Chapter 5
Force and Motion—I

5-1 NEWTON'S FIRST AND SECOND LAWS

Newton's Three Laws

- Newton's 3 laws define some of the most fundamental things in physics including:
  - Why things fall down
  - Why objects move
  - And much more!

5.a Point mass

1- An object can be considered as a point object if during motion in a given time, it covers distance much greater than its own size.
2- Object with zero dimension considered as a point mass.
3- Point mass is a mathematical concept to simplify the problems.

5.b Inertia

1- Inherent property of all the bodies by virtue of which they cannot change their state of rest or uniform motion along a straight line by their own is called inertia.
2- Inertia is not a physical quantity; it is only a property of the body which depends on mass of the body.
3- Inertia has no units and no dimensions
4- Two bodies of equal mass, one in motion and another is at rest, possess same inertia because it is a factor of mass only and does not depend upon the velocity.

5.c Frame of Reference

1- A frame in which an observer is situated and makes his observations is known as his "Frame of reference".
2- The reference frame is associated with a co-ordinate system and a clock to measure the position and time of events happening in space. We can describe all the physical quantities like position, velocity, and acceleration etc. of an object in this coordinate system.
3- Frame of reference are of two types:
i- Inertial frame of reference and
ii- Non-inertial frame of reference.

(i) **Inertial frame of reference:**

a- A frame of reference which is at rest or which is moving with a uniform velocity along a straight line is called an inertial frame of reference.
b- In inertial frame of reference Newton's laws of motion holds good.
c- Inertial frame of reference are also called unaccelerated frame of reference or Newtonian or Galilean frame of reference.
d- Ideally no inertial frame exist in universe. For practical purpose a frame of reference may be considered as inertial if it's acceleration is negligible with respect to the acceleration of the object to be observed.
e- To measure the acceleration of a falling apple, earth can be considered as an inertial frame.
f- To observe the motion of planets, earth can not be considered as an inertial frame but for this purpose the sun may be assumed to be an inertial frame.

Example: The lift at rest, lift moving (up or down) with constant velocity, car moving with constant velocity on a straight road. we can assume that the ground is an inertial frame provided we can neglect Earth's astronomical motions (such as its rotation).

(ii) **Non inertial frame of reference:**

a- Accelerated frame of references are called non-inertial frame of reference.

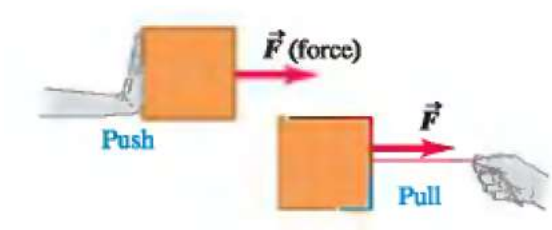
b- Newton's laws of motion are not applicable in non-inertial frame of reference.

Example: Car moving in uniform circular motion, lift which is moving upward or downward with some acceleration, plane which is taking off.

Newton's Laws: Forces and Motion

<u>Some properties of Forces</u>:

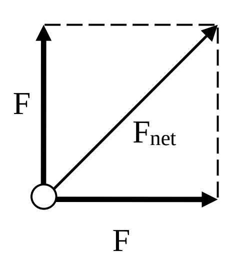
1- A force is a push or pull exerted by one object on another.
2- A force is an interaction between two objects or between an oject and its environment.
3- A force is a <u>vector</u>: it has a magnitude and a direction. Forces add like vectors, not like scalars.
4- The force may, or may not, be in contact with the body, e.g. magnetic and gravitational fields. Whenever we are pushing or pulling, lifting or binding, twisting or tearing, stretching or squeezing, we are exerting a force.



The forces can:

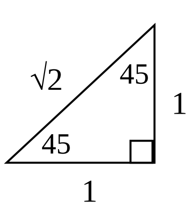
1. Change the state of rest or motion of a body (speed of a body).
2. Change the direction of movement of a body.
3. Change the size or shape of a body.

