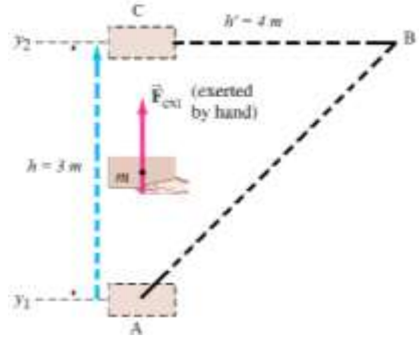
Chapter 8
Potential Energy and Conservation of Energy

Important Terms (For chapters 7 and 8)

- **conservative force:** a force which does work on an object which is independent of the path taken by the object between its starting point and its ending point
- **conserved properties:** any properties which remain constant during a process
- **energy:** the non-material quantity which is the ability to do work on a system
- **joule:** the SI unit for energy, and work, equal to one Newton-meter
- **kinetic energy:** the energy a mass has by virtue of its motion
- **law of conservation of energy:** the total energy of a system remains constant during a process
- **mechanical energy:** the sum of the potential and kinetic energies in a system
- **potential energy:** the energy an object has because of its position
- **power: t**he rate at which work is done or energy is dissipated
- **watt:** the SI unit for power equal to one joule of energy per second
- **work:** the scalar product of force and displacement

$W = \mathbf{F}\cdot\mathbf{s} = (F\cos\theta)s$	where
$W = \int \mathbf{F}\cdot d\mathbf{s} = \int F ds$	W = work
$KE = \frac{1}{2}mv^2$	F = force
$W = \Delta K = KE_f - KE_i = \frac{1}{2}m{v_f}^2 - \frac{1}{2}m{v_i}^2$	s = displacement
$PE = mgh$	**F. s** = scalar product of force and displacement
$\Delta U_g = -W_g = mg(h_f - h_i)$	KE = kinetic energy
$\Delta E_{mech} = E_f - E_i = 0$	v = velocity or speed
$(PE_f + KE_f) - (PE_i + KE_i) = 0$	m = mass
$W_f = \Delta E_{mech}$	PE = potential energy (denoted as U)
$P = \frac{W}{t} = \frac{\mathbf{F}\cdot\mathbf{s}}{t} = \mathbf{F}\cdot\frac{\mathbf{s}}{t} = \mathbf{F}\cdot\mathbf{v} = (F\cos\theta)v$	g = acceleration due to gravity
$P = m\,a\cdot\mathbf{v} = (F\cos\theta)v$	h = height above some reference point
	P = power
	t = time
	i = initial state
	f = final state

8-1 POTENTIAL ENERGY (Done in Chapter 7)



H.W. Use the force $\vec{F}_{ext} = 10\,\hat{j}$ to calculate the work done, W_{ext}, through the path AC and the path ABC. Ans (30 J).
What is your comment?

H.W. What is the work done, W_G, by the gravitational field?
H.W. What is the relation between W_{ext} and W_G? Define the P.E. U_G.
Answer: Define: ext = external force, G = gravitation field

$$W_{ext} + W_G = 0$$

$$U_G = W_{ext}$$

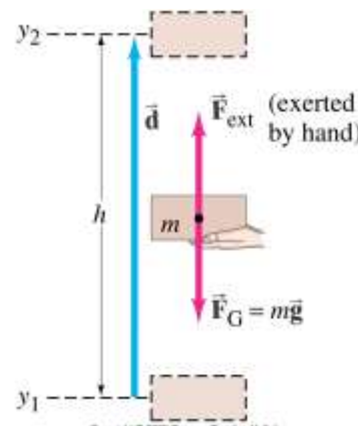
In summary: The work done could be calculated in two different ways:

$$W_{ext} = \begin{cases} \vec{F}_{ext} \cdot \vec{d} & \text{(I)} \\ \Delta K = \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) & \text{(II)} \end{cases}$$

In rising (lowering) a mass *m* to (from) a height *h*, the work done by the gravitational force W_G is

$$W_G \uparrow = -mgh$$

$$W_G \downarrow = \ mgh$$



Compare this with the change in the gravitational potential energy:

$$\Delta U_G \uparrow = \ mgh$$

$$\Delta U_G \downarrow = -mgh$$

This implies:

$$\boxed{\Delta U_G \uparrow\downarrow = -W_G \uparrow\downarrow}$$

This is also applied for the spring:

$$U_S = -W_S = \tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2$$

8-2 CONSERVATION OF MECHANICAL ENERGY

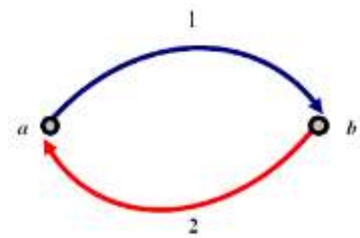
Conservative force

A force is conservative if

"***the work done by this force when acting on a moving object between two points is independent of the path the particle takes between these two points***."

The work done by conservative force depends only on the coordinates of the initial and final points and independent of the path taken between these two points. (see the following figure).

Let $W_{ab}(1)$ be the work done in moving from point ***a*** to point ***b*** through the path **(1),** and $W_{ab}(2)$ be the work done in moving from point ***a*** to point ***b*** through the path **(2).** If the force is conservative then



$$W_{ab}(1) = W_{ab}(2)$$

The work done by conservative force in a reversed path equal the negative of the work done by this force in the non-reversed path.

$$W_{ab}(1) = -W_{ba}(1) = -W_{ba}(2)$$

$$W_{ab}(1) = -W_{ba}(2)$$

$$W_{ab}(1) + W_{ba}(2) = 0$$

Since the path aba is a closed path then, we can say that:

"***the work done by conservative force when the object moves in a closed path is zero***".

The work done by gravitational force

1- When a body of mass m moves from a point of height $\mathbf{h_i}$ to a point of height $\mathbf{h_f}$ from the earth surface, under the effect of gravity, the work done by the gravity is given by

$$W_G = -mg(h_f - h_i)$$

It is negative because the motion is against the weight mg.

2- If the body makes a round trip and returns to its initial height, $h_f = h_i$, then the work done by gravity is zero. Since the work done by any force equal the change in kinetic energy, then in this case the change in kinetic energy is zero. The final kinetic energy equal the initial kinetic energy and the final speed equal the initial speed.

$$K_f = K_i$$

$$v_f = v_i$$

Nonconservative force

A force is non-conservative if the work done by this force on a particle moving between two points depends on the path taken e.g. friction forces. The property of non-conservative force is that

$$W_{ab}(1) \neq W_{ab}(2)$$

$$W_{ab}(1) + W_{ba}(2) \neq 0$$

Potential energy and the work done by conservative force

The work done by the conservative force equal the decrease in potential energy. If the work is W_c, then

$$W_c = -\Delta U$$

i.e. the work done by the conservative force equal the negative of the change in potential energy.

Conservation of mechanical energy

When a conservative force acts on a moving object, then from the work energy theorem

$$W_c = \Delta K \qquad \text{(i)}$$

Where W_c is the work done and ΔK is the change in kinetic energy.
Since the force is conservative then

$$W_c = -\Delta U \qquad \text{(ii)}$$

is the change in potential energy. From (i) and (ii) we get.

$$\Delta K = -\Delta U \quad \Rightarrow \quad \Delta K + \Delta U = 0$$

$$\Rightarrow \quad \Delta(K+U) = 0$$

$K + U = E$ is the total energy. K_i is the initial kinetic energy, U_i is the initial potential energy, K_f is the final kinetic energy, and U_f is the final potential energy

$$K_i + U_i = K_f + U_f$$

$$E_i = E_f$$

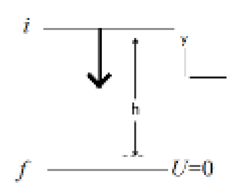
The law of conservation of mechanical energy

This law states that

"***the total mechanical energy of a system remains constant if the only force acting on the body is conservative force.***"

If the potential energy decreases (or increases) the kinetic energy increases (or decreases) by the same amount.

Example1: Use conservation of total mechanical energy to compute the final velocity of an object dropped from some height, h, just before it strikes the ground.



Answer: Here, only conservative forces are present. Consequently, we have:

$$E_i = E_f \quad \Rightarrow \quad K_i + U_i = K_f + U_f$$

Define the zero potential energy reference level at the final position f, see figure. The conservation of mechanical energy proceeds as follows:

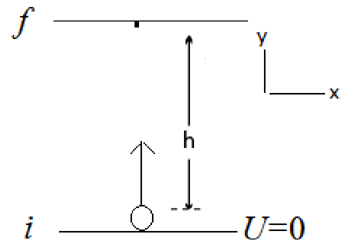
At point *i*, $K_i = 0, \quad U_i = mgh \quad \Rightarrow \quad E_i = K_i + U_i = mgh$

At point *f*, $K_f = \frac{m}{2} v_f^2, \quad U_f = 0 \quad \Rightarrow \quad E_f = K_f + U_f = \frac{m}{2} v_f^2$

Use the conservation of mechanical energy proceeds as $\Delta E = 0$

$$\Rightarrow \left(E_f - E_i\right) = \frac{m}{2} v_f^2 - mgh = 0 \quad \Rightarrow \quad v_f = \sqrt{2gh}$$

Which is the same as the kinematic equation.



Example2: To what height will a ball rise if tossed upwards with an initial velocity of 10 m/s?

Answer: Again, only conservative forces are present:

$$E_i = E_f$$

$$K_i + U_i = K_f + U_f$$

or

$$\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f \,.$$

If we define the ground as being at height $h_i = 0$, (zero potential energy reference level), then $h_f = h$ and conservation of mechanical energy proceeds as follows:

At point *i*, $K_i = \frac{1}{2}mv_i^2, \quad U_i = 0 \quad \Rightarrow \quad E_i = K_i + U_i = \frac{1}{2}mv_i^2$

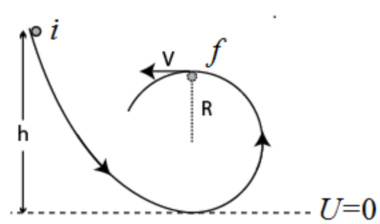
At point *f*, $K_f = 0, \quad U_f = mgh \quad \Rightarrow \quad E_f = K_f + U_f = mgh$

$$\frac{1}{2}mv_i^2 + 0 = 0 + mgh = \quad \Rightarrow \quad h = \frac{v_i^2}{2g} = 5.1 \text{ m.}$$

Note: We could also use Work-Energy theorem:

$$W = \Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = +m\vec{g}.\vec{h}$$

$$\Rightarrow \quad 0 - \frac{1}{2}mv_i^2 = -gh_f \quad \Rightarrow \quad h_f = 5.1 \text{ m.}$$



Example: A ball slides without friction around a loop-the-loop (see the figure). A ball is released, from rest, at a height *h* from the left side of the loop of radius *R*. What is the speed of the ball at the highest point of the loop? (g = acceleration due to gravity)

Answer: Apply the conservation of the mechanical energy at point I and at the highest point of the loop, we find:

At point i **,** $K_i = 0, \quad U_i = mgh \quad \Rightarrow \quad E_i = K_i + U_i = mgh$

At point f **,** $K_f = \frac{m}{2}\text{v}^2, \quad U_f = mg(2R) \quad \Rightarrow \quad E_f = K_f + U_f = \frac{m}{2}\text{v}^2 + 2mgR$

Applying the conservation of mechanical energy:

$$\Delta E = 0 \quad \Rightarrow \left(E_f - E_i\right) = \frac{m}{2}\text{v}^2 + 2mgR - mgh = 0 \quad \Rightarrow \quad \text{v} = \underline{\sqrt{2g(h-2R)}}$$

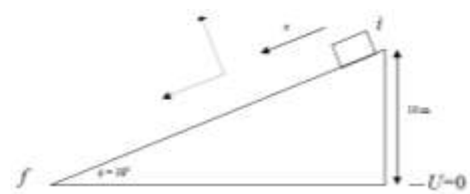
Example: A projectile is fired from the top of a 40 m high building with a speed of 20 m/s. What will be its speed when it strikes the ground?

