Chapter 7
Kinetic Energy and Work

Energy: Measure of the ability of a body or system to do work or produce a change, expressed usually in joules or kilowatt hours (kWh). No activity is possible without energy and its total amount in the universe is fixed. In other words, it cannot be created or destroyed but can only be changed from one type to another. The two basic types of energy are (1) Potential: energy associated with the nature, position, or state (such as chemical energy, electrical energy, nuclear energy). (2) Kinetic: energy associated with motion (such as a moving car or a spinning wheel).

Read more: http://www.businessdictionary.com/definition/energy.html

7-1 KINETIC ENERGY

For an object with mass *m* and speed *v*, the **kinetic energy** is defined as

$$K = \tfrac{1}{2} m v^2 .$$

The SI unit of kinetic energy (and all types of energy) is the **joule** (J), named for James Prescott Joule, an English scientist of the 1800s and defined as

$$1 \text{ joule} = 1 \text{ J} = 1 \text{ kg m}^2/\text{s}^2.$$

Kinetic energy is a scalar (it has magnitude but no direction); it is always a positive number.

Example: Calculate the kinetic energy of a 4.0 kg particle and its velocity is **v** = (4.0 **i** - 3.0 **j**) m/s.

Answer:

$$K = \tfrac{1}{2} m v^2 = \tfrac{1}{2} m(\mathrm{v}\cdot\mathrm{v}) = \frac{1}{2}4\left(4^2 + 3^2\right) = \underline{50 \text{ J}}$$

7-2 WORK AND KINETIC ENERGY

Energy is difficult to define because it comes in many different forms. It is hard to find a single definition which covers all the forms.

Some types of energy:
kinetic energy (KE) = energy of motion
thermal energy = energy of "atomic jiggling"
potential energy(PE) = stored energy of position/configuration
various kinds of PE are:

- gravitational
- electrostatic
- elastic (actually a form of electrostatic PE)
- chemical (another form of electrostatic PE)
- nuclear

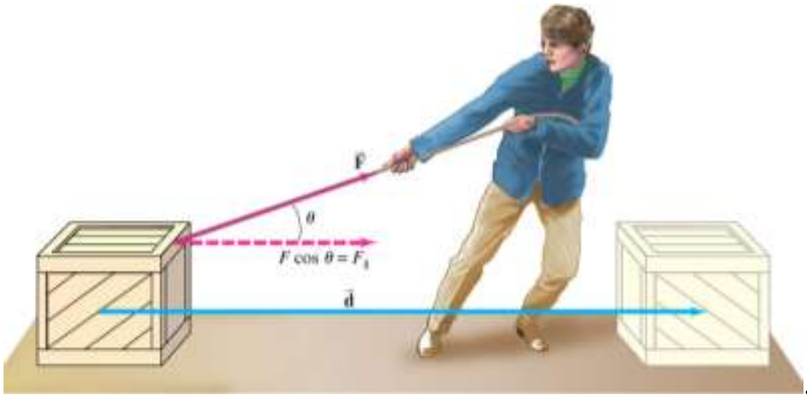
Radiant energy = energy of light
Mass energy (Einstein's Relativity Theory says mass is a form of energy.)
Almost all forms of energy on earth can be traced back to the Sun:

Example: Lift a book (gravitational PE) ← chemical PE in muscles ← chemical PE in food ← cows ← grass ← sun (through photosynthesis)!

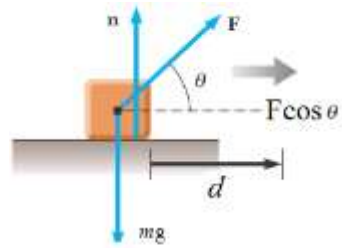
Some textbooks say that energy is the ability to do work (not a bad definition, but rather abstract). A key idea that we will use over and over again is this: **Whenever work is being done, energy is being changed from one form to another or being transferred from one body to another. The amount of work done on a system is the change in energy of the system.**

WORK

When an object moves while a force is being exerted on it, then **work** is being done on the object by the force.



If an object moves through a displacement $\vec{d}$ while a constant force $\vec{F}$ is acting on it, the force does an amount of work equal to



$$W_F = \vec{F}.\vec{d} = \left(F\cos\theta\right)d = F_\parallel\, d$$

where θ is the angle between the force $\vec{F}$ and the displacement $\vec{d}$.

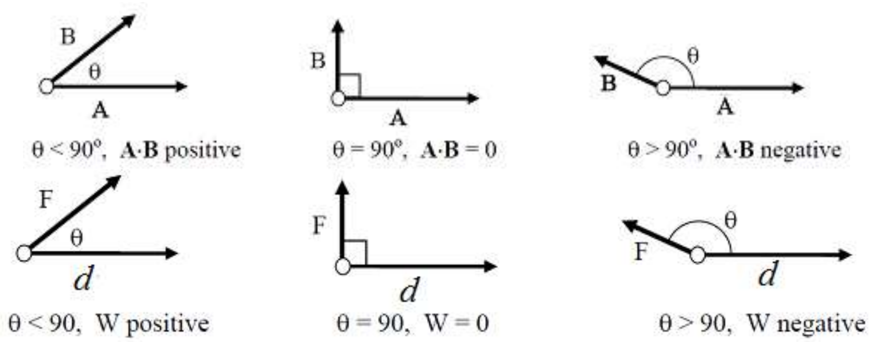
$F_\parallel$ = component of force along the direction of displacement,

$W_F = F_\parallel \times$distance .