Example: Two forces, labeled $\mathbf{F}_1$ and $\mathbf{F}_2$, are both acting on the same object. The forces have the same magnitude $|\vec{F}_1| = |\vec{F}_2| = F$ and are 90° apart in direction:



$$\vec{F}_{net} = \vec{F}_{total} = \sum \vec{F} = \vec{F}_1 + \vec{F}_2$$

($\sum \vec{F}$ means "sum of all the forces on the object")



$$\Rightarrow F_{net} = \sqrt{2}\, F \quad \text{(NOT 2F)}$$

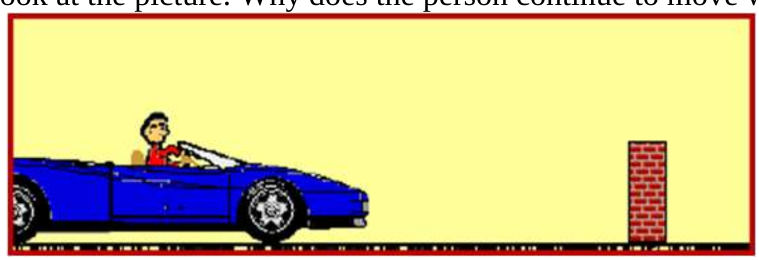
Isaac Newton (British, 1642-1727) first figured out the precise relationship between forces and motion.

• **Newton's First Law** (**NI**):"*If no force acts on a body,* $\vec{F}_{net} = 0$*, the body's velocity cannot change; that is, the body cannot accelerate"*. If the net force acting on an object is zero, then it has constant velocity. $\vec{F}_{net} = 0 \Leftrightarrow \vec{v} = \text{constant}$

- ➢ IF the object experiences NO net external force….
- Resting objects remain at rest.
- Moving objects move at a constant velocity.

- ❑ ***"An object will continue in a state of rest or uniform motion in a straight line unless an external force acts upon it". This is also called the Law of Inertia.***

Example: Take a look at the picture. Why does the person continue to move when the car stops?



Answer: The person keeps moving because no force pushed on it. The force of the wall only acted on the car.

• **Newton's Second Law** (**NII**): The net force on a body is equal to the product of the body's mass and its acceleration.

$$\vec{F}_{net} = m\,\vec{a}$$

(Notice! F_{net}, not F, in this equation. There may be many forces acting on an object, but there is only one *net* force.) Only forces that act on *that* body are to be included in the vector sum, not forces acting on other bodies that might be involved in the given situation.

The net force on an object causes the object to accelerate (change its velocity).

- ➢ The acceleration of an object is:
- Directly proportional to the net external force acting on it, and…
- Inversely proportional to its mass

Units of force: $[F]=[m][a]=kg\cdot m/s^2 = 1$ newton = 1 N
A force of 1 N is about 0.22 pounds. A small plum weighs about 1 Newton.

Things to notice about NII:

- If $\vec{F}_{net}=0$, then $\vec{a}=0$ and velocity = constant. (1^{st} Law).
- The vector $\vec{a}$ has the same direction as $\vec{F}_{net}$.
- The magnitude of the acceleration *a* is proportional to 1/m (at constant F_{net}).
- $\vec{F}_{net}=\sum\vec{F} = m\,\vec{a}$ is a **vector** equation
  $\Rightarrow \sum F_x = m\,a_x \;,\; \sum F_y = m\,a_y$

Before Newton, everyone thought (incorrectly!): "Force causes motion." **WRONG**!
(You can have motion without any force causing the motion. Example: glider on an air track.)

After Newton, "Force causes *changes* in motion." RIGHT!

Example: Imagine you are riding on a sled. You are accelerating at a rate of 4 m/s² and have a total mass of 40 kg. **What is the net force exerted?** (Remember: **F=*ma***)
Answer:

$$F = ma \quad \Rightarrow \quad F = (40\text{ kg})(4\text{m/s}^2) = 160\text{ N}$$

Example: While two forces act on it, a particle of mass *m* = 3.2 kg is to move continuously with velocity (3 **i**−4 **j) m/s**. One of the forces is F_1 = (2 **i**−6 **j) N**. What is the other force?
Answer: Newton's Second Law tells us that if **a** is the acceleration of the particle, then (as there are only two forces acting on it) we have:

$$\boldsymbol{F}_1 + \boldsymbol{F}_2 = m\,\boldsymbol{a}$$

But here the acceleration of the particle is zero!! (Its velocity does not change.) This tells us that

$$\boldsymbol{F}_1 + \boldsymbol{F}_2 = 0 \qquad \Rightarrow \qquad \boldsymbol{F}_1 = -\boldsymbol{F}_2$$

and so the second force is

$$\boldsymbol{F}_1 = -\boldsymbol{F}_2 = (-2\text{ N})\,\mathbf{i} + (6\text{ N})\,\mathbf{j}$$

This was a simple problem just to see if you're paying attention!