Answer:

$$E_i = E_f \quad \Rightarrow \quad \frac{1}{2}mv_i^2 + mgh = \frac{1}{2}mv_f^2$$

$$\Rightarrow v_f = \sqrt{v_i^2 + 2gh} = \sqrt{20^2 + 2\times 9.8\times 40} = \underline{34.4 \text{ m/s}}$$

Example: A 10 kg block slides down a smooth surface. It is released from rest at the top of an incline (38° to the horizontal) at a height 10 meters. What is its speed at the bottom of the incline?



Answer:

We've seen this problem before. This method of solving the problem yields a solution but it depends upon resolving vectors, summing forces, determining acceleration, and finally kinematics!

Now consider using conservation of mechanical energy to address the same problem:

Since only conservative forces are present:

$$K_i + U_i = K_f + U_f$$

If we define the ground as being at height $h_f = 0$, (zero potential energy reference level), then $h_i = h$

At point *i*, $K_i = 0, \quad U_i = mgh_i \quad \Rightarrow \quad E_i = K_i + U_i = mgh$

At point *f*, $K_f = \frac{1}{2}mv_f^2, \quad U_f = 0 \quad \Rightarrow \quad E_f = K_f + U_f = \frac{1}{2}mv_f^2$

Use the conservation of mechanical energy proceeds as $\Delta E = 0$

$$\Rightarrow \left(E_f - E_i\right) = \frac{m}{2}v_f^2 - mgh = 0 \quad \Rightarrow \quad v_f = \sqrt{2gh}$$

Then

$$\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f \,.$$

If we define the ground as being at height $h_f = 0$, (zero potential energy reference level), then $h_i = h$

$$\frac{1}{2}mv_f^2 + 0 = 0 + mgh \quad \Rightarrow \quad v_f = \sqrt{2gh} = 14.1 \text{ m/s.}$$

This is a much more compact and elegant method of solution.

Conservation of Mechanical Energy with non-conservative forces

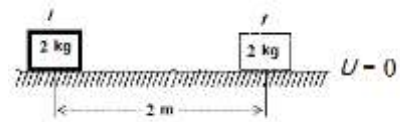
When non-conservative forces are present, total mechanical energy is not conserved, i.e., the total energy of the system changes ($E_f \neq E_i$). The only non-conservative force we will deal with is friction.

Frictional Forces

Frictional forces result in energy being dissipated from systems. Frictional forces reduce the kinetic energy of a system by some amount that generally is easy to compute. The result is that the initial and final energies of a non-conservative system are not the same. The energies, in fact, differ by the amount of work done by the non-conservative force (friction):

$$W_{nc} = W_f = \vec{f}_k.\vec{d} = \Delta KE \quad \text{or} \quad W_{nc} = \vec{f}_k.\vec{d} = \Delta E$$

Example: A 2.0 kg block is initially moving to the right on a rough horizontal surface, with $\mu_K = 0.5$, at a speed of 5.0 m/s. Calculate its speed after moving 2.0 m.



Answer:

At state *i*: $K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(2)(5)^2, \quad U_i = 0, \quad E_i = 25$ J.

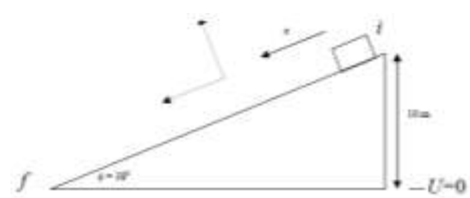
At state *f*: $K_f = \frac{1}{2}mv_f^2 = v_f^2, \quad U_i = 0, \quad E_i = v_f^2.$

$$W_{nc} = \begin{cases} \vec{f}_k.\vec{d} = -\mu_k mgd = -\frac{1}{2}(2)(9.8)(2) & \text{(I)} \\ E_f - E_i = v_f^2 - 25 & \text{(II)} \end{cases}$$

Equating (I) and (II), we get

$$v_f = \sqrt{25 - 19.6} = 2.3 \text{ m/s}$$

Example: A 10 kg block slides down a rough surface (μ_k =0.2). It is released from rest at the top of an incline (38^o to the horizontal) at a height 10 meters. What is its speed at the bottom of the incline?



Answer: Using conservation of energy. Since a non-conservative force is present

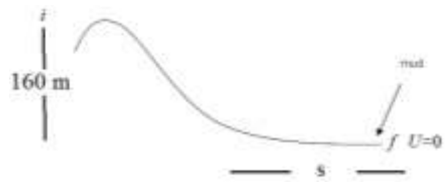
$W_f = \vec{f}_k.\vec{d} = \Delta KE$ or $W_f = \vec{f}_k.\vec{d} = \Delta E$. Using the latter:

$$W_{nc} = \begin{cases} \vec{f}_k.\vec{d} = -\mu_k mgd\cos\theta & \text{(I)} \\ \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) + \left(mgh_f - mgh_i\right) = \frac{1}{2}mv_f^2 - mgh_i & \text{(II)} \end{cases}$$

Equating (I) and (II), we get

$$v_f^2 = \sqrt{2gh - 2\mu_k mgd\cos\theta} = 12.1 \text{ m/s}$$

Example: A skier starts from rest at the top of a 160 m tall hill. The portion of the hill covered with snow is essentially frictionless but flat section at the base of the hill covered with mud. If the skier weighs 80 kg, and the coefficient of kinetic friction between rental skis and mud is 0.8, how much distance is required for the skier to come to rest?



Answer:

A non-conservative force, friction, is present here, $E_i \neq E_f$, and total mechanical energy is not conserved.

$$E_i = mgh = (80\text{ kg})(9.8\text{ m/s}^2)(160\text{ m}) = 125440\text{ J}$$

$$E_f = 0$$

$$W_{nc} = \begin{cases} f_s.s = -\mu_k mgs & \text{(I)} \\ \Delta E = -125440\text{ J}_i & \text{(II)} \end{cases}$$

Equating (I) and (II), we get

$$\Rightarrow s = \frac{125440\text{ J}}{(80\text{ kg})(9.8\text{ m/s}^2)(0.8)} = 200\text{ m}$$

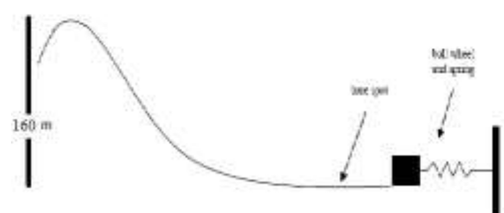
We could arrive at the same result by looking at the skier's velocity at the base of the hill and computing the change in *kinetic energy* to get the work done by the non-conservative force.

$$v_{bottom} = \sqrt{2gh} = 56 m \cdot s^{-1}$$

$$v_{final} = 0$$

$$\Delta KE = -125440 J \therefore s = 200 m$$

Example: This time our 80 kg skier starts from rest at the top of a 160 meter tall hill. The downhill portion of the run is snow covered and essentially frictionless but a flat section at the base of the hill has a bare spot 2 meters in length. The skier skis straight down the hill, over the bare spot at the bottom, hits the counterweight on the bull-wheel compressing the damping spring 50 centimeters in the process. If the spring constant is 1×10^6 N/m, what is the coefficient of kinetic friction between rental skis and bluegrass?



Answer: A non-conservative force, friction, is present here, $E_i \neq E_f$, and total mechanical energy is not conserved.

$$E_i = mgh = (80\text{ kg})(9.8\text{ m/s}^2)(160\text{ m}) = 125440\text{ J}$$

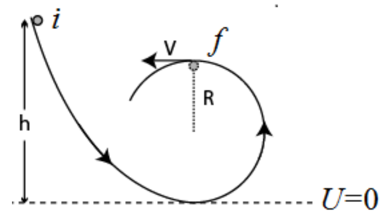
$$E_f = \frac{1}{2}kx^2 = (0.5)(10^6\text{ N/m})(0.5\text{ m})^2 = 125000\text{ J}$$

$$W_f = \Delta E = -440\text{ J} = f_s.s = -\mu_k mgs$$

$$\Rightarrow \mu_k = \frac{440\text{ J}}{(80\text{ kg})(2\text{ m})(9.8\text{ m/s}^2)} = 0.28$$

Extra Problems

Example: A ball slides without friction around a loop-the-loop (see the figure). A ball is released, from rest, at a height h from the left side of the loop of radius R. What is the ratio (h/R) so that the ball has a speed $\text{v} = \sqrt{Rg}$ at the highest point of the loop? (g = acceleration due to gravity)



Answer: Apply the conservation of the mechanical energy at the beginning of the trip and at the highest point of the loop, we find:

At point *i*, $K_i = 0, \quad U_i = mgh \quad \Rightarrow \quad E_i = K_i + U_i = mgh$

At point *f*, $K_f = \frac{m}{2}\text{v}^2 = \frac{m}{2}(Rg), \quad U_f = mg(2R) \quad \Rightarrow \quad E_f = K_f + U_f = \frac{5}{2}mgR$

$$\Delta E = 0 \quad \Rightarrow \left(E_f - E_i\right) = \frac{5}{2}mgR - mgh = 0 \quad \Rightarrow \quad \frac{h}{R} = \underline{\frac{5}{2}}$$

Example: A worker does 500 J of work in moving a 20 kg box a distance D on a rough horizontal floor. The box starts from rest and its final velocity after moving the distance D is 4.0 m/s. Find the work done by the friction between the box and the floor in moving the distance D.

Answer: Total work by the man is used to change the K.E. of the body and to overcome the friction. So,

$$\begin{aligned} \Delta K &= W_{man} + W_f \\ \Rightarrow \quad W_f &= \Delta K - W_{man} \\ &= \frac{1}{2}mv_f^2 - 500 = \frac{1}{2}20 \times 4^2 - 500 = -340 \text{ J.} \end{aligned}$$

Example: A 2.0 kg block is released from rest 60 m above the ground. Take the gravitational potential energy of the block to be zero at the ground. At what height above the ground is the kinetic energy of the block equal to half its gravitational potential energy? (Ignore air resistance)

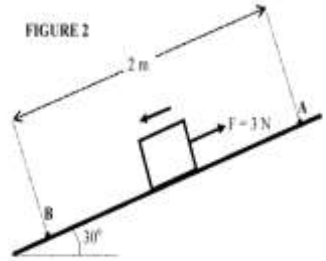
Answer: Apply the conservation of the mechanical energy at the beginning of the trip and at the x-position, we find:

$$K_i + U_i = K_x + U_x$$

$$\Rightarrow 0 + mgh = \frac{1}{2}mgx + mgx \Rightarrow x = \frac{2}{3}h = \underline{40 \text{ m}}$$

Example: A 2.2-kg block starts from rest on a rough inclined plane that makes an angle of 25° with the horizontal. The coefficient of kinetic friction is 0.25. As the block goes 2.0 m down the plane, find the change in the mechanical energy of the block.