SI units of work: [W] = [F][d] = 1 N·m = 1 joule = 1 J

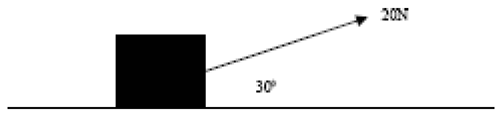
Work is a scalar quantity: Note that *work is a scalar quantity that has* no direction. In fact, all types of energy are scalar quantities. This is one of the nice features of the energy approach. It is easier to deal with scalar quantities than with vector quantities!

Note: The dot product is positive, negative, or zero depending on the relative directions of the vectors **A** and **B**. When **A** and **B** are at right angles (θ = 90°), the dot product is zero. When the angle θ is greater then 90°, then the dot product is negative.



Why do we define work this way? **Answer:** Whenever work is done, energy is being transformed from one form to another. The amount of work done is the amount of energy transformed. (This is the **First Law of Thermodynamics.** (Phys 102).)

Example 1: Compute the work done by a 20 N force applied to a 10 kg block as shown below if the block moves 20 meters as a result of the application of the force. The surface is smooth.



Answer: Before we begin the computation of work, let's examine what happens when this system is set into motion. The application of the 20 N force causes the block to accelerate to the right since there is no balancing force to the left (the magnitude of this acceleration is 1.73 m/s^2).

$$W = F_{\parallel} d = (F\cos\theta)d = Fd\cos\theta = (20\text{N})(20\text{ m})(\cos 30^\circ) = 346\text{ J}.$$

How would this problem be different if the 20 N force were applied at an angle of zero degrees with the horizontal? What about 90^o?

Zero work: The work is zero if

i- the force is zero,

ii- the displacement is zero, or

iii- the angle between F and d is 90^o.

When an object is displaced on a frictionless, horizontal surface, the normal force **n** and the gravitational force ***mg*** do no work on the object. In the situation shown in the figure, $F\cos\theta$ is the only force doing work on the object.

Negative work: The work is negative if the force or one of its components is opposite to the displacement. In the figure below the **displacement is to the right**, therefore:

(a) Represents no work is done,

(b) most negative work,

(c) most positive work

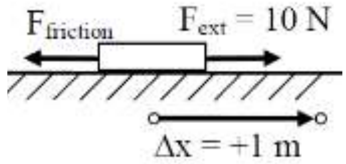
(d) negative work

(a) (b) (c) (d)

Note that work is energy transfer.

- If W is positive energy is transferred to the system and the stored energy of the system increases;
- If W is negative energy is transferred from the system and the stored energy of the system decreases.

Example of work: Move book at constant velocity along a rough table with a constant horizontal force of magnitude F_{ext} = 10 N. Total displacement is Δx = 1 m.



Answer:

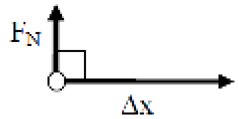
- work done *by external force* = W_{Fext} = + $F_{ext} \cdot \Delta x$ = 10 N · 1 m = 10 N·m = +10 J

Since velocity = constant, F_{net} = 0, so $|F_{ext}| = |F_{fric}|$ = 10 N

- Work done *by force of friction* = $W_{Ffric} = -|F_{fric}| \cdot |\Delta x|$ = – 10 J (since cos 180° = –1)

- Work done *by normal force* F_N is zero: $W_{FN} = 0$

(since normal force is perpendicular to displacement, cos 90° = 0.)

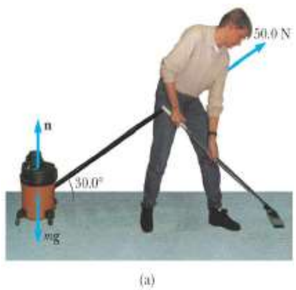


Work done *by the net force* is zero. Since v = constant ⇒ $F_{net} = 0$ ⇒ $W_{net} = 0$.

Moral of this example: Whenever you talk about the work done, you must be <u>very careful</u> to specify *which force* does the work.

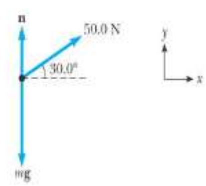
Example: A vacuum cleaner being pulled at an angle of 30.0° from the horizontal by a force of 50 N;

a. Calculate the work done as the vacuum cleaner moves a distance of 3.0 m.
b. What is the work done by the normal force ***n***? and
c. What is the work done by the weight mg?



Solution

a. Input: F = 50 N, d = 3m, $\theta = 30.0°$
   W= F d cos θ = (50 N)(3 m)(cos30.0°) = 130 J



b. Since the angle between ***n*** and ***d*** is 90° and cos 90° is zero then the work is zero.
c. Since the angle between ***mg*** and ***d*** is 270° (- 90°), and since cos 270° = 0, then the work done by ***n*** is zero.

Example: A block is pushed through a displacement of **d** = (15 m) **i** − (12 m) **j**, by a force **F** = (210 N) **i** − (150 N) **j** exert on the block. How much work does the force do on the block during the displacement?

Answer: Here we have the simple case of a straight–line displacement **d** and a constant force **F**. Then the work done by the force is $W = \mathbf{F} \cdot \mathbf{d}$. We are given all the components, so we can compute the dot product using the components of **F** and **d**:

$$W = \mathbf{F} \cdot \mathbf{d} = F_x d_x + F_y d_y = (210\,\mathrm{N})((15\,\mathrm{m}) + (-150\,\mathrm{N})(-12\,\mathrm{m}) = 4950\,\mathrm{J}$$

The work-energy theorem

This theory states that:

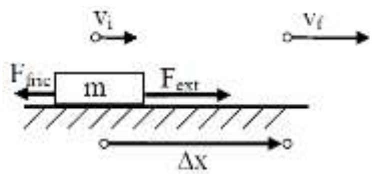
"the work done on an object by the resultant force (sum of all forces) acting upon it, equal the change in its kinetic energy".