Example: A 3.0 kg mass undergoes an acceleration given by a = (2.0 i + 5.0 j) m s^2. Find the resultant force F and its magnitude.
Answer: Newton's Second Law tells us that the resultant (net) force on a mass *m* is $\vec{F}_{net}=m\,\vec{a}$. So here we find:

$$\mathbf{F}_{net} = m\mathbf{a} = (3.0\,\text{kg})(2.0\mathbf{i}+5.0\mathbf{j})\,\frac{\text{m}}{\text{s}^2} = (6.0\mathbf{i}+15.\mathbf{j})\,\text{N}$$

The magnitude of the resultant force is

$$|F_{net}| = \sqrt{(6.0)^2+(15.)^2} = 16.2\text{ N}$$

Example: A 4.0 kg object has a velocity of 3.0i m/s at one instant. Eight seconds later, its velocity is (8.0i+10.0j) m/s. Assuming the object was subject to a constant net force, find (a) the components of the force and (b) its magnitude.

Answer:

(a) We are told that the (net) force acting on the mass was constant. Then we know that its acceleration was also constant, and we can use the constant–acceleration results from the previous chapter. We are given the initial and final velocities so we can compute the components of the acceleration:

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{[(8.0\,\frac{m}{s}) - (3.0\,\frac{m}{s})]}{(8.0\,s)} = 0.63\,\frac{m}{s^2}$$

and

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{[(10.0\,\frac{m}{s}) - (0.0\,\frac{m}{s})]}{(8.0\,s)} = 1.3\,\frac{m}{s^2}$$

We have the mass of the object, so from Newton's Second Law we get the components of the force:

$$F_x = ma_x = (4.0\,kg)(0.63\,\tfrac{m}{s^2}) = 2.5\,N$$

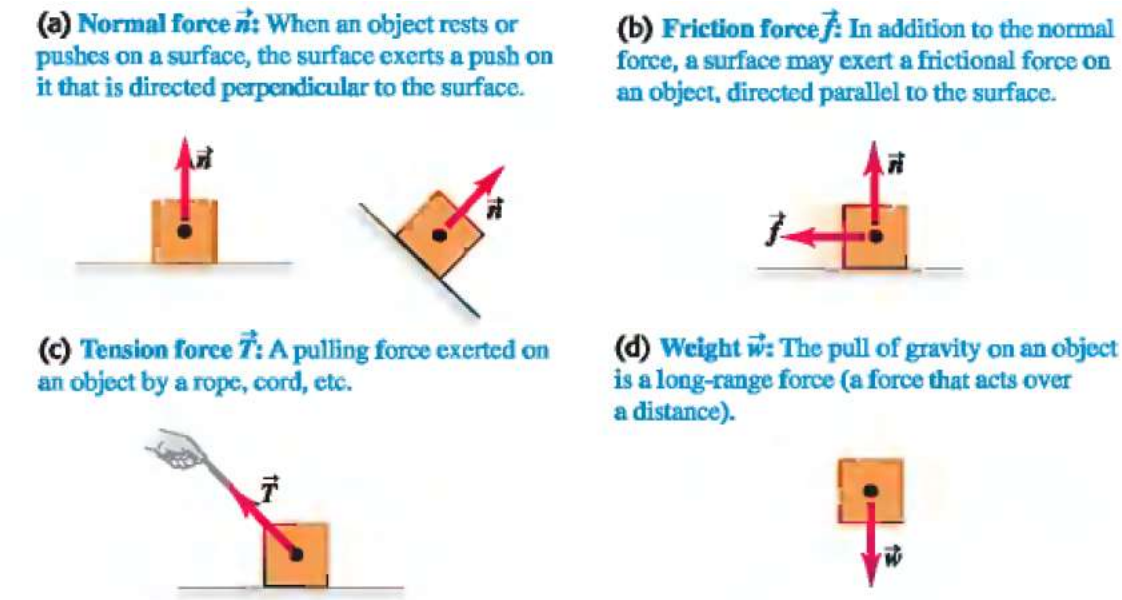
$$F_y = ma_y = (4.0\,kg)(1.3\,\tfrac{m}{s^2}) = 5.0\,N$$

(b) The magnitude of the (net) force is $F = \sqrt{F_x^2 + F_y^2} = \sqrt{(2.5\,N)^2 + (5.0\,N)^2} = 5.6\,N$ and its direction θ is given by

$$\tan\theta = \frac{F_y}{F_x} = \frac{5.0}{2.5} = 2.0 \quad \Longrightarrow \quad \theta = \tan^{-1}(2.0) = 63.4^\circ$$

(The question didn't ask for the direction but there it is anyway!)