FIGURE 2



Answer:

$$\begin{aligned} \Delta E &= W_{nc} = -\mu N d = -\mu mg \cos 30^\circ d \\ &= -0.25 \times 2.2 \times 9.8 \times 0.866 \times 2 = \underline{-9.3 \text{ J}} \end{aligned}$$

Example: A 0.50 kg block attached to an ideal spring with a spring constant of 80 N/m oscillates on a horizontal frictionless surface. The speed of the block is 0.50 m/s, when the spring is stretched by 4.0 cm. The maximum speed the block can have is: (Ans: 0.71 m/s)

Answer:

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 =$$
$$= \frac{1}{2}0.5\times 0.5^2 + \frac{1}{2}80\times 0.04^2 = 0.1265$$
$$\Rightarrow \quad v_f = \sqrt{\frac{2}{0.5}0.1265} = \underline{0.71\text{ m/s}}$$

Example: A 10.0 kg block is released from rest at 100 m above the ground. When it has fallen 50 m, its kinetic energy is:

Answer: $$\Delta K = -\Delta U_g = mgh = 10\times 9.8\times 50 = \underline{4900\text{ J}}.$$

Example: A 4.0 kg block is initially moving to the right on a horizontal frictionless surface at a speed of 5.0 m/s. It then compresses a horizontal spring of spring constant 200 N/m. At the instant when the kinetic energy of the block is equal to the potential energy of the spring, the mechanical energy of the block-spring system is:

Answer:

$$\Delta E = 0 \Rightarrow E_i = E_f \text{ (or at any place)} \Rightarrow E_i = \frac{1}{2}mv_i^2 = \frac{1}{2}\times 4\times 5^2 = \underline{50\text{ J}}.$$

Example: A 5.0 kg block starts up a 30° incline with 198 J of kinetic energy. The block slides up the incline and stops after traveling 4.0 m. The work done by the force of friction between the block and the incline is:

Answer:

$$W_f = \Delta E = \Delta K + \Delta U = (K_f - K_i) + (U_f - U_i)$$
$$= (0-198) + (5\times 9.8\times 4\sin 30^\circ) = \underline{-100\text{ J}}.$$

Example: A 2.0 kg object is connected to one end of an unstretched spring which is attached to the ceiling by the other end and then the object is allowed to drop. The spring constant of the spring is 196 N/m. How far does it drop before coming to rest momentarily? (Ans: 0.20 m)

Answer: Apply the conservation of the mechanical energy:

$$\Delta E = 0 \quad \Rightarrow \quad \frac{1}{2}kx^2 = mgh \quad \Rightarrow \quad h = \frac{2mg}{k} = 0.2\text{ m}.$$

Example: An ideal spring (compressed by 7.00 cm and initially at rest,) fires a 15.0 g block horizontally across a frictionless table top. The spring has a spring constant of 20.0 N/m. The speed of the block as it leaves the spring is:

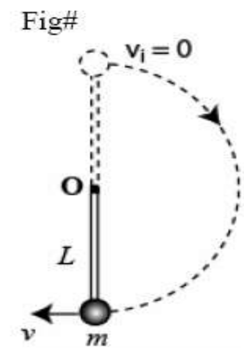
Answer: Apply the conservation of the mechanical energy:

$$\Delta E = 0 \quad \Rightarrow \quad \left(K_f - K_i\right) + \left(U_{Gf} - U_{Gi}\right) = 0$$

$$\Rightarrow (\frac{1}{2}mv^2 - 0) + \left(0 - \frac{1}{2}kx^2\right) = 0$$

$$\Rightarrow v = \sqrt{\frac{k}{m}}x = \sqrt{\frac{20}{0.015}}(0.07) = 2.56 \text{ m/s.}$$

Example: A small object of mass m on the end of a massless rod of length L is held vertically, initially. The rod is pivoted at the other end **O**. The object is then released from rest and allowed to A small object of mass m on the end of a massless rod of length L is held vertically, initially. The rod is pivoted at the other end **O**. The object is then released from rest and allowed to swing down in a circular path as shown in the figure. What is the speed (v) of the object at the lowest point of its swing? (Assume no friction at the pivot)



Answer: Apply the conservation of the mechanical energy at the beginning of the trip and at the lowest point of the motion, we find:

$$\Delta E = 0 \quad \Rightarrow \quad \left(K_f - K_i\right) + \left(U_{Gf} - U_{Gi}\right) = 0$$

$$\Rightarrow (\frac{1}{2}mv^2 - 0) + \left(mg\,2L - 0\right) = 0$$

$$\Rightarrow v = \sqrt{4Lg}$$

Example: A projectile is fired from the top of a 40 m high building with a speed of 20 m/s. What will be its speed when it strikes the ground?

Answer:

$$E_i = E_f \quad \Rightarrow \quad \frac{1}{2}mv_i^2 + mgh = \frac{1}{2}mv_f^2$$

$$\Rightarrow v_f = \sqrt{v_i^2 + 2gh} = \sqrt{20^2 + 2\times 9.8\times 40} = \underline{34.4 \text{ m/s}}$$

Example: A projectile of mass 0.20 kg is fired with an initial speed of 20 m/s at an angle of 60 degrees above the horizontal. The kinetic energy of the projectile at its highest point is:

Answer:

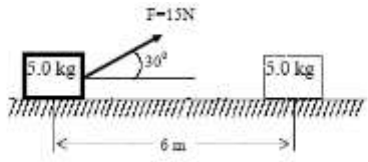
$$v_{yf}^{2} = v_{yi}^{2} - 2gd;$$

$$\Rightarrow 0 = \left(20\sin 60^{\circ}\right)^{2} - 2gd \Rightarrow d = 15.3 \text{ m}$$

$$v_{f}^{2} = v_{i}^{2} - 2gd = 20^{2} - 2\times 9.8\times 15.3 = 100$$

$$\Rightarrow K_{f} = \frac{1}{2}mv_{f}^{2} = \frac{1}{2}\times 0.2\times 100 = \underline{10 \text{ J}}$$

Example1: A 5.0-kg object is pulled along a rough horizontal surface at constant speed by 15 N force acting 30° above the horizontal (see Figure). (a) How much work is done by the friction force as the object moves 6.0 m? (b) What is the increase in thermal energy of the block–floor system?



Answer:

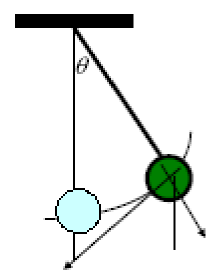
(a) constant speed $\Rightarrow \vec{a} = \Delta K = 0$

$$\Delta K = \text{Total work done} = W_{f} + W_{F} = 0$$

$$\Rightarrow \quad W_{f} = -W_{F} = -F\cos 30^{\circ}\,\text{d} = -15\times 0.866\times 6 = -\underline{77.9 \text{ J}}$$

(b) The increase in thermal energy will be $\Delta E_{\text{th}} = \left|W_{f}\right| = 77.9$ J

Example: A 0.6-kg ball is suspended from the ceiling at the end of a 2.0-m string. As this ball swings, it has a speed of 4.0 m/s at the lowest point of its path. What maximum angle does the string make with the vertical as the ball swings?



Answer: Apply the conservation of the mechanical energy we find:

$$\frac{1}{2}mv^{2} = mgh \quad \Rightarrow \quad h = \frac{v^{2}}{2g} = 0.816 \text{ m}$$

With

$$h = \ell - \ell\cos\theta \quad \Rightarrow \quad \cos\theta = 1 - \frac{h}{\ell} = 10.408 = 0.592$$

$$\Rightarrow \quad \theta = \cos^{-1} 0.592 = \underline{54^{\circ}}$$

Example: A 2.0 kg block is pulled at a constant speed of 1.1 m/s across a horizontal rough surface by an applied force of 12 N directed 30° above the horizontal. At what rate is the frictional force doing work on the block?

Answer: The acceleration here is zero, since we have constant velocity. The work done by the applied force is used to overcome the friction work, so:

$$\Delta K = W_{f} + W_{F} = 0 \quad \Rightarrow \quad W_{f} = -W_{F} = -F\cos 30^{\circ}\,\text{d}$$

$$\therefore P = -\frac{dW_{F}}{dt} = -F\cos 30^{\circ}\,\text{v}_{f} = -12\times 0.866\times 1.1 = \underline{-11.4 \text{ W}}$$

Example: A 10 kg object is dropped vertically from rest. After falling a distance of 50 m, it has a speed of 26 m/s. How much work is done by the air resistance on the object during this descent?

Answer:

$$W_{nc} = E_f - E_i = (0 + \frac{1}{2} m \times 26^2) - (mg \times 50 + 0) = \underline{-1520 \text{ J}}$$

Example: As a particle moves from point A to point B only two forces act on it: one force is non-conservative and does work = -30 J, the other force is conservative and does +50 J work. The change of the kinetic energy of the particle is:

Answer:

$$\Delta K = W_{nc} + W_c = -30 + 50 = \underline{20 \text{ J}}$$

Q: A 16 *kg* crate falls from rest from a height of 1.0 *m* onto a spring scale with a spring constant of 2.74×10^3 N/m. Find the maximum distance the spring is compressed.

Answer: If the spring is compressed a distance x, we can apply the conservation of mechanical energy;

$$\Delta E = E_f - E_i = \left(\cancel{K_f} - \cancel{K_i} \right) + \left(U_{Gf} - U_{Gi} \right) + \left(U_{Sf} - U_{Si} \right) = 0;$$

$$mg(1 + x) - 0 + \left(0 - \frac{1}{2} k x^2 \right) = 0$$

Solve of x , will find: $x \approx \underline{0.4 \text{ m}}$;

We took the final position of the spring as a reference of our potential energy.

Q4. A block is sliding down on a rough inclined plane. A man applies a force to reduce the acceleration of the block. Let W_f be the work done by the friction force, W_m the work done by the man, and W_g the work done by the gravitational force. While the block is sliding down, which of the following is TRUE?

A) $W_f < 0$, $W_m < 0$, $W_g > 0$
B) $W_f < 0$, $W_m > 0$, $W_g < 0$
C) $W_f < 0$, $W_m < 0$, $W_g < 0$
D) $W_f < 0$, $W_m > 0$, $W_g > 0$
E) $W_f > 0$, $W_m > 0$, $W_g > 0$

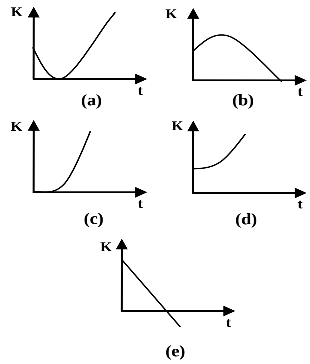
Ans:

A

Q5. A ball is launched upward from the edge of a cliff. Which of the graphs shown in **Figure 3** could possibly represent how the kinetic energy of the ball changes during its flight?