The work-energy theorem is valid for constant force as well as for varying forces.

Prove that (Simple but not needed)

$$W = \Delta K = K_f - K_i = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

"Proof" of $W_{net} = \Delta$ KE. Here we show that the Work-KE theorem is true for a special case. I push a book of mass m along a table with a constant external force of magnitude $F_{ext.}$ The force of friction on the book has magnitude F_{fric}. The book starts with an initial velocity v_i and ends with a final velocity v_f. While the force is applied, the book moves a displacement Δx. We show that $W_{net} = \Delta KE$ in this case.



$F_{net} = F_{ext} - F_{fric}$ (the normal force and force of gravity cancel). $W_{net} = +F_{net}\,\Delta x$

What is ΔKE? KE involve v^2, so we look for a formula involving v^2. Since F_{net} = constant, the acceleration is constant, and so we can use the 1D constant acceleration formula

$$v^2 = v_0^2 + 2a(x - x_0).$$

So we have

$v_f^2 - v_i^2 = 2a(x_f - x_i) = 2(F_{net}/m)(\Delta x)$ [using $a = F_{net}/m$]

$\Delta KE = KE_f - KE_i = (1/2)mv_f^2 - (1/2)mv_i^2 = (1/2)m \cdot 2(F_{net}/m)(\Delta x) = +F_{net}\,\Delta x = W_{net}$.

Done!

Note that:

According to the work-energy theorem

1- work done by all forces = ΔK
2- if the work is positive $K_f > K_i$ and the velocity increases, $\boldsymbol{v_f > v_i}$
3- if the work is negative $K_f < K_i$ and the velocity decreases $\boldsymbol{v_f < v_i}$
4- only when external force acts on an object its energy increases by an amount equal to the work done by this force.
5- we can consider the kinetic energy of a body as the work that the body do in coming to rest.

Example: A 4.0 kg cart starts up an incline with a speed of 3.0 m/s and comes to rest 2.0 m up the incline. The net work done on the cart is:

Answer: Use the work-energy theorem, we have

$$W = \Delta K = K_f - K_i = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \frac{1}{2} \times 4 \times (0 - 3^2) = \underline{-18\text{ J}}$$

Example: A particle moves along a straight path through displacement $\vec{d} = (8\hat{i} + c\hat{j})$ m while force $\vec{F} = (2\hat{i} - 4\hat{j})$ N acts on it. (other forces also act on the particle.) What is the value of c if the work done by $\vec{F}$ on the particle is (a) zero, (b) positive, and (c) negative.

<u>Solution</u>

(a) $W = \vec{F} \cdot \vec{d} = (2\hat{i} - 4\hat{j}) \cdot (8\hat{i} + c\hat{j}) = 16 - 4c$, which, if equal zero, implies c = 16/4 = 4 m.

(b) If $W > 0$ then $16 > 4c$, which implies $c < 4$ m.

(c) If $W < 0$ then $16 < 4c$, which implies $c > 4$ m.

Example: What acceleration is required to stop a mass m = 1000 kg car traveling 28 m/s in a distance of 100 meters?

Answer: Use the equation $W = \Delta K = K_f - K_i = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = Fd\cos\theta$, one finds

$$\Rightarrow \frac{1}{2}\times 1000\times(0-28^2) = F(100\text{ m}) \quad \Rightarrow F = -3925\text{ N}$$

$$\therefore a = F/m = -\frac{3925}{1000} = -\underline{3.92\text{ m/s}^2}$$

What is the significance of the minus sign in the answer?

Example: At time t = 0 a single force F acts on a 2.0 kg particle and changes its velocity from at t = 0 to $\mathbf{v}_i$ = (4.0 **i** - 3.0 **j**) m/s to $\mathbf{v}_f$ = (4.0 **i** +3.0 **j**) m/s at t = 3.0 s. During this time the work done by F on the particle is:

Answer: $v_f^2 = (4.0\,\hat{i} + 3.0\,\hat{j})^2 = 4.0^2 + 3.0^2 = 25$, $v_i^2 = (4.0\,\hat{i} - 3.0\,\hat{j})^2 = 4.0^2 + 3.0^2 = 25$

$$W = \Delta K = K_f - K_i = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = \frac{1}{2}\times 2\times(25-25) = \underline{0\text{ J}}$$

Chapter 7
Kinetic Energy and Work

Important Terms (For chapters 7)

- **conserved properties:** any properties which remain constant during a process
- **energy:** the non-material quantity which is the ability to do work on a system
- **joule:** the SI unit for energy, and work, equal to one Newton-meter
- **kinetic energy:** the energy a mass has by virtue of its motion
- **mechanical energy:** the sum of the potential and kinetic energies in a system
- **potential energy:** the energy an object has because of its position
- **power: t**he rate at which work is done or energy is dissipated
- **watt:** the SI unit for power equal to one joule of energy per second
- **work:** the scalar product of force and displacement

$W = \mathbf{F}\cdot\mathbf{s} = \left(F\cos\theta\right)s$ $W = \int \mathbf{F}\cdot d\mathbf{s} = \int F ds$ $KE = \frac{1}{2}mv^2$ $W = KE_f - KE_i = \frac{1}{2}mv_f^{\,2} - \frac{1}{2}mv_i^{\,2}$ $PE = mgh$ $W_g = \Delta PE_g = mg\left(h_i - h_f\right)$ $PE_i + KE_i = PE_f + KE_f$ $P = \frac{W}{t}$ $P = \mathbf{F}\cdot\mathbf{v} = \left(F\cos\theta\right)v$	where *W* = work *F* = force *s* = displacement **F. s** = scalar product of force and displacement *KE* = kinetic energy *v* = velocity or speed *m* = mass *PE* = potential energy (denoted as *U*) *g* = acceleration due to gravity *h* = height above some reference point *P* = power *t* = time