5-2 SOME PARTICULAR FORCES

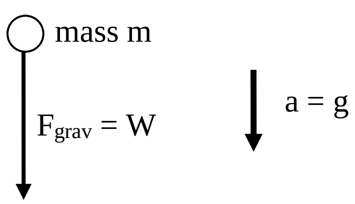


(I) **<u>Gravitational force</u>**:

Definition: The WEIGHT of an object = the force of gravity on the object.

$$W = mg \quad \text{Why?}$$

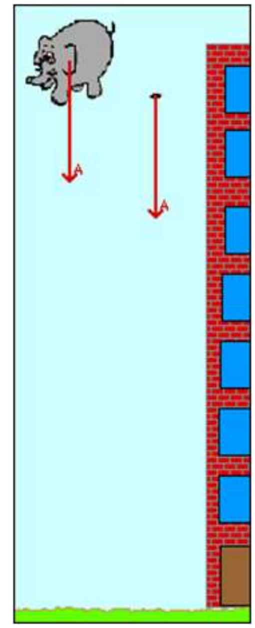
Recall this experimental fact: when object is in *free-fall*, meaning $F_{net} = F_{gravity}$, then $a = g$.
So in this situation (free-fall), $F_{net} = ma \Rightarrow F_{grav} = mg$



How big a force is 1 N? If m = 1 kg, $W = F_{grav} = mg = (1\ kg)(9.8\ m/s^2) = 9.8$ N. A kilogram mass has a weight of about 10 N, which is about 2.2 lbs. The pound (lb) is the English unit of force: 1 lb = 4.44 N.

<u>Summary of weight and mass</u>

	Mass (m)	Weight (W = mg)
Definition	The quantity of a matter contained in a body (scalar quantity)	The force of gravity due to the pull of earth (vector quantity)
Measured	With a beam balance or an electronic balance	With a spring balance
Properties	Remains constant everywhere	Changes from one place to another
SI Units	Kilograms (kg)	Newton (N)



Example: Look at the figure. Why do the elephant and feather hit the ground at the same time? (assume no air resistance)
Answer: The elephant and feather hit the ground at the same time because they have equal accelerations- gravity (9.8 m/s²).
Q: What will be different between the elephant and the feather when they hit the ground?
Answer: Let's assume the mass of the elephant is 100,000 kg and the mass of the feather is 1 kg. They both accelerate (due to gravity) at a rate of 9.8 m/s².
What is their net forces?

(elephant)	(feather)
F=ma	F=ma
F=(100,000kg)(9.8 m/s²)	F=(1kg)(9.8 m/s²)
F=980,000 N	F=9.8 N

This means the elephant will hit the ground with a force 100,000 times bigger than the feather. That will definitely leave a mark!

Example: If a man weighs 875N on Earth, what would he weigh on Jupiter, where the free–fall acceleration is 25.9 m/s^2?
Answer: The weight of a mass *m* on the earth is *W* = *mg* where *g* is the free–fall acceleration on Earth. The mass of the man is:

$$m = \frac{W}{g} = \frac{875\,\mathrm{N}}{9.80\,\frac{\mathrm{m}}{\mathrm{s}^2}} = 89.3\,\mathrm{kg}$$

His weight on Jupiter is found using g_{Jupiter} instead of *g*:

$$W_{\mathrm{Jupiter}} = m g_{\mathrm{Jupiter}} = (89.3\,\mathrm{kg})(25.9\,\tfrac{\mathrm{m}}{\mathrm{s}^2}) = 2.31 \times 10^3\,\mathrm{N}$$

The man's weight on Jupiter is 2.31×10^3 N.
(The statement of the problem is a little deceptive; Jupiter has no solid surface! The planet will indeed pull on this man with a force of 2.31 ×103 N, but there is no "ground" to push back!)