A) a
B) b
C) c
D) d
E) e

Figure 3



Ans:

A

Q6: A man pushes a block up an incline at a constant speed. As the block moves up the incline, only one statement is true:

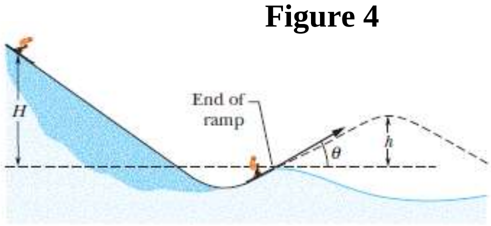
A) Its kinetic energy remains constant and the potential energy of block-earth system increases
B) Its kinetic energy and the potential energy of block-earth system both increase
C) Its kinetic energy increases and the potential energy of block-earth system remains constant
D) Its kinetic energy decreases and the potential energy of block-earth system remains constant
E) Its kinetic energy decreases and the potential energy of block-earth system decreases

Ans:

$\Delta E = 0 \qquad \Delta K + \Delta U = 0 \qquad \Delta K = 0 \qquad \Delta V = mg(h - h_0)$

Q8: A 80 kg skier starts from rest at height *H* = 25 m above the end of a frictionless ski-jump ramp and leaves the ramp at angle θ= 30°, as shown in **Figure 4**. What is the maximum height *h* of his jump above the end of the ramp? Neglect the effects of air resistance.

Figure 4



A) 6.3 m
B) 5.6 m
C) 7.1 m
D) 4.3 m
E) 8.1 m

Ans:

$$\Delta E = 0 \Rightarrow mgH = \frac{1}{2}mv^2$$

$$v_o = \sqrt{2gH}$$

$$v_f^2 = v_o^2 - 2gh$$

$$0 = 2gH\sin^2(30) - 2gh$$

$$\Rightarrow \frac{H}{4} = h = \frac{25}{4} \cong 6.3 \text{ m}$$

Q6: A 2.00 kg ball is thrown with an initial velocity of $\vec{v}_o = 18\hat{i} + 10\hat{j}$, where v_o is in m/s. What is the maximum change in the potential energy of the ball-Earth system during its flight?

Ans:

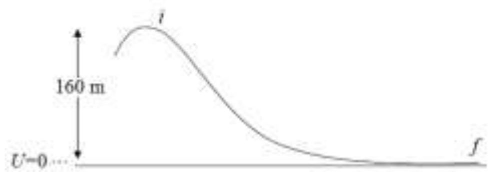
$$\Delta K = \frac{1}{2}m(v^2 - v_0^2) = -\Delta U \rightarrow \Delta U = \frac{1}{2}m(v_0^2 - v^2) = \frac{1}{2}2(18^2 + 10^2 - 18^2)$$
$$= 100\, J$$

Q8: The only force acting on a particle is a conservative force $\vec{F}$. If the particle is at point A, the potential energy of the system associated with $\vec{F}$ and the particle is 80 J. If the particle moves from point A to point B, the work done on the particle by $\vec{F}$ is + 20 J. What is the potential energy of the system with the particle at B?

Ans:

$$W = -\Delta U = -(U_B - U_A) \rightarrow U_B = U_A - W = 60\, J$$

Example: An 80 kg skier starts from rest at the top of a 160 meter tall hill and attains a velocity of 36 m/s by the time they reach the bottom of the 200 meter run. What is the coefficient of kinetic friction between their skis and the snow?



Answer:

A non-conservative force, friction, is present here, ($E_f \neq E_i$), and total mechanical energy is not conserved.

$$E_i = K_i + U_i = 0 + mgh = (80\text{ kg})(9.8\text{ m/s}^2)(160\text{ m}) = 125440\text{ J}$$

$$E_f = K_f + U_f = \tfrac{1}{2}mv^2 + 0 = (0.5)(80\text{ kg})(36\text{ m/s}^2) = 51840\text{ J}$$

$$W_f = \Delta E = -73600\text{ J} = f_s.s = -\mu_k mhs$$

$$\Rightarrow \mu_k = \frac{73600\text{ J}}{(80\text{ kg})(9.8\text{ m/s}^2)(200\text{ m})} = 0.47$$

Chapter 8
Review Problems (Spring)

WORK DONE BY A SPRING FORCE

1- We are dealing with massless spring, which implies $K_{\text{spring}} = 0$

2- Hooke's "Law": $F_{\text{spring}} = -k\ x$

3- The work done by an external force F_{ext} (such as the force from my hand which is opposite to F_{spring}) to stretch or compress a spring by an amount x is given by

$$W_{ext} = \int_{x_i}^{x_f} F_{ext}\, dx = \int_{x_i}^{x_f} k\, x\, dx = = \tfrac{1}{2} k\, x^2 \Big|_{x_i}^{x_f} = \tfrac{1}{2} k \left(x_f^2 - x_i^2 \right)$$

This is the general expression for work done by an external force F_{ext} that can be used in all cases, if we know x_i and x_f .

Note that:

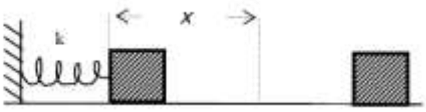
Starting from equilibrium position, i.e. $x_i = 0$, then the above equation reduces to $W_{ext} = \tfrac{1}{2} k \left(x_f^2 \right)$. This condition implies:

i- The work done by the external force is always positive, regardless of whether the spring is stretched (x_f positive) or compressed (x_f negative).

ii- The expression $\Delta W_{Field} = -\Delta W_{ext}$ means that the work done by the spring is always negative, regardless of whether the spring is stretched (x_f positive) or compressed (x_f negative).

iii- The expression $\Delta U_S = -\Delta W_S = \Delta W_{ext} = \frac{1}{2} k \left(x_f^2 \right)$ means that the potential energy stored in the spring is always positive, regardless of whether the spring is stretched (x_f positive) or compressed (x_f negative).

Q: A 0.450 kg block, resting on a frictionless surface is pushed 8.00 cm into a light spring, $k = 111\ \text{N/m}$. It is then released. What is the velocity of the block as it just leaves the spring?



Answer:

We can solve this by analyzing the energy situation. There are only two terms we need worry about, the potential energy stored in the spring and the kinetic energy when the spring uncoils and the block is released.

$$i-\text{state}: K_{i,m} = \frac{1}{2} m v_i^2 = 0, \quad U_{i,s} = \frac{1}{2} k x^2 \quad \Rightarrow \quad E_i = \frac{1}{2} k x^2$$

$$f-\text{state}: K_{f,m} = \frac{1}{2} m v_f^2 = 0, \quad U_{f,s} = 0 \quad \Rightarrow \quad E_f = \frac{1}{2} m v_f^2 + 0$$

Conservation of mechanical energy implies:

$$E_f = E_f \quad \Rightarrow \quad \frac{1}{2} k x^2 = \frac{1}{2} m v^2$$

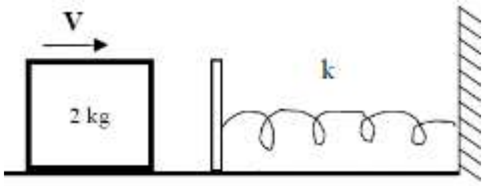
$$v = \sqrt{\frac{1}{m} k x^2} = \sqrt{\frac{1}{0.450\ \text{kg}} \left(111 \frac{\text{kg} \times \text{m}}{\text{s}^2} \right) (0.080\ \text{m})^2} = 1.26 \frac{\text{m}}{\text{s}}$$

Q: A 255 g block is traveling along a smooth surface with a velocity of 12.5 m/s. It runs head on into a spring ($k = 325$ N/m). How far is the spring compressed?

Answer:

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \quad \Rightarrow \quad x = v\sqrt{\frac{m}{k}} \quad = 12.5\frac{\text{m}}{\text{s}}\sqrt{\frac{0.255\ \text{kg}}{325\ \frac{\text{kg}\times\text{m}}{\text{m}\times\text{s}^2}}} \quad = \quad 0.350\ \text{m}$$

Q: In the figure, a block (M = 2.0 kg) slides on a frictionless horizontal surface towards a spring with a spring constant $k = 2000$ N/m. The speed of the block just before it hits the spring is 6.0 m/s. How fast is the block moving at the instant the spring has been compressed 15 cm?



Answer:

$$i-\text{state}: K_{i,m} = \tfrac{1}{2}mv_i^2, \quad U_{i,s} = 0 \quad \Rightarrow \quad E_i = \tfrac{1}{2}mv_i^2 = \frac{1}{2}2(6^2)$$

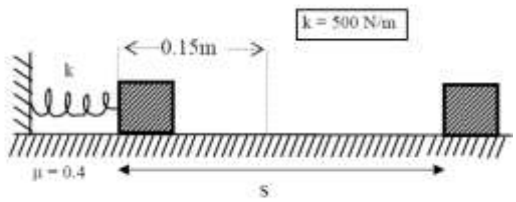
$$f-\text{state}: K_{f,m} = \tfrac{1}{2}mv_f^2, \quad U_{f,s} = \tfrac{1}{2}kx^2 \quad \Rightarrow \quad E_f = \tfrac{1}{2}mv_f^2 + \tfrac{1}{2}kx^2$$

Conservation of mechanical energy implies:

$$E_f - E_i = 0 \quad \Rightarrow \quad \frac{1}{2}2\left(v_f^2 - 6^2\right) + \tfrac{1}{2}kx^2 = 0$$

$$v_f^2 = 6^2 - \frac{1}{2}2000\left(0.15^2\right) = 36 - 22.5 = 13.5 \quad \Rightarrow \quad v_f = \underline{3.67\ \text{m/s}}$$

Q: A block of mass m=3.0 kg is kept at rest after it has compressed a horizontal massless spring ($k = 500$ N/m) by 0.15 m, as shown in the figure. When the block is released, it travels a distance S on a horizontal rough surface ($\mu_k = 0.4$) before stopping. Calculate the distance S.



Answer:

$$i-\text{state}: K_{i,m} = \tfrac{1}{2}mv_i^2 = 0, \quad U_i = \tfrac{1}{2}kx^2 \quad \Rightarrow \quad E_i = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(500)(0.15)^2 = 5.625$$

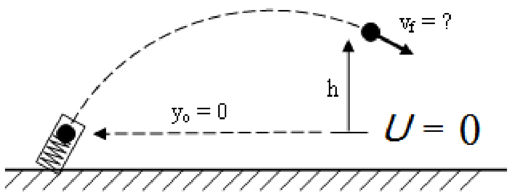
$$f-\text{state}: K_{f,m} = \tfrac{1}{2}mv_f^2 = 0, \quad U_f = 0 \quad \Rightarrow \quad E_f = 0 + 0$$

$$W_{nc} = \begin{cases} \vec{f}_k . \vec{d} = -\mu_k mg\text{S} = -0.4(3)(9.8)(\text{S}) & \text{(I)} \\ E_f - E_i = -5.625 & \text{(II)} \end{cases}$$

Using the equation: $E_f - E_i = W_{nc}$ and equating (I) and (II), we get

$$\text{S} = \frac{5.625}{0.4(3)(9.8)} = \underline{0.478\ \text{m}}$$

Q: A spring-loaded gun fires a dart at an angle θ from the horizontal. The dart gun has a spring with spring constant *k* that compresses a distance *x*. Assume no air resistance. What is the speed of the dart when it is at a height *h* above the initial position?