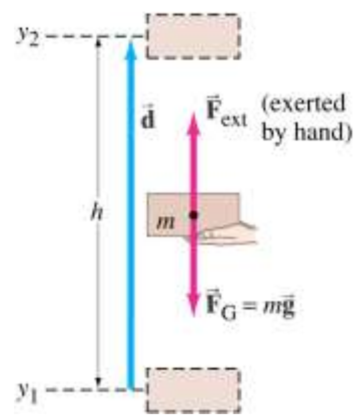
7-3 WORK DONE BY THE GRAVITATIONAL FORCE

Potential energy: The potential energy is "***the energy associated with the position of an object.***" It can be considered as a stored energy of the object that can be converted into work or kinetic energy.

- The concept of potential energy can be used only when the object is acted by a type of force called conservative force (See chapter 8). The gravitational force, the electric force and the force of a spring are all conservative force.

In raising a mass *m* to a height *h*, the work done by the external force (extremal agent) is

$$W_{ext} = F_{ext} d \cos 0^{\circ} = mgh = mg(y_2 - y_1)$$



We therefore define the gravitational potential energy:

$$PE_{gravity} = U = mgy$$

Note that:

$$W_G = F_G d \cos 180^{\circ} = mgh = mg(y_1 - y_2) = -W_{ext}$$

i.e.

$$\boxed{W_G + W_{ext} = 0}$$

This potential energy can become kinetic energy if the object is dropped. Potential energy is a property of a system as a whole, not just of the object (because it depends on external forces).

If $PE_{gravity} = mgy$, where do we measure *y* from 0 . It turns out not to matter, as long as we are consistent about where we choose *y* = 0. Only changes in potential energy can be measured.

- Potential energy can also be stored in a spring when it is compressed; the figure below shows potential energy yielding kinetic energy.

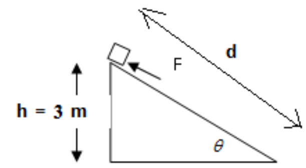
(a) (b) (c)

Example: A 4-kg box is pushed across a level floor with a force of 60 N for a displacement of 20 m, and then lifted to a height of 2 m. What is the total work done **on** the box?

Answer:

The total work done = work done to push the block for the displacement of 20 m
+ work done to lift the block to a height of 2 m.

$$W_{total} = Fs + mgh = (60\text{ N})(20\text{ m}) + (4\text{ kg})(10\text{ m/s}^2)(2\text{ m}) = 1280\text{ J}$$



Example: A 20-kg cart is pushed up the inclined plane shown by a force **F** to a height of 3 m. What is the potential energy of the cart when it reaches the top of the inclined plane?

Answer:

1- Work done by the force F $\left(W_F\right)$ = **F. d** = $|F|d\sin 0 = |F|d$

Important Note: What is F? Answer: From F.B.D. $|F| = mg\sin\theta$, then

$$\left(W_F\right) = \mathbf{F.\,d} = W_F = |F|d = mg\left(d\sin\theta\right) = mgh$$

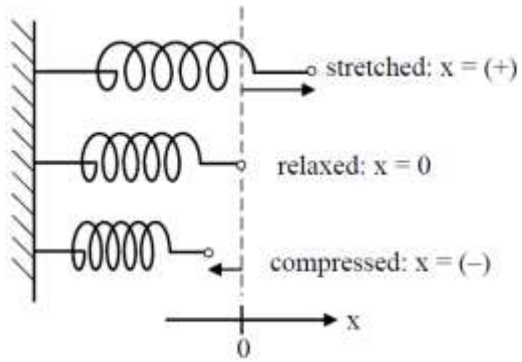
The potential energy is equal to the work done against gravity:

$$W = mgh = \left(20\text{ kg}\right)\left(10\text{ m/s}^2\right)\left(3\text{ m}\right) = 600\text{ J}$$

7-4 WORK DONE BY A SPRING FORCE

We want to derive an expression for the work done to stretch or compress a spring, so we take a little detour and talk about springs.

Most springs obey "Hooke's Law" which says that the force exerted by a spring is proportional to the displacement from the equilibrium (relaxed) position.



x is the amount that the spring is stretched (+) or compressed (–)

$|F_{spring}| \propto |x|$

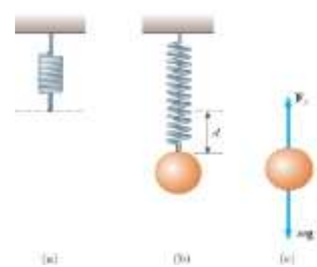
double $|x|$ $\Rightarrow$ double $|F_{spring}|$

Hooke's "Law" : $F_{spring} = -kx$

k = spring constant = measure of stiffness, big k ⇔ stiff spring, small k ⇔ floppy spring

units of k = [k] = [F]/[x] = N/m (newtons per meter)

The determination of the elastic force constant of a spring



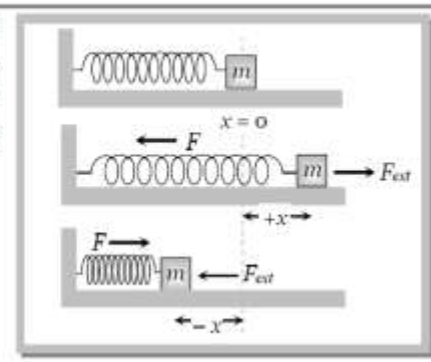
The elastic force constant of a spring is determined experimentally as follows:

1- A mass m is suspended to a spring vertically.
2- Due to the weight of the mass the spring will be elongated by a distance d. Then the restoring force $F_{spring} = -k\,d$.
3- At equilibrium or balance. $kd = mg$,

$$k = mg/d$$

Elastic Potential Energy

(1) **Restoring force and spring constant :** When a spring is stretched or compressed from its normal position ($x = 0$) by a small distance x, then a restoring force is produced in the spring to bring it to the normal position.According to Hooke's law this restoring force is proportional to the displacement x and its direction is always opposite to the displacement.



i.e. $\vec{F} \propto -\vec{x}$

or $\vec{F} = -k\vec{x}$ .....(i)

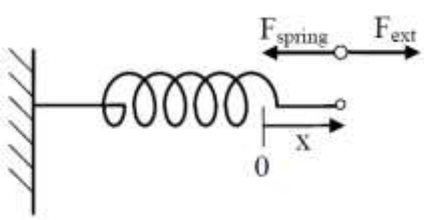
where k is called spring constant.

- Why the (–) sign in Hooke's Law? It's because the direction of the force exerted by the spring is opposite direction of displacement. When displacement is to the right (x +), the spring pulls back to the left (F–) i.e. F is (–).; when x is (–), F is (+).

Why Hooke's "Law" in quotes? Because it is not really a law, it is only approximately true for most springs as long as the extension is not too great. If a spring is stretched past its "elastic limit", the spring will permanently deform, and it will not obey Hooke's Law.

- We now show that the work done by an external force F_{ext} (such as the force from my hand) to stretch or compress a spring by an amount x is given by

$$W_{ext} = \frac{1}{2} k x^2$$



To hold the spring stretched a displacement x, We have to exert an external force $F_{ext} = -k x$. To slowly stretch the spring from x_i to x_f , We have to apply a force that increases from zero to kx_f. Our general definition of work in 1D is: $W_F = \int F\, dx$.

$$W_{ext} = \int_{x_i}^{x_f} F_{ext}\, dx = \int_{x_i}^{x_f} k\, x\, dx = \tfrac{1}{2} k\, x^2 \Big|_{x_i}^{x_f} = \tfrac{1}{2} k \left(x_f^2 - x_i^2 \right)$$

Done.

Notice that the work done by the external force is always positive, regardless of whether the spring is stretched (x positive) or compressed (x negative).

Example: An ideal spring is hung vertically from the ceiling. When a 2.0 kg mass hangs at rest from it, the spring is extended 6.0 cm from its relaxed length. A downward external force is now applied to the mass to extend the spring an additional 10 cm. Calculate

a- the spring constant,

b- the work done by the **spring.** Note that: $W_{ext} + W_s = 0$.

Answer:

a- Calculate the spring constant. $k = \left|\frac{F}{x}\right| = \frac{2.0\times 9.8}{0.06} = 326.666$ N/m

b- While the spring is being extended by the external force, the work done by the spring is:

$$W_s = \frac{1}{2}k\left(x_i^2 - x_f^2\right) = \frac{1}{2}\times\left|\frac{F}{x}\right|\times(\underbrace{0.06^2 - 0.16^2}_{-0.022}) = \underline{-3.6\text{ J}};$$

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(2) **Expression for elastic potential energy :** When a spring is stretched or compressed from its normal position ($x = 0$), work has to be done by external force against restoring force. $\vec{F}_{ext} = \vec{F}_{restoring} = k\vec{x}$

Let the spring is further stretched through the distance dx, then work done

$$dW = \vec{F}_{ext}.\vec{dx} = F_{ext}.dx\cos 0^o = kx\,dx \qquad [\text{As } \cos 0^o = 1]$$

Therefore total work done to stretch the spring through a distance x from its mean position is given by

$$W = \int_0^x dW = \int_0^x kx\,dx = k\left[\frac{x^2}{2}\right]_0^x = \frac{1}{2}kx^2$$

This work done is stored as the potential energy of the stretched spring.

$\therefore$ Elastic potential energy $U = \frac{1}{2}kx^2$

$$U = \frac{1}{2}Fx \qquad \left[\text{As } k = \frac{F}{x}\right]$$

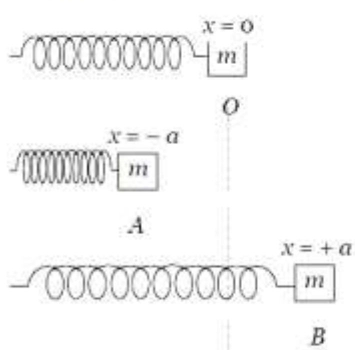
$$U = \frac{F^2}{2k} \qquad \left[\text{As } x = \frac{F}{k}\right]$$

$\therefore$ Elastic potential energy $U = \frac{1}{2}kx^2 = \frac{1}{2}Fx = \frac{F^2}{2k}$

Note : ❑If spring is stretched from initial position x_1 to final position x_2 then work done

= Increment in elastic potential energy $= \frac{1}{2}k(x_2^2 - x_1^2)$

(3) **Energy graph for a spring :** If the mass attached with spring perf[orms] simple harmonic motion about its mean position then its potential ener[gy] at any position (x) can be given by



$$U = \frac{1}{2}kx^2 \qquad ....(i)$$

Problem . A long spring is stretched by 2 *cm*, its potential energy is *U*. If the spring is stretched by 10 *cm*, the potential energy stored in it will be