Rules for drawing "Free-body diagram" or force diagram:

1) Draw a blob representing the object.
2) Draw only the forces acting **on** the object (not the forces which the object exerts on others).
3) Indicate strength and direction of forces on the object by drawing arrows coming out of the object.
4) Use symbols to represent the magnitudes of the forces (Don't worry about +/- signs. The forces arrows show the directions of the forces already.)

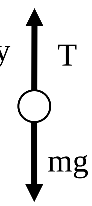
Example: Object with mass m, hanging by a cord. Magnitude of force exerted by the cord = tension T

Three different situations:

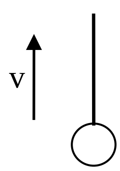
I. Hanging stationary: velocity v = 0 $\Rightarrow$

$a = 0 \Rightarrow F_{net} = 0 \Rightarrow T = mg$

free-body diagram:



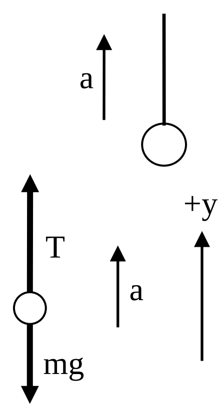
II. Pulled upward with constant velocity **v**.



$\mathbf{v}$ = constant $\Rightarrow a = 0 \Rightarrow F_{net} = 0 \Rightarrow T = mg$ (again!)

same free-body diagram as in case I

III. Object is **accelerated** upward (may be <u>moving</u> upward OR downward!)



Question: What is the magnitude of the tension T in the cord?
To analyze this situation:
Step 1: Draw free-body diagram showing forces (show direction of acceleration off to one side of diagram.)

Step 2: Choose a coordinate system and a + direction (always best to choose the direction of the acceleration as the +direction)

Step 3: Write down equations $\sum F_x = m a_x$, $\sum F_y = m a_y$
In this example, just the y-equation is needed:
$+T - mg = ma \Rightarrow T = mg + ma = m(a+g)$

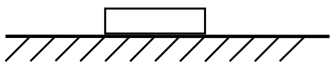
Example:

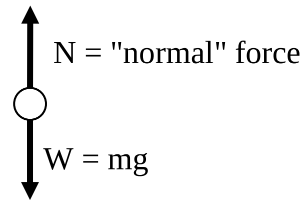
Glider on an air track with **v** = constant OR book at rest on a table. In both cases,

$a = 0 \Leftrightarrow F_{net} = 0$

N = force exerted on book by table OR force on glider by air track

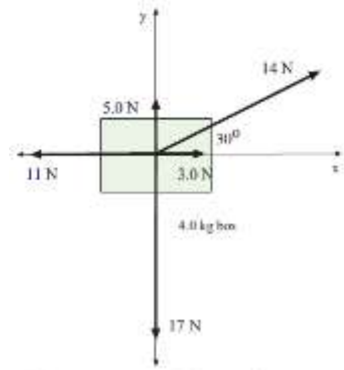
"normal" means "perpendicular"





Since $F_{net} = 0 \Rightarrow N = mg \Leftrightarrow$ upward force on book exactly cancels downward force on book. How does the table know to exert an upward force that is exactly the right size to cancel the weight of the book? Answer: the table is not perfectly rigid; it is flexible, it is slightly springy. The table is slightly compressed by the weight of the book, and it pushes back on the book, exactly like a compressed spring. The heavier the book, the more the table-spring compresses, and the more it pushes upward on the book.

Example: Five forces pull on the 4.0 kg box in figure. Find the box's acceleration (a) in unit–vector notation and (b) as a magnitude and direction.