Answer:

$$E_f = E_i \Rightarrow K_i + U_{g,i} + U_{elas,i} = K_f + U_{g,f} + U_{elas,f}$$

$$0 + 0 + \tfrac{1}{2}\mathrm{kx}^2 = \tfrac{1}{2}\mathrm{mv}^2 + \mathrm{mgh} + 0$$

$$\mathrm{v} = \sqrt{\left(\frac{\mathrm{k}}{\mathrm{m}}\right)\mathrm{x}^2 - 2\,\mathrm{g}\,\mathrm{h}}$$

Notice that the angle θ never entered into the solution.

Q: A 25 kg box is pushed along a horizontal smooth surface with a force of 35 N for 1.5 m. It then slides into a spring $(k = 1500 \text{ N/m})$ and compresses it. Find the distance that the spring is compressed.

Answer:

Find the acceleration:

$$F = ma \quad \Rightarrow \quad a = \frac{F}{m} \quad = 35\frac{\mathrm{kg}\times\mathrm{m}}{\mathrm{s}^2}\left(\frac{1}{25\,\mathrm{kg}}\right) \quad = \quad 1.4\,\frac{\mathrm{m}}{\mathrm{s}^2}$$

Find the final velocity:

$$v^2 = v_0^{\,2} + 2ax \quad \Rightarrow v = \sqrt{2ax} \quad = \sqrt{2(1.4)1.5\,m} \quad = \quad 2.05\,\mathrm{m/s}$$

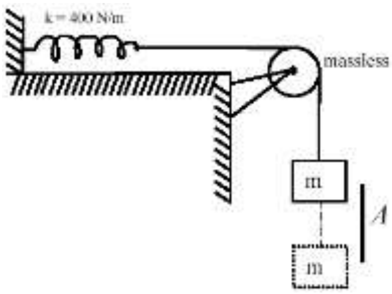
Now we can find the kinetic energy of the thing before compressing:

$$K_o = \frac{1}{2}mv^2 \quad = \frac{1}{2}(25\,kg)\left(2.05\,\frac{m}{s}\right)^2 \quad = \quad 52.5\,J$$

We can now find the displacement of the spring using the kinetic energy we just found:

$$K_0 = \frac{1}{2}kx^2 \quad \Rightarrow \quad x = \sqrt{\frac{2K_0}{k}} \quad = \sqrt{\frac{2(52.5\,\mathrm{N}\times\mathrm{m})}{1\,500\,\mathrm{N/m}}} \quad = \quad 0.26\,\mathrm{m}$$

Example: A block of mass m = 10 kg is connected to un-stretched spring $(k = 400 \text{ N/m})$ (see Figure). The block is released from rest. If the pulley is mass less and frictionless, what is the maximum extension of the spring?

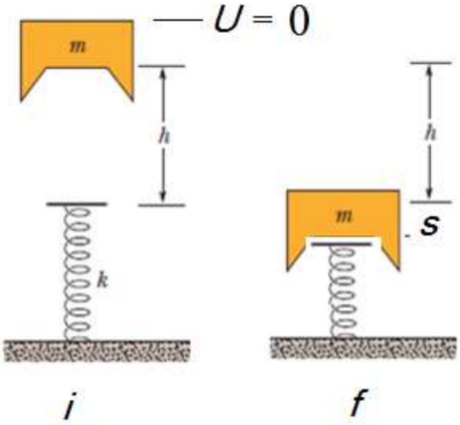


Answer: Take our reference point for the gravitational potential energy at the initial position of the block, and apply the conservation of the mechanical energy using the maximum extension of the spring is *A*, one can finds:

$$E_f - E_i = 0 \quad \Rightarrow \quad \tfrac{1}{2}kA^2 - mgA = 0$$

$$\Rightarrow \quad A = \frac{2mg}{k} = \frac{2\times 10\times 9.8}{400} = \underline{0.49\ \mathrm{m}}$$

Example: A block of mass m = 2.0 kg is dropped from height h = 40 cm onto a spring of spring constant $(k = 1960\text{ N/m})$ (see Figure). Find the maximum distance, s, the spring is compressed.



Answer: We denote m as the mass of the block, h = 0.40 m as the height from which it dropped (measured from the relaxed position of the spring), and s the compression of the spring (measured downward so that it yields a positive value). Our reference point for the gravitational potential energy is the initial position of the block.

At the initial state,

$$K_i = K_{i,\text{mass}} + K_{i,\text{spring}} = 0 + 0, \quad U_i = U_{i,\text{mass}} + U_{i,\text{spring}} = 0 + 0$$

$$\Rightarrow \quad E_i = 0 + 0$$

At the final state,

$$K_f = K_{f,\text{mass}} + K_{f,\text{spring}} = 0 + 0, \quad U_f = U_{f,\text{mass}} + U_{f,\text{spring}} = -mg(h+s) + \frac{1}{2}ks^2$$

$$\Rightarrow \quad E_f = -mg(h+s) + \frac{1}{2}ks^2$$

The block drops a total distance $h+s$, and the final gravitational potential energy is $-mg(h+s)$. The spring potential energy is $\frac{1}{2}ks^2$ in the final situation, and the kinetic energy is zero both at the beginning and end. Since energy is conserved

$$\Delta E = E_f - E_i = 0 \Rightarrow \quad 0 = -mg(h+s) + \frac{1}{2}ks^2$$

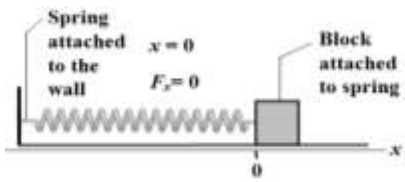
which is a second degree equation in x. Using the quadratic formula, its solution is

$$s = \frac{mg \pm \sqrt{(mg)^2 + 2mghk}}{k}$$

Now mg = 19.6 N, h = 0.40 m, and $k = 1960\text{ N/m}$, and we choose the positive root so that s > 0.

$$s = \frac{19.6 \pm \sqrt{(19.6)^2 + 2(19.6)(0.4)(1960)}}{1960} = 0.10\text{ m}$$

Q: A spring and a block are in the arrangement of **Figure**. When the block is pulled out to x = 4.00 cm, we must apply a force of magnitude 360 N to hold it there. The block is pulled to x = 11.0 cm and then released. How much work does the spring do on the block as the block moves from x = 5.00 cm to x = 3.00 cm?

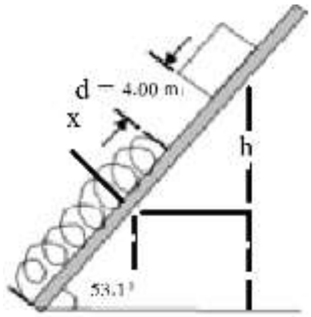


Answer: First use Hook's law to calculate the constant k:

$$|F| = kx \quad \Rightarrow \quad k = \frac{F}{x} = \frac{360}{4\times10^{-2}} = \underline{9000 \text{ N/m}};$$

$$W_s = \tfrac{1}{2}k\left(x_i^2 - x_f^2\right) = \tfrac{1}{2}9000(25-9)\times10^{-4} = \underline{7.2 \text{ J}}$$

Example: A 2.00 kg package is released on a rough 53.1° incline at d = 4.00 m from a long spring of force constant $(k = 120 \text{ N/m})$. The spring is attached to the bottom of the incline as shown in Figure. If the maximum compression of the spring is x = 1.00 m, what is the work done by the friction force?



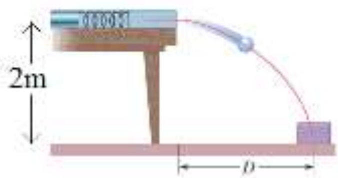
Answer: Take our reference point for the gravitational potential energy at the initial position of the block, and apply the rule:

$$\Delta K + \Delta U = W_f$$

$$\Delta K = 0, \quad \Delta U = U_f - U_i = \frac{1}{2}kx^2 - mg(h)$$

$$W_f = \frac{1}{2}\times120\times1^2 - 2\times9.8\times(d+x)\sin(53.1^\circ) = -18.4 \text{ J}$$

Example: A 1.0 kg ball is launched from a spring $(k = 135 \text{ N/m})$ that has been compressed a distance of 25 cm. The ball is launched horizontally by the spring, which is 2.0 m above the deck. (a) What is the velocity of the ball just after it leaves the spring? (b) What is the horizontal distance the ball travels before it hits? (c) What is the kinetic energy of the ball just before it hits?



Answer:

(a) $\frac{1}{2}kx^2 = \frac{1}{2}mv^2 \quad v = x\sqrt{\frac{k}{m}} \quad = (0.25\, m)\sqrt{\frac{1}{1.0\, kg}(135)} \quad = \quad 2.9 \text{ m/s}$

(b) Find the time to fall: $y = \frac{1}{2}at^2 \quad \Rightarrow t = \sqrt{\frac{2y}{a}} \quad = \sqrt{\frac{2(2.0)}{9.8}} \quad = \quad 0.639\text{ s}$

$$D = vt \quad = 2.9(0.639) \quad = \quad 1.9\text{ m}$$

(c) To directly calculate the kinetic energy of the ball, we would have to calculate what it's velocity is just before it hits. This would be a complicated problem – vectors, ***x*** and ***y*** components etc. Much easier to calculate it using conservation of energy. Its energy at the top, which will be the potential energy in the spring plus the gravitational potential energy because of the ball's height, has to equal its kinetic energy at the bottom. We could also use its kinetic energy plus its gravitational potential energy at the top just before it leaves the table (this is because its kinetic energy must equal the potential energy stored in the spring before launch).

$$mgy_0 + \frac{1}{2}kx_0{}^2 = K_{final}$$

$$K_{final} = mgy_0 + \frac{1}{2}kx_0{}^2 = 1.0\,(9.8)(2.0) + \frac{1}{2}(135)(0.25)^2 = 24 \text{ J}$$

Example: In the figure, A 2.00 kg block situated on a rough incline, with $\theta = 37.0^\circ$, is connected to a spring of negligible mass and spring constant 100 N/m. The block is released from rest when the spring is un-stretched. If the pulley is massless and frictionless and the block moves 20.0 cm down the incline before coming to rest, then find the coefficient of kinetic friction between the block and incline.

Answer: use the relation

$$\Delta K + \Delta U_g + \Delta U_s = W_{nc} \qquad (1)$$

with the conditions:

$$\begin{aligned} \Delta K &= K_f - K_i = 0, \\ \Delta U_g &= -mgd \sin 37^\circ = -2.36, \\ \Delta U_g &= \tfrac{1}{2}kx^2 = 2 \\ W_{nc} &= -\mu_k mgd \cos 37^\circ = -3.13\mu_k \end{aligned} \qquad (2)$$

From (2) to (1), we find:

$$\mu_k = 0.115$$

Example: A 2.0 kg object is connected to one end of an unstretched spring which is attached to the ceiling by the other end and then the object is allowed to drop. The spring constant of the spring is 196 N/m. How far does it drop before coming to rest momentarily?