(a) $U/25$ (b) $U/5$ (c) $5U$ (d) $25U$

Solution : (d) Elastic potential energy of a spring $U = \frac{1}{2}kx^2$ $\therefore U \propto x^2$

So $\frac{U_2}{U_1} = \left(\frac{x_2}{x_1}\right)^2 \Rightarrow \frac{U_2}{U} = \left(\frac{10\,cm}{2\,cm}\right)^2 \Rightarrow U_2 = 25U$

Problem . A spring of spring constant $5 \times 10^3 N/m$ is stretched initially by 5 *cm* from the unstretched position. Then the work required to stretch it further by another 5 *cm* is

(a) 6.25 *N-m* (b) 12.50 *N-m* (c) 18.75 *N-m* (d) 25.00 *N-m*

Solution : (c) Work done to stretch the spring from x_1 to x_2

$W = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2}5 \times 10^3[(10 \times 10^{-2})^2 - (5 \times 10^{-2})^2] = \frac{1}{2} \times 5 \times 10^3 \times 75 \times 10^{-4} = 18.75\,N.m$.

Problem . Two springs of spring constants $1500\,N/m$ and $3000\,N/m$ respectively are stretched with the same force. They will have potential energy in the ratio

(a) 4 : 1 (b) 1 : 4 (c) 2 : 1 (d) 1 : 2

Solution : (c) Potential energy of spring $U = \frac{F^2}{2k} \Rightarrow \frac{U_1}{U_2} = \frac{k_2}{k_1} = \frac{3000}{1500} = 2:1$ [If *F* = constant]

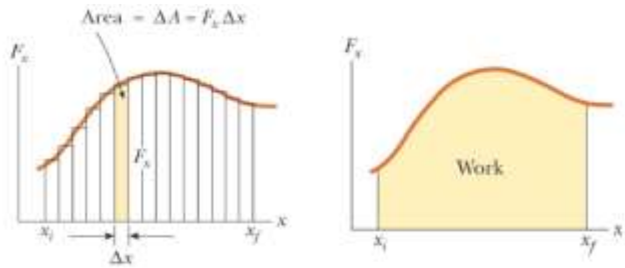
7-5 WORK DONE BY A GENERAL VARIABLE FORCE

When the force acting on an object is varying force i.e it depends on the position of the object the work done by this force while the object moves from an initial position x_1 to a final position x_2 is given by

$$W = \int_{x_1}^{x_2} F_x dx$$

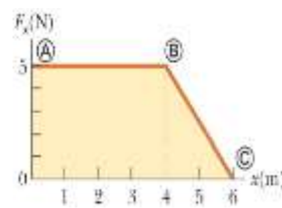
where F_x is the component of the force in x direction i.e. parallel to motion.

The work done by the force component F_x for the small displacement Δx is $F_x \ \Delta x$, which equals the area under the force displacement graph. In the given figure it is the area of the shaded rectangle. Figure (a) shows the total work done, for the displacement from x_i to x_f is approximately equal to the sum of the areas of all the rectangles. In figure (b) the work done by the component F_x of the varying force as the particle moves from x_i to x_f is *exactly* equal to the area under this curve.



(a) **(b)**

Example: The force acting on a particle is constant for the first 4.0 m of motion and then decreases linearly with x from x_B = 4.0 m to x_C = 6.0 m. as shown in the figure. Find the net work done by these forces as the object moves from A to C.



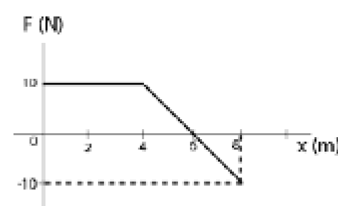
Solution: The net work done by these forces is the area under the curve.

$$W(A \to B) = 5 \times 4 = 20 \text{ J}$$

$$W(B \to C) = \tfrac{1}{2}(2)\,(5) = 5 \text{ J}$$

$$W(A \to C) = 20 + 5 = 25 \text{ J}$$

Example: A varying force F(N) acts on a particle of mass m = 2.0 kg as shown in Figure. Find the speed of the particle at x = 8.0 m, if the kinetic energy at x = 0 is 9.0 J.



Answer: $W = \text{Area under the curve} = 4 \times 10 + \frac{1}{2} \times 2 \times 10 - \frac{1}{2} \times 2 \times 10 = 40 \text{ J}$

$\because W = \Delta K = K_f - K_i = 40 \text{ J}$

$\therefore K_f = 40 \text{ J} + K_i = 40 \text{ J} + 9 \text{ J} = 49 \text{ J} \quad \Rightarrow \quad v_f = \sqrt{2\frac{K_f}{m}} = \sqrt{2\frac{49}{2}} = \underline{7.0 \text{ m/s}}$

7-6 POWER

Power is simply the rate at which you do work. Do work fast, you get a lot of work done, and you are very powerful. If your power output is small, however, it takes a lot of time to do the work. Work can be done slowly or quickly, but the time taken to perform the work doesn't affect the amount of work which is done, since there is no element of time in the definition for work. However, if you do the work quickly, you are operating at a higher power level than if you do the work slowly. *Power is defined as the rate at which work is done.* Oftentimes we think of electricity when we think of power, but it can be applied to mechanical work and energy as easily as it is applied to electrical energy. The equation for power is

$$P = \frac{Work}{time}$$

and has units of joules/second or *watts* (W). A machine is producing one watt of power if it is doing one joule of work every second.

The watt is named after James Watt, a British engineer who perfected the steam engine. Watt himself developed his own unit for power, one that is still in use. This is the beloved **horsepower**. The symbol for horsepower is ***hp***. The watt is often used for appliances that work off electricity – toasters, refrigerators, waffle irons, coffee makers, heaters, etc.