Five forces pull on a box

Answer:

(a) Newton's Second Law will give the box's acceleration, but we must first find the sum of the forces on the box. Adding the *x* components of the forces gives:

$$\sum F_x = -11\,\text{N} + 14\,\text{N}\cos 30^\circ + 3.0\,\text{N} = 4.1\,\text{N}$$

(two of the forces have only *y* components). Adding the *y* components of the forces gives:

$$\sum F_y = +5.0\,\text{N} + 14\,\text{N}\sin 30^\circ - 17\,\text{N} = -5.0\,\text{N}$$

So the net force on the box (in unit-vector notation) is

$$\sum \mathbf{F} = (4.1\,\text{N})\mathbf{i} + (-5.0\,\text{N})\mathbf{j}\ .$$

Then we find the *x* and *y* components of the box's acceleration using $\vec{a} = \vec{F}_{net}/m$

$$\mathbf{a} = \sum \mathbf{F}/m: \quad a_x = \frac{\sum F_x}{m} = \frac{(4.1\,\text{N})}{(4.0\,\text{kg})} = 1.0\,\frac{\text{m}}{\text{s}^2}, \qquad a_y = \frac{\sum F_y}{m} = \frac{(-5.0\,\text{N})}{(4.0\,\text{kg})} = -1.2\,\frac{\text{m}}{\text{s}^2}$$

So in unit–vector form, the acceleration of the box is

$$\mathbf{a} = (1.0\,\tfrac{\text{m}}{\text{s}^2})\mathbf{i} + (-1.2\,\tfrac{\text{m}}{\text{s}^2})\mathbf{j}$$

(b)The acceleration found in part (a) has a magnitude of

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(1.0\,\tfrac{\text{m}}{\text{s}^2})^2 + (-1.2\,\tfrac{\text{m}}{\text{s}^2})^2} = 1.6\,\tfrac{\text{m}}{\text{s}^2}$$

and to find its direction θ we calculate

$$\tan\theta = \frac{a_y}{a_x} = \frac{-1.2}{1.0} = -1.2$$

which gives us:

$$\theta = \tan^{-1}(-1.2) = -50^\circ$$

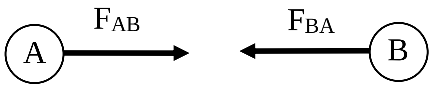
Here, since a_y is negative and a_x is positive, this choice for θ lies in the proper quadrant.

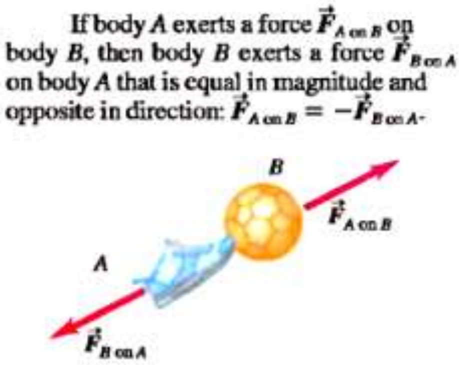
Chapter 5
Force and Motion—II

5-3 APPLYING NEWTON’S LAWS

• **Newton's Third Law (NIII)**: If body A exerts a force on body B ($=\vec{F}_{BA}=\vec{F}_{\text{on B by A}}$), then B exerts an equal and opposite force on A ($=\vec{F}_{AB}=\vec{F}_{\text{on A by B}}$).

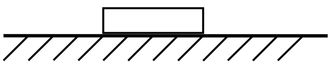
$$\underbrace{\vec{F}_{BA}=-\vec{F}_{AB}}_{\text{2 forces on 2 different objects}}$$

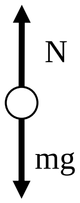




Forces always act between ***pairs*** of bodies. A force on one body is always being caused by a second body. NIII says that the force from one body on the other is always accompanied by a force from the other on the first, and the two forces of this "**action-reaction**" pair are always equal in magnitude and opposite in direction. Notice that the 2 forces of the action-reaction pair act on ***different*** bodies.
Example of equal and opposite forces that are NOT the action-reaction pair of NIII:

Book on a table:



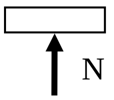


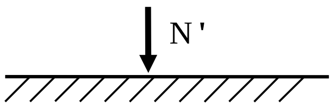
N = mg
(these are NOT the 2 forces of NIII, since both act on the same object.)

So which forces make up the action-reaction pairs in this situation ?

$\vec{N}$ = force on book due to table

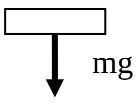
$\vec{N}'$ = force on table due to book



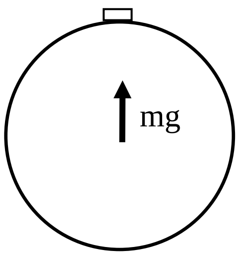


$\vec{N}=-\vec{N}'$, says NIII

$m\vec{g}$ = force on book due to planet earth (gravity)



Book exerts same size force upward on the whole earth.