Answer: Apply the conservation of the mechanical energy:

$$\Delta E = 0 \quad \Rightarrow \quad \frac{1}{2}kx^2 = mgh \quad \Rightarrow \quad h = \frac{2mg}{k} = 0.2 \text{ m.}$$

Example: An ideal spring (compressed by 7.00 cm and initially at rest,) fires a 15.0 g block horizontally across a frictionless table top. The spring has a spring constant of 20.0 N/m. The speed of the block as it leaves the spring is:

Answer: Apply the conservation of the mechanical energy:

$$\Delta E = 0 \quad \Rightarrow \quad (K_f - K_i) + (U_{Gf} - U_{Gi}) = 0$$

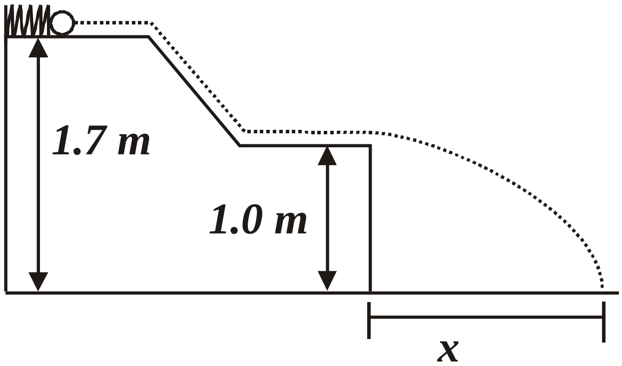
$$\Rightarrow (\frac{1}{2}mv^2 - 0) + \left(0 - \frac{1}{2}kx^2\right) = 0$$

$$\Rightarrow v = \sqrt{\frac{k}{m}}x = \sqrt{\frac{20}{0.015}}(0.07) = 2.56 \text{ m/s.}$$

Extra Problems

Example: A spring is compressed a distance of 12 cm by a 675 g ball. The spring constant is $(k = 225\text{ N/m})$. The ball is on a smooth surface as shown. The spring is released and sets the ball into motion. Find: (a) the speed of the ball when it leaves the spring, (b) the speed of the ball just before it leaves the edge of the table, (c) the kinetic energy of the ball just before it hits the deck, (d) the horizontal distance, ***x***, it travels when it leaves the table until it hits the deck. Assume that the ball rolls down the ramp.

Answer:



(a) $\frac{1}{2}kx_0^2 = \frac{1}{2}mv^2 \quad \Rightarrow v = x_0\sqrt{\frac{k}{m}} \quad = (0.12)\sqrt{\frac{1}{0.675}(225)} \quad = \quad 2.2\text{ m/s}$

(b) $\frac{1}{2}mv_0^2 + mgy_0 = \frac{1}{2}mv^2 \quad v = \sqrt{2\left(\frac{1}{2}v_0^2 + gy_0\right)}$

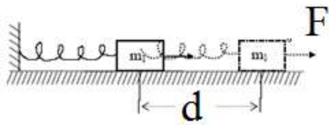
$$v_f = \sqrt{2\left(\tfrac{1}{2}(2.2)^2 + (9.8)(1.7 - 1.0\ )\right)} \quad = 4.3\text{ m/s}$$

(c) $mgy_0 + \frac{1}{2}kx_0^2 = \frac{1}{2}kx^2$

$$K = mgy_0 + \tfrac{1}{2}kx_0^2 = 0.675\,kg\,(9.8)(1.7\ ) + \tfrac{1}{2}(225)(0.12\ )^2 = \quad 13\text{ J}$$

(d) $y = \frac{1}{2}at^2 \quad \Rightarrow t = \sqrt{\frac{2y}{a}} \quad = \sqrt{\frac{2(1.0\ )}{9.8}} \quad = \quad 0.452\text{ s}$

$$x = vt \quad = 4.3\frac{\text{m}}{\text{s}}(0.452\text{ s}) \quad = \quad 1.9\text{ m}$$



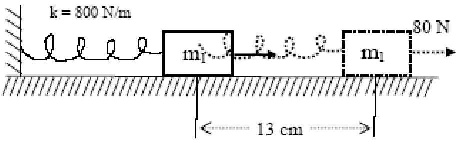
Example: A block, of mass m, is resting on a horizontal surface with kinetic friction coefficient μ_k. The block is attached to an un-stretched spring with spring constant k (see Figure). A force F parallel to the surface is applied to the block. What is the speed of the block when its displaced is d from its initial position?

Answer:

Method 1: use $W_{spring} = -U_{spring} = -\frac{1}{2}kx^2$

$$\Delta K = \frac{1}{2}m(v_f^2 - \cancel{v_i^2}) = W_{spring} + W_F - W_{friction} = Fd - \frac{1}{2}kx^2 - \mu_k(mg)$$

Method 2: **U**se the equation of motion ($ma = F - \int_0^d kxdx - \mu_k(mg) = F - \frac{1}{2}kx^2 - \mu_k(mg)$), then use the kinematic equation $v_f^2 - v_i^2 = 2\vec{a}.\vec{d} = 2\left(\frac{1}{m}\left[F - \frac{1}{2}kx^2 - \mu_k(mg)\right]\right)d = \frac{2}{m}\left(W_F - U_{spring} + W_f\right)$



Example: A 12-kg block is resting on a horizontal frictionless surface. The block is attached to an un-stretched spring $(k = 800 \text{ N/m})$ (see Figure). A force F = 80 N parallel to the surface is applied to the block. What is the speed of the block when it is displaced by 13 cm from its initial position?

Answer: **Note that:** $W_{spring} = -U_{spring} = -\frac{1}{2}kx^2$, or use the equation of motion ($a = F - \int_0^d kxdx$)!

$$\Delta K = \frac{1}{2}m(v_f^2 - \cancel{v_i^2}) = W_{spring} + W_F = Fd - \frac{1}{2}kx^2$$

$$\Rightarrow \frac{1}{2}mv_f^2 = 80 \times 0.13 - \frac{1}{2} \times 800 \times (0.13)^2 = 3.64 \text{ J}$$

$$\Rightarrow v_f = \sqrt{\frac{2 \times 3.64}{12}} = \underline{0.78 \text{ m/s}}$$

Example: A 0.50 kg block attached to an ideal spring with a spring constant of 80 N/m oscillates on a horizontal frictionless surface. The speed of the block is 0.50 m/s, when the spring is stretched by 4.0 cm. The maximum speed the block can have is:

Answer:

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}0.5 \times 0.5^2 + \frac{1}{2}80 \times 0.04^2 = 0.1265$$

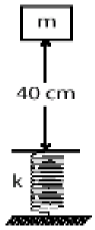
$$\Rightarrow \quad v_f = \sqrt{\frac{2}{0.5}0.1265} = \underline{0.71 \text{ m/s}}$$

Example: A 4.0 kg block is initially moving to the right on a horizontal frictionless surface at a speed of 5.0 m/s. It then compresses a horizontal spring of spring constant 200 N/m. At the instant when the kinetic energy of the block is equal to the potential energy of the spring, the mechanical energy of the block-spring system is:

Answer:

$$\Delta E = 0 \Rightarrow E_i = E_f \text{ (or at any place)} \Rightarrow E_i = \frac{1}{2}mv_i^2 = \frac{1}{2}\times 4\times 5^2 = \underline{50\text{ J}}.$$

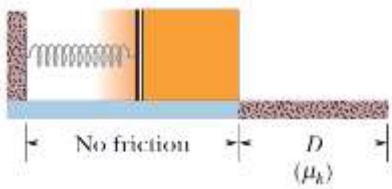
Q: A 3.00 kg block is dropped from a height of 40 cm onto a spring of spring constant k (see Figure). If the maximum distance the spring is compressed = 0.130 m, find k.



Answer:

$$\Delta E = E_f - E_i$$
$$= \left(\cancel{K_f} - \cancel{K_i}\right) + \left(U_{Gf} - U_{Gi}\right) + \left(U_{Sf} - U_{Si}\right) = 0;$$
$$\Rightarrow \left(mg\times 40 - (-mg\times x)\right) + \left(0 - \frac{1}{2}kx^2\right) = 0$$
$$k = \frac{2\times 3\times 9.8\times\left(40+0.13\right)}{0.13^2} = \underline{1844\text{ N/m}}$$

Example: As shown in **Figure**, a 3.50 kg block is accelerated from rest by compressing against a spring by a distance *x*. The other end of the spring is fixed to the wall and the spring has a spring constant of 134 N/m. The block leaves the spring and then travels over a rough horizontal floor with a coefficient of kinetic friction $\mu_k = 0.250$. The frictional force stops the block in distance *D* = 7.80 m. Find the distance *x*?



Answer: Draw the initial and final states and find the following

Initial state: (compressed spring and stationary mass),

$$K_{i,S} = 0,\quad U_{i,S} = \tfrac{1}{2}kx^2,\quad K_{i,m} = 0,\quad U_{i,m} = 0 \quad\Rightarrow\quad E_i = \tfrac{1}{2}kx^2$$

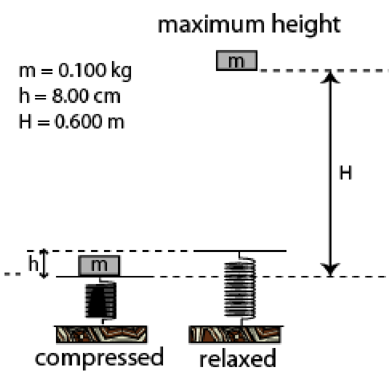
Final state: (un-stretched spring and stationary mass at the end of the rough horizontal floor),

$$K_{f,S} = 0,\quad U_{f,S} = 0,\quad K_{f,m} = 0,\quad U_{f,m} = 0 \quad\Rightarrow\quad E_f = 0$$

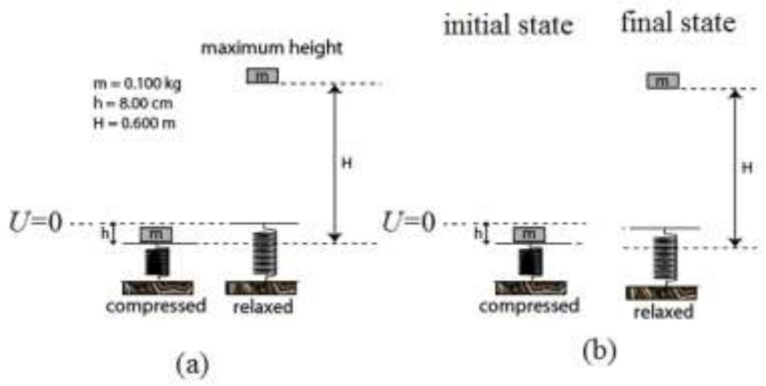
Then use the equation

$$W_f = \vec{f}_s.\vec{D} = -\mu_k mgD = \Delta E = -\tfrac{1}{2}kx^2 \quad\Rightarrow\quad -\mu_k mgD = -\tfrac{1}{2}kx^2 \quad\Rightarrow\quad x = 1.00\text{ m.}$$

Example8: A block (mass = 0.100 kg) is pushed against a vertical spring compressing the spring a distance of h = 8.00 cm (see the figure a). The block is not attached to the spring. When released from rest, the block rises to a maximum height of H = 0.600 m. Calculate the spring constant.