The units of power are:

1- Joule/second = Watt.
2- k watt = 1000 watt,
3- hp = 746 watt.
4- A unit of work and energy is kwh (kilowatt- hour).

Example: It takes an engine 25 seconds to do 1 700 J of work. How much power did it develop?

Answer:

$$P = \frac{W}{\Delta t} = \frac{1\,700\text{ J}}{25\text{ s}} = \underline{68\text{ W}}$$

Example: What is the energy consumed by 100 Watt bulb in one hour?

Answer:

$$p = W/t \Rightarrow W = pt = 100 \times 3600 = 3.6 \times 10^{5} \text{ Joules.}$$

Variant Power Equations:

Start with the equation for average power in the form: $P_{avg} = \dfrac{W}{\Delta t}$

We will usually drop the "*avg*" bit and just use:

$$P = \frac{W}{\Delta t} \qquad or \qquad P = \frac{W}{t}$$

Recall that there are several different equations for work:

$$P = \frac{W}{t} = \frac{\vec{\mathrm{F}}.\vec{\mathrm{d}}}{t} = \vec{\mathrm{F}}.\frac{\vec{\mathrm{d}}}{t} = \vec{\mathrm{F}}.\vec{\mathrm{v}} = m\left(\vec{\mathrm{a}}.\vec{\mathrm{v}}\right)$$

This is the different forms of the second power equation.

Example: An elevator cab has a mass of 4500 kg and can carry a maximum load of 1800 kg. If the cab is moving upward of a full load at 3.80 m/s, what power is required of the force moving the cab to maintain that speed?

Solution: A convenient approach is provided by $P = \vec{F} \cdot \vec{v}$

$P = F\ v$ =(1800 kg + 4500 kg)(9.8 m/s^2)(3.80 m/s)= 235 kW.

Note that we have set the applied force equal to the weight in order to maintain constant velocity (zero acceleration).

Example: A 2000 kg elevator moves 20 m upward in 4.9 sec at a constant speed. At what average rate does the force from the cable do the work on the elevator?

Answer: The acceleration is zero, since the velocity is constant.

$$\therefore P = \frac{TS}{t} = \frac{mgs}{t} = \frac{2000 \times 9.8 \times 20}{4.9} = \underline{8.0 \times 10^4 \text{ W}}.$$

Example: A helicopter lifts an 80 kg man vertically from the ground by means of a cable. The upward acceleration of the man is 2.0 m/s^2. Find the rate at which the work is being done on the man by the tension of the cable when the speed of the man is 1.5 m/s.

Answer:

$$ma = T - mg \Rightarrow T = m(a+g)$$

$$P = Tv = m(a+g)v = 80(2+9.8) \times 1.5 = \underline{1416 \text{ W}}$$

Example: A force **F** = (3.00 i +7.00 j) N acts on a 2.00 kg object that moves from an initial position $\mathbf{r}_1$=(3.00i - 2.00j) m to a final position $\mathbf{r}_2$ = (5.00 i + 4.00 j) m in 4.00 s . What is the average power due to the force during that time interval?

Answer:

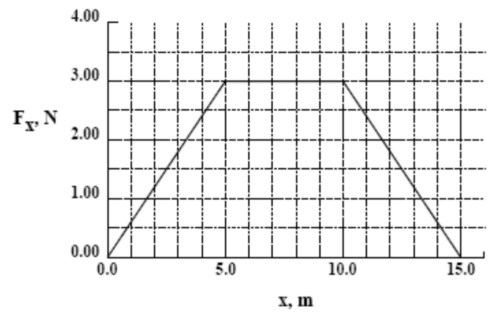
$$P = \frac{dW}{dt} = \mathbf{F}.\frac{d\mathbf{r}}{dt} = (3.00\,\hat{i} + 7.00\,\hat{j}) . \frac{[(5.00\,\hat{i} + 4.00\,\hat{j}) - (3.00\,\hat{i} - 2.00\,\hat{j})]}{4}$$

$$= \frac{(3.00\,\hat{i} + 7.00\,\hat{j}).(2.00\,\hat{i} + 6.00\,\hat{j})}{4} = \frac{6+42}{4} = \underline{12 \text{ W}}$$

Extra Problems

Example: A particle is subject to a force F_x that varies with position as shown in the figure. Find the work done by the force on the body as it moves

(a) from $x = 0$ to $x = 5.0$ m,
(b) from $x = 5.0$ m to $x = 10$ m and
(c) from $x = 10$ m to $x = 15$ m.
(d) What is the total work done by the force over the distance $x = 0$ to $x = 15$ m?



Answer:

(a) Here the force is not the same all through the object's motion, so we can't use the simple formula $W = F_x x$. We must use the more general expression for the work done when a particle moves along a straight line,

$$W = \int_{x_i}^{x_f} F_x dx$$

Of course, this is just the "area under the curve" of F_x vs. x from x_i to x_f . In part (a) we want this "area" evaluated from $x = 0$ to $x = 5.0$ m. From the figure, we see that this is just half of a rectangle of base 5.0 m and height 3.0 N. So the work done is

$$W(0 \rightarrow 5) = (3.0\text{ N})(5.0\text{m})/2 = 7.5\text{ J}.$$

(Of course, when we evaluate the "area", we just keep the units which go along with the base and the height; here they were meters and newtons, the product of which is a joule.) So the work done by the force for this displacement is 7.5 J.

