteristic of any laminar (non-turbulant) falling stream because the gravitational force increases the speed of the stream. Here the indicated cross-sectional areas are $A_0 = 1.2\ \text{cm}^2$ and $A = 0.35\ \text{cm}^2$. The two levels are separated by a vertical distance $h = 45$ mm. What is the volume flow rate from the tap?

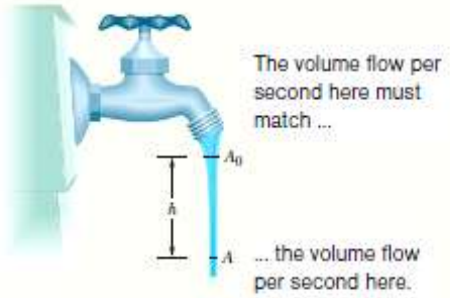


Figure 14-18 As water falls from a tap, its speed increases. Because the volume flow rate must be the same at all horizontal cross sections of the stream, the stream must "neck down" (narrow).

KEY IDEA

The volume flow rate through the higher cross section must be the same as that through the lower cross section.

Calculations: From Eq. 14-24, we have

$$A_0 v_0 = A v, \qquad (14\text{-}26)$$

where v_0 and v are the water speeds at the levels corresponding to A_0 and A. From Eq. 2-16 we can also write, because the water is falling freely with acceleration g,

$$v^2 = v_0^2 + 2gh. \qquad (14\text{-}27)$$

Eliminating v between Eqs. 14-26 and 14-27 and solving for v_0, we obtain

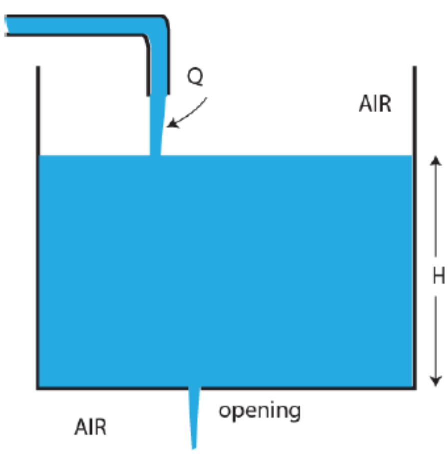
$$v_0 = \sqrt{\frac{2ghA^2}{A_0^2 - A^2}}$$
$$= \sqrt{\frac{(2)(9.8\ \text{m/s}^2)(0.045\ \text{m})(0.35\ \text{cm}^2)^2}{(1.2\ \text{cm}^2)^2 - (0.35\ \text{cm}^2)^2}}$$
$$= 0.286\ \text{m/s} = 28.6\ \text{cm/s}.$$

From Eq. 14-24, the volume flow rate R_V is then

$$R_V = A_0 v_0 = (1.2\ \text{cm}^2)(28.6\ \text{cm/s})$$
$$= 34\ \text{cm}^3/\text{s}. \qquad \text{(Answer)}$$

T121_Q25. Water pours into a very large open tank at a volume flow rate of Q (**Figure 10**). The tank has an opening at the bottom. The area of this opening for the water level in the tank to be maintained at a fixed level H is given by:

Figure 10



Answer:

$$Q = v_2 A$$

To find v_2:

$$\cancel{P_1} + \frac{1}{2}\rho \cancelto{0}{v_1}^2 + \rho g y_1 = \cancel{P_2} + \frac{1}{2}\rho v_2^2 + \cancelto{0}{\rho g y_2}$$

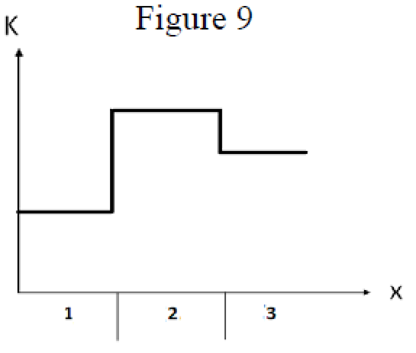
$$\cancel{\rho} g H = \frac{1}{2}\cancel{\rho} v_2^2$$

$$v_2 = \sqrt{2gH}$$

$$\therefore A = \frac{Q}{\sqrt{2gH}}$$

T131_**Q21.** Water flows smoothly in a horizontal pipe. **Figure 9** shows the kinetic energy K of a water element as it moves along the x-axis that runs along the pipe. Rank the numbered sections of the pipe according to the pipe radius, smallest first.

Figure 9



Answer:

$$K = \frac{1}{2}\rho v^2;\ but\ R_v = Av\ \Rightarrow v = \frac{R_v}{A}$$

$$K = \frac{\rho}{2}\frac{R_v^2}{A^2} = \left(\frac{\rho R_v^2}{2}\right)\frac{1}{A^2}$$

$$K \propto \frac{1}{A^2}$$

Fluid Flow (free reading)

An ideal fluid is a fluid that is *incompressible* (that is, its density can't change) and has no internal friction (called viscosity). Liquids are approximately incompressible in most situations, and we may also treat a gas as incompressible if the pressure differences from one region to another are not too great.
The path of an individual particle in a moving fluid is called a **flow line**. If the overall flow pattern does not change with time, the flow is called **steady flow**.
A streamline is a curve whose tangent at any point is in the direction of the fluid velocity at that point.
We will consider only steady-flow situations, for which flow lines and streamlines are identical. The flow lines passing through the edge of an imaginary element of area A (Fig.14.19) form a tube called a **flow tube**. From the definition of a flow line, in steady flow no fluid can cross the side walls of a flow tube; the fluids in different flow tubes cannot mix.

14.19 A flow tube bounded by flow lines. In steady flow, fluid cannot cross the walls of a flow tube.

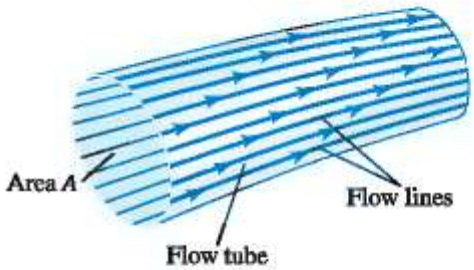
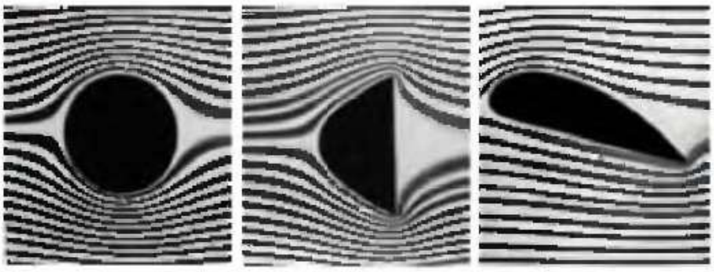


Figure 14.20 shows patterns of fluid flow from left to right around a number of obstacles. These patterns are typical of **laminar flow**, in which adjacent layers of fluid slide smoothly past each other and the flow is steady. At sufficiently high flow rates, or when boundary surfaces cause abrupt changes in velocity, the flow can become irregular and chaotic. This is called **turbulent flow** (Fig. 14.21). In turbulent flow there are no steady-state pattern; the flow pattern changes continuously.

14.20 Laminar flow around obstacles of different shapes.



14.21 The flow of smoke rising from these incense sticks is laminar up to a certain point, and then becomes turbulent.