Answer: If the spring is compressed a distance h, we can apply the conservation of mechanical energy;



At the initial state,

$$K_i = K_{i,\text{mass}} + K_{i,\text{spring}} = 0+0, \quad U_i = U_{i,\text{mass}} + U_{i,\text{spring}} = -mgh + \tfrac{1}{2}kh^2$$

$$\Rightarrow \quad E_i = -mgh + \tfrac{1}{2}kh^2$$

At the final state,

$$K_f = K_{f,\text{mass}} + K_{f,\text{spring}} = 0+0, \quad U_f = U_{f,\text{mass}} + U_{f,\text{spring}} = mg(H-h)+0$$

$$\Rightarrow \quad E_f = mg(H-h)$$

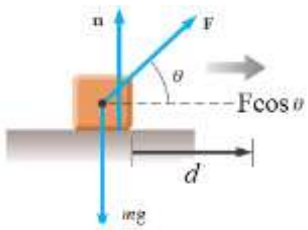
$$\therefore \Delta E = 0 \quad \Rightarrow \left(E_f - E_i\right) = mg\left(H-h\right) - \left(-mgh + \tfrac{1}{2}kh^2\right) = 0$$

$$\Rightarrow k = \frac{2mgH}{h^2} = \frac{2\times 0.1\times 9.8\times 0.6}{0.08^2} = \frac{1.176}{0.0064} = \underline{184\text{ N/m}}$$

Chapter 8
Review Problems

The following three problems are related to Sec 8.4.

The increase in thermal energy $\Delta E_{\text{th}} = |f_k d|$



Work-energy theorem again!

Start with $v_f^2 = v_i^2 + 2\vec{a}\,.\,\vec{d}$, and use the Newton's second law $\vec{a} = \sum_i \vec{F}_i$,

to find

$$v_f^2 - v_i^2 = \frac{2}{m}\left(\vec{F}_1 + \vec{F}_2 + \cdots\right).\,\vec{d}$$

$$\Rightarrow \Delta K = K_f - K_i = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right) = W_1 + W_2 + \cdots$$

With constant velocity, $\Rightarrow \quad \Delta K = 0 \quad \Rightarrow \quad W_1 + W_2 + \cdots = 0$

1- Work done, W_F, by applied force $\vec{\text{F}}$ is given by the relation

$$W_F = \vec{\text{F}}.\,\vec{\text{d}} = \left(F\cos\theta\right)d = F_p\ d$$

where θ is the angle between $\vec{\text{F}}$ and the displacement $\vec{\text{d}}$.
F_p = component of force along the direction of displacement,

$$W_F = F_p\ .\,\text{distance}\ .$$

2- The work-energy theorem is written as: $\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \sum_j W_j$

Proof: Start with $v_f^2 = v_i^2 + 2\vec{a}\,.\,\vec{d}$, and use the Newton's second law $\vec{a} = \sum_i \vec{F}_i$, to find

$$v_f^2 - v_i^2 = \frac{2}{m}\left(\vec{F}_1 + \vec{F}_2 + \cdots\right).\,\vec{d} \quad \Rightarrow \quad \Delta K = K_f - K_i = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right) = W_1 + W_2 + \cdots$$

Constant velocity, $\Rightarrow \quad \Delta K = 0 \quad \Rightarrow \quad W_1 + W_2 + \cdots = 0$

Note that: According to the work-energy theorem

i- work done by all forces = ΔK
ii- if the work is positive $K_f > K_i$ and the velocity increases, $\boldsymbol{v_f > v_i}$
iii- if the work is negative $K_f < K_i$ and the velocity decreases $\boldsymbol{v_f < v_i}$
iv- only when external force acts on an object its energy increases by an amount equal to the work done by this force.
v- we can consider the kinetic energy of a body as the work that the body do in coming to rest.

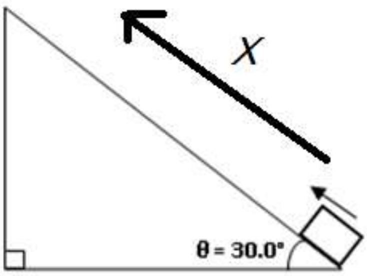
Remember that: In the following, field could be refer to gravity, spring, electric,…

$$\Delta W_{Field} = -\Delta W_{ext}$$

$$\Delta U_{Field} = -\Delta W_{Field} = \Delta W_{ext}$$

$$E_f - E_i = W_{non-conservative}$$

Q: A 5.0 kg block starts up a 30° incline with 130 J of kinetic energy. How far will it slide up the incline if the coefficient of kinetic friction between the block and the incline is 0.30?



Answer:

We will suppose that the block slide up a distance X along the incline

$\Delta k + \Delta ug = Wnc$

$-130 + m\,g\,X\,sin\theta = -(\mu k)\,mg\,cos\theta\,(X)$

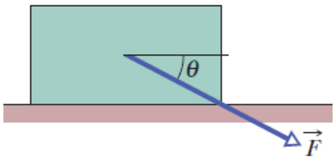
$-130 + 24.5\,X = -(0.3)\,(5.0)\,(9.8)\,(.866)\,X$

$-130 + 24.5\,X = -12.73\,X$

$37.23\,X = 130$

$X = 130/37.23 = 3.49\,m$

Q: A worker pushed a 27 kg block 9.2 m along a level floor at constant speed with a force, $\vec{F}$, directed with angle $\theta = 32^\circ$ below the horizontal, see the figure. If the coefficient of kinetic friction between block and floor was 0.20, what were (a) the work done by the worker's force and (b) the increase in thermal energy of the block–floor system?



Answer:

Since the velocity is constant, $\vec{a} = 0$ and the *x* and the *y* component of Newton's second law are:

$$y: \quad F_N = F\sin\theta + mg, \qquad \text{(i)}$$

$$x: \quad f_k = F\cos\theta \qquad \text{(ii)}$$

$$f_k = \mu_k F_N = \mu_k\left(F\sin\theta + mg\right) \qquad \text{(iii)}$$

where *m* is the mass of the block, $\vec{F}$ is the force exerted by the rope, and θ is the angle between that force and the horizontal. Equating (ii) and (iii), one finds:

$$F = \frac{\mu_k mg}{\cos\theta - \mu_k\sin\theta} = \frac{0.2\times 27\times 9.8}{\cos 32^\circ - 0.2\sin 32^\circ} \approx 71\text{ N}$$

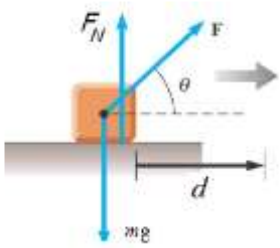
(a) The work done on the block by the worker is,

$$W = Fd\cos\theta = (71\,\text{N})(9.2\,\text{m})\cos 32° = 5.54\times 10^2\text{ J}.$$

(b) Since $f_k = \mu_k F_N = \mu_k\left(F\sin\theta + mg\right) = 60$ N, we find

$$\Delta E_{\text{th}} = f_k d = (60\,\text{N})(9.2\,\text{m}) = 5.54\times 10^2\text{ J}.$$

Q: A rope is used to pull a 3.57 kg block at constant speed 4.06 m along a horizontal floor. The force on the block from the rope is $F = 7.68$ N and directed $\theta = 15.0^\circ$ above the horizontal. What are (a) the work done by the rope's force, (b) the increase in thermal energy of the block-floor system, and (c) the coefficient of kinetic friction between the block and floor?



Answer:

(a) The work done on the block by the force in the rope is,

$$W = Fd\cos\theta = (7.68\,\text{N})(4.06\,\text{m})\cos 15.0° = 30.1\text{J}.$$

(b) Using *f* for the magnitude of the kinetic friction force, reveals that the increase in thermal energy is

$$\Delta E_{th} = fd = (7.42\,\text{N})(4.06\,\text{m}) = 30.1\text{J}.$$

(c) We can use Newton's second law of motion to obtain the frictional and normal forces, then use $\mu_k = f / F_N$ to obtain the coefficient of friction. Place the *x* axis along the path of the block and the *y* axis normal to the floor. The *x* and the *y* component of Newton's second law are

$$x: \quad F\cos\theta - f = 0$$
$$y: \quad F_N + F\sin\theta - mg = 0,$$

where *m* is the mass of the block, *F* is the force exerted by the rope, and θ is the angle between that force and the horizontal. The first equation gives

$$f = F\cos\theta = (7.68\text{ N})\cos 15.0° = 7.42\text{ N}$$

and the second gives

$$F_N = mg - F\sin\theta = (3.57\text{ kg})(9.8\text{ m/s}^2) - (7.68\text{ N})\sin 15.0° = 33.0\text{ N}.$$

Thus,

$$\mu_k = \frac{f}{F_N} = \frac{7.42\text{ N}}{33.0\text{ N}} = 0.225\,.$$

Example1: A 40 kg box initially at rest is pushed 5.0 m along a rough horizontal floor with a constant applied horizontal force of 130 N. If the coefficient of friction between the box and floor is 0.30, find

(a) the work done by the applied force,
(b) the energy lost due to friction,
(c) the change in kinetic energy of the box, and
(d) the final speed of the box.

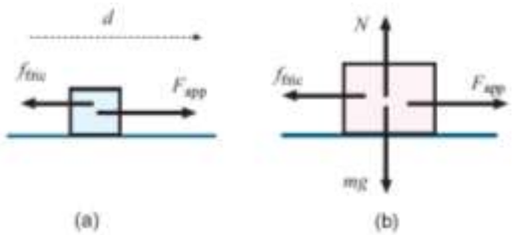


Figure 1: (a) Applied force and friction force both do work on the box. (b) Diagram showing all the forces acting on the box.

Answer:

(a) The motion of the box and the forces which do work on it are shown in Fig. 1(a). The (constant) applied force points in the same direction as the displacement. Our formula for the work done by a constant force gives

$$W_{\text{app}} = Fd\cos\phi = (130\,\text{N})(5.0\,\text{m})\cos 0° = 6.5\times 10^2\text{ J}$$

The applied force does 6.5×10^2 J of work.

(b) Fig. 1(b) shows all the forces acting on the box.
The vertical forces acting on the box are gravity (*mg*, downward) and the floor’s normal force (*N*, upward). It follows that *N* = *mg* and so the magnitude of the friction force is

$$f_{\text{fric}} = \mu N = \mu mg = (0.30)(40\,\text{kg})(9.80\,\tfrac{\text{m}}{\text{s}^2}) = 1.2\times 10^2\text{ N}$$

The friction force is directed opposite the direction of motion $\left(\theta = 180^\circ\right)$ and so the work that it does is

$$\begin{aligned} W_{\text{fric}} &= Fd\cos\phi \\ &= f_{\text{fric}}\, d\cos 180° = (1.2\times 10^2\,\text{N})(5.0\,\text{m})(-1) = -5.9\times 10^2\text{ J} \end{aligned}$$

or we might say that 5.9×10^2 J is lost to friction, and The increase in thermal energy will be $\Delta E_{\rm th} = |f_k d| = 5.9\times10^2$ J

(c) Since the normal force and gravity do no work on the box as it moves, the net work done is

$$W_{\rm net} = W_{\rm app} + W_{\rm fric} = 6.5 \times 10^2\ {\rm J} - 5.9 \times 10^2\ {\rm J} = 62\ {\rm J}\ .$$

By the Work–Kinetic Energy Theorem, this is equal to the change in kinetic energy of the box:

$$\Delta K = K_f - K_i = W_{\rm net} = 62\ {\rm J}\ .$$

(d) Here, the initial kinetic energy K_i was zero because the box was initially at rest. So we have $K_f = 62$ J. From the definition of kinetic energy, $K = ½\, mv^2$, we get the final speed of the box:

$$v_f^2 = \frac{2K_f}{m} = \frac{2(62\ {\rm J})}{(40\ {\rm kg})} = 3.1\ \frac{\rm m^2}{\rm s^2}$$

so that

$$v_f = 1.8\ \frac{\rm m}{\rm s}$$

Example6: At $t = 0$ a 1.0 kg ball is thrown from a tall tower with $\vec{v}_0 = \left(18\,\hat{\rm i} + 24\,\hat{\rm j}\right)$m/s . What is ΔU of the ball–Earth system between $t = 0$ and $t = 6.0$ s (still free fall)?