(b) The region under the curve from $x = 5.0$m to $x = 10.0$m is a full rectangle of base 5.0 m and height 3.0 N. The work done for this movement of the particle is

$$W(5 \rightarrow 10) = (3.0\text{ N})(5.0\text{ m}) = 15.\text{ J}$$

(c) For the movement from $x = 10.0$m to $x = 15.0$m the region under the curve is a half rectangle of base 5.0m and height 3.0 N. The work done is

$$W(10 \rightarrow 15) = (3.0\text{ N})(5.0\text{m})/2 = 7.5\text{J}.$$

(d) The total work done over the distance $x = 0$ to $x = 15.0$m is the sum of the three separate "areas",

$$W\text{total}(0 \rightarrow 15) = 7.5\text{ J} + 15.\text{ J} + 7.5\text{ J} = 30.\text{ J}$$

Example: A 200 kg box is pulled along a horizontal surface by an engine. The coefficient of friction between the box and the surface is 0.400. The power the engine delivers to move the box at constant speed of 5.00 m/s is:

Answer: Here, we use $f = \mu_k mg = 0.4 \times 200 \times 9.8 = 784$ N.

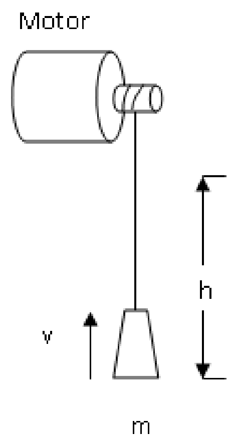
$$P = \frac{dW}{dt} = \vec{f} . \vec{v} = 784 \times 5 = \underline{3920\text{ W}}.$$

Example: A 2.0 kg block is pulled at a constant speed of 1.1 m/s across a horizontal rough surface by an applied force of 12 N directed 30° above the horizontal. At what rate is the frictional force doing work on the block?
Answer: The acceleration here is zero, since we have constant velocity. The work done by the applied force is used to overcome the friction work, so:

$$\Delta K = W_f + W_F = 0 \quad \Rightarrow \quad W_f = -W_F = -F\cos 30^\circ \, \mathrm{d}$$

$$\therefore P = -\frac{dW_F}{dt} = -F\cos 30^\circ \, \mathrm{v}_f = -12 \times 0.866 \times 1.1 = \underline{-11.4\ \mathrm{W}}$$

Example: A motor raises a mass of 3 kg to a height *h* at a constant speed of 0.05 m/s. The battery (not shown) which provides energy to the motor originally stores 4 J of energy, all of which can be used to lift the mass.
(a) What is the power developed in the motor?
(b) To what maximum height can the motor lift the mass using its stored energy?



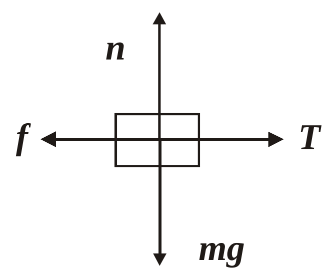
Solution
(a)

$$P = \frac{W}{t} = \frac{Fh}{t} = \frac{(mg)h}{t} = (mg)v = (3\,\mathrm{kg})(10\ \mathrm{m/s^2})(0.05\ \mathrm{m/s}) = 1.5\ \mathrm{W}$$

(b)

$$W = mgh \quad \Rightarrow \quad h = \frac{W}{mg} = \frac{4\ \mathrm{J}}{(3\mathrm{kg})(10\ \ \mathrm{m/s^2})} = 0.13\ \mathrm{m}$$

Example: A 15.5 kg block is pulled across a flat deck at a constant speed of 3.0 m/s with a rope. The rope is horizontal to the deck. The coefficient of kinetic friction is 0.330. How much power does it take to do this?



Answer: We can find the force of friction once that is found we can then calculate the force. Since the block is moving at a constant speed, the sum of the horizontal forces must be zero. Also since it isn't falling or rising, the sum of the vertical forces must be zero.
Up is positive as is going to the right.

x – direction: $T - f = 0 \quad \Rightarrow \quad T = f$ (1)

y – direction: $n - mg = 0 \quad \Rightarrow \quad n = mg$ (2)

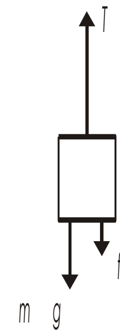
By definition, the frictional force is: $f = \mu_k n = \mu_k mg$ (3)

Substitute in the value for ***f***:

$$f = T = \mu_k mg \quad = 0.330(15.5\,\mathrm{kg})\left(9.8\frac{\mathrm{m}}{\mathrm{s}^2}\right) \quad = \quad 50.1\ \mathrm{N}$$

Now we can find the power it takes to do this:

$$P = F \cdot v = 50.1\ \mathrm{N} \times 3.0\ \mathrm{m/s} \approx 150\ \mathrm{W.}$$



Example: A 1250 kg Elevator carries a maximum load of 955 kg. A constant frictional force of 3850 N exists. What minimum power (in hp) for the motor is needed to lift the thing at a constant speed of 3.50 m/s?

Answer: We need to find the force needed to lift the elevator. We draw a FBD and examine the sum of the forces. They must equal zero since the elevator will move at a constant speed.

$$T - mg - f = 0 \qquad \Rightarrow \qquad T = mg + f \qquad (1)$$

$$P = F\,v = T\,v \qquad (2)$$

Substitute from (1) into (1):

$$P = (mg + f)v$$

$$P = \left((995\,kg + 1250\,kg)\left(9.8\frac{m}{s^2}\right) + 3850\,N\right)\left(3.50\,\frac{m}{s}\right) \quad = 90\,480\,W$$

$$P = 90480\,\text{W}\left(\frac{1\,\text{hp}}{746\,\text{W}}\right) \quad = \quad \underline{122\ \text{hp}}$$