Answer: Since time does not directly enter into the energy formulations, we return to the kinematic equations to find the change of height during this $t = 6.0$ s flight.

$$\Delta y = v_{0y} t - \frac{1}{2} g t^2$$

This leads to $\Delta y = -32$ m. Therefore, $\Delta U = mg\Delta y = -318\ {\rm J} \approx -3.2\times10^{-2}\ {\rm J}$.

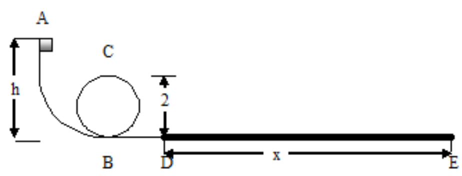
Example7: A 2.00 kg ball is thrown with an initial velocity of $\vec{v}_i = \left(18\,\hat{\rm i} + 10\,\hat{\rm j}\right)$m/s . What is the maximum change in the potential energy of the ball-Earth system during its flight?

Answer:

$$\Delta K = \frac{1}{2} m(\vec{v}_f^2 - \vec{v}_i^2) = -\Delta U_G$$

$$\Rightarrow \Delta U_G = \frac{1}{2} m(\vec{v}_i^2 - \vec{v}_f^2) = \frac{1}{2} 2(18^2 + 10^2 - 18^2) = \underline{100\ {\rm J}}$$

Example: A small block of mass m begins from rest at the top of a curved track at a height h and travels around a circular loop of radius R. There is negligible friction between the block and the track between points A and D, but the coefficient of kinetic friction on the horizontal surface between points D and E is μ. The distance between points D and E is x. **Answer** all of the following questions in terms of the given quantities and fundamental constants.



(a) Determine the speed of the block at point B, at the bottom of the loop.
(b) Determine the kinetic energy of the block at point C, at the top of the loop.
(c) After the block slides down the loop from point C to point D, it enters the rough portion of the track. The speed of the block at point E is half the speed of the block at point D.

Determine the speed of the block at point D, just before it enters the rough portion of the track.
(d) Determine the amount of work done by friction between points D and E.
(e) Find an expression for the coefficient of kinetic friction μ.

Solution

Since there is no friction on the track between points A and B, there is no loss of energy. Thus,

$$U_A = K_B \quad \Rightarrow \quad mgh = \frac{1}{2}mv_B{}^2 \quad \Rightarrow \quad v_B = \sqrt{2gh}$$

(b) Conservation of energy:

$$U_A = U_C + K_C$$
$$mgh = mg(2R) + K_C \quad \Rightarrow \quad K_C = mgh - 2mgR$$

(c) There is no energy lost on the track between points A and D, so the speed of the block at point D is the same as the speed at point B:

$$v_D = \sqrt{2gh}$$

(d) The work done by friction is the product of the frictional force and the displacement through which it acts.

$$W_f = -fx = -\mu F_N x = -\mu mgx$$

No use $$W_f = \Delta K = \frac{m}{2}\left(v_f^2 - v_D^2\right) = -\mu mgx \quad \Rightarrow \quad \mu = \frac{v_D^2}{2mgx} = \frac{2gh}{2mgx} = \underline{\frac{h}{x}}$$

(e) The frictional force causes the block to have a negative acceleration according to Newton's second law:

$$a = \frac{f}{m} = \frac{-\mu mg}{m} = -\mu g$$

Using a kinematic equation,

$$v_E{}^2 = v_D{}^2 + 2ax$$
$$0 = \left(\sqrt{2gh}\right)^2 + 2(-\mu g)x \quad \Rightarrow \quad \mu = \frac{2gh}{2gx} = \underline{\frac{h}{x}}$$

Example: A worker does 500 J of work in moving a 20 kg box a distance *D* on a rough horizontal floor. The box starts from rest and its final velocity after moving the distance *D* is 4.0 m/s. Find the work done by the friction between the box and the floor in moving the distance *D*.

Answer: Total work by the man is used to change the K.E. of the body and to overcome the friction. So,

$$\Delta K = W_{man} + W_f$$

$$\Rightarrow \quad W_f = \Delta K - W_{man} = \frac{1}{2}mv_f^2 - 500 = \frac{1}{2}20 \times 4^2 - 500 = -340 \text{ J.}$$

Example: A 2.0 kg block is released from rest 60 m above the ground. Take the gravitational potential energy of the block to be zero at the ground. At what height above the ground is the kinetic energy of the block equal to half its gravitational potential energy? (Ignore air resistance)

Answer: Apply the conservation of the mechanical energy at the beginning of the trip and at the highest point of the loop, we find:

$$K_i + U_i = K_f + U_f \Rightarrow 0 + mgh = \frac{1}{2}mgx + mgx$$

$$\Rightarrow x = \frac{2}{3}h = \underline{40\ \text{m}}$$

Example: A 2.2 kg block starts from rest on a rough inclined plane that makes an angle of 30° above the horizontal. The coefficient of kinetic friction is 0.25. As the block moves 3.0 m down the plane, the change in the mechanical energy of the block is:

Answer:

$$\Delta E = W_f = -\mu_k mg\cos 30^\circ d = -0.25\times 2.2\times 9.8\times 0.866\times 3 = \underline{-14.0\ \text{J}}$$

Example: A 10.0 kg block is released from rest at 100 m above the ground. When it has fallen 50 m, its kinetic energy is:

Answer:

$$\Delta K = -\Delta U_g = mgh = 10\times 9.8\times 50 = \underline{4900\ \text{J}}.$$

Example: A 4.0 kg block is initially moving to the right on a horizontal frictionless surface at a speed of 5.0 m/s. It then compresses a horizontal spring of spring constant 200 N/m. At the instant when the kinetic energy of the block is equal to the potential energy of the spring, the mechanical energy of the block-spring system is:

Answer:

$$\Delta E = 0 \Rightarrow E_i = E_f \text{ (or at any place)} \Rightarrow E_i = \frac{1}{2}mv_i^2 = \frac{1}{2}\times 4\times 5^2 = \underline{50\ \text{J}}.$$

Example: A 5.0 kg block starts up a 30° incline with 198 J of kinetic energy. The block slides up the incline and stops after traveling 4.0 m. The work done by the force of friction between the block and the incline is:

Answer:

$$W_f = \Delta E = \Delta K + \Delta U = (K_f - K_i) + (U_f - U_i)$$

$$= (0-198) + (5\times 9.8\times 4\sin 30^\circ) = \underline{-100\ \text{J}}.$$

Example: Conservation of Mechanical Energy

Q: A 49 kg projectile has a kinetic energy of 825 kJ when it is fired at an angle of 23.0°. Find

(a) the time of flight for the projectile and

(b) the maximum range of the projectile.

Answer:

We need to know the initial velocity using its kinetic energy:

$$K = \frac{1}{2}mv^2 \quad v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(825\,000\ \text{kg}\times\text{m}^2)}{49\ \text{kg}\times\text{s}^2}} = 184\ \frac{\text{m}}{\text{s}}$$

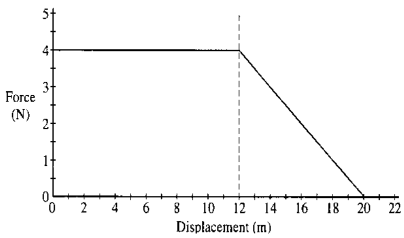
$$v_{y0} = v\sin\theta \quad = \left(184\,\frac{\text{m}}{\text{s}}\right)\sin 23.0^o \quad = \quad 71.9\,\frac{\text{m}}{\text{s}}$$

$$v_{x0} = v\cos\theta \quad = \left(184\,\frac{\text{m}}{\text{s}}\right)\cos 23.0^o \quad = \quad 169\,\frac{\text{m}}{\text{s}}$$

(a) Find the time: $v_y = v_{y0} + gt \quad \Rightarrow t = \dfrac{v_y - v_{y0}}{g} \quad = \dfrac{(-71.9) - 71.9}{-9.8} \quad = 14.7\,\text{s}$

(b) $x = vt \quad = \left(169\,\frac{\text{m}}{\text{s}}\right)15\,\text{s} \quad = 2\,500\,\text{m}$

Example: A 0.20 kg object moves along a straight line. The net force acting on the object varies with the object's displacement as shown in the graph. The object starts from rest at displacement ***x*** = 0 and time ***t*** = 0 and is displaced a distance of 20 m. Determine each of the following.



a. The acceleration of the particle when its displacement ***x*** = 6 *m*.

The force is constant from time zero till its displacement is 6 meters, so we can use the second law.

$$F = ma \quad \Rightarrow a = \frac{F}{m} = 4\,\frac{\text{kg}\times\text{m}}{\text{s}^2}\left(\frac{1}{0.20\,\text{kg}}\right) \quad = 20\,\frac{\text{m}}{\text{s}^2}$$

b. The time taken for the object to be displaced the first 12 m.

Again, the force is constant from the start till the displacement is 12 m. This means that the acceleration is also constant, so we can use one of the kinematic equations to find the time.

$$x = x_o + v_o t + \frac{1}{2}at^2 \qquad x = \frac{1}{2}at^2 \qquad t = \sqrt{\frac{2x}{a}} \quad = \sqrt{\frac{2(12\,\text{m})}{20\,\frac{\text{m}}{\text{s}^2}}} \quad = \quad 1.1\,\text{s}$$

c. The amount of work done by the net force in displacing the object the first 12 *m*.

The work done is the area under the curve:

$$W(0 \rightarrow 12) \quad = (4\,N)(12\,m) \quad = \quad 48\,\text{J}$$

d. The speed of the object at displacement ***x*** = 12 *m*. We can use conservation of energy to solve this bit.

$$W(0 \rightarrow 12) = \frac{1}{2}mv^2 \qquad v = \sqrt{\frac{2W}{m}} \quad = \sqrt{\frac{2(48\,\frac{\text{kg}\times\text{m}\times\text{m}}{\text{s}^2})}{0.20\text{kg}}} \quad = \quad 21.9\,\frac{\text{m}}{\text{s}}$$

e. The final speed of the object at displacement ***x*** = 20 *m*.

We can use conservation of energy again. The work is equal to the area under the curve, so we add the area of the rectangle to the area of the triangle.

$$W = \left[\frac{1}{2}(4N)(20 - 12\text{m})\right] + \left[(4\text{N})(12\text{m})\right] = 64\text{J}$$

$$W = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{\frac{2W}{m}} \quad = \sqrt{\frac{2(64\,\frac{\text{kg}\times\text{m}\times\text{m}}{\text{s}^2})}{0.20\,\text{kg}}} \quad = \quad 25.3\,\frac{\text{m}}{\text{s}}$$