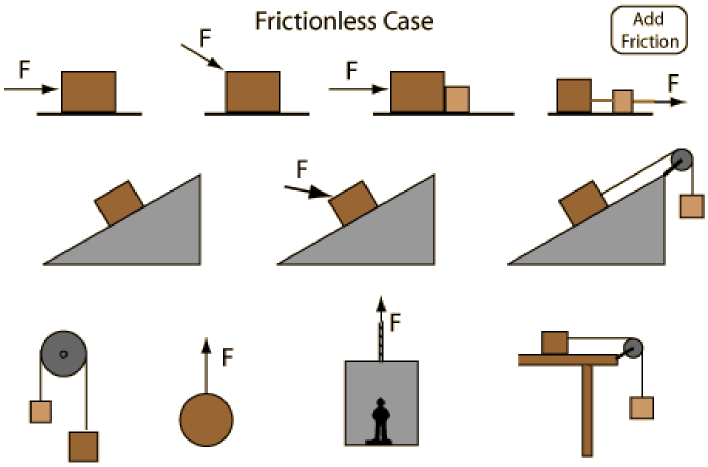
Where are we so far?

We have introduced Newton's 3 **Laws.** Laws are statements which are true <u>always</u>. There are no derivations of Newton's Laws; in particular there is no derivation of $\mathbf{F}_{net} = m\,\mathbf{a}$. These laws are taken as <u>assumptions</u> or <u>axioms</u> of the theory of Newtonian mechanics. We believe these laws are correct because all of the consequences of these laws are found to agree with experiment. Remember, the philosophy of science is this: "The final test of the validity of any idea is experiment."

In Physics, the only statements that are true <u>always</u> are definitions (like $\bar{a} = \frac{d\bar{v}}{dt}$) and laws. You should memorize definitions and laws.

An equation like $x = x_o + v_o t + \frac{1}{2} a t^2$ is not a law, because it is not always true. (This particular equation is only true when acceleration *a* = constant.)

Some examples of standard "building block" problems which help build understanding of the principles of mechanics.

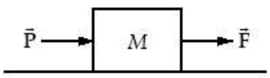


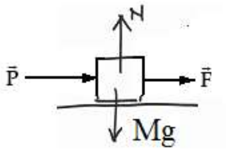
A Newton pair of forces has the following properties:
(a) The two forces act on two ***different*** bodies.
(b) Both forces are always of the ***same type*** (i.e. both gravitational, both electrostatic, etc.).
(c) The forces are ***equal*** in magnitude.
(d) The forces act in ***opposite*** directions.

<u>Action</u>	<u>Reaction</u>	
You push back on **F**loor	**F**loor pushes forward on **Y**ou	(explains how a person walks)
Tires push back on **R**oad	**R**oad pushes forward on **T**ires	(explains how a car moves)
Rocket pushes back on **G**as	**G**as pushes forward on **R**ocket	(explains how a jet moves)
Book pushes down on **T**able	**T**able pushes up on **B**ook	
Earth pulls down on **M**oon	**M**oon pulls up on **E**arth	
Magnet pulls left on **N**ail	**N**ail pulls right on **M**agnet	

Worked Examples

Example: A heavy packing case of mass M is pulled by a force **F** and pushed by a force **P**, both acting horizontally. The equation of motion of the packing case is:





Answer: Use the NII M **a** = **F** + **P** for the free body diagramme

$\Rightarrow$ M a_x = F + P

M a_y = N - Mg = 0 (i.e. no motion in y-direction) $\Rightarrow$ N = Mg

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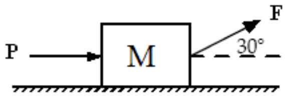
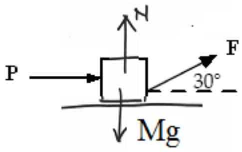
Example: A heavy packing case, of mass M, is pulled by a force **F** exerted at a 30° angle, and pushed by a force **P** exerted horizontally. The equation of motion of the packing case is





Answer: Use the NII M **a** = **F** + **P** for the free body diagramme

$\Rightarrow$ M a_x = F cos 30° + P and

M a_y = F sin 30° + N - Mg = 0 (i.e. no motion in y-direction) $\Rightarrow$ N = Mg- F sin 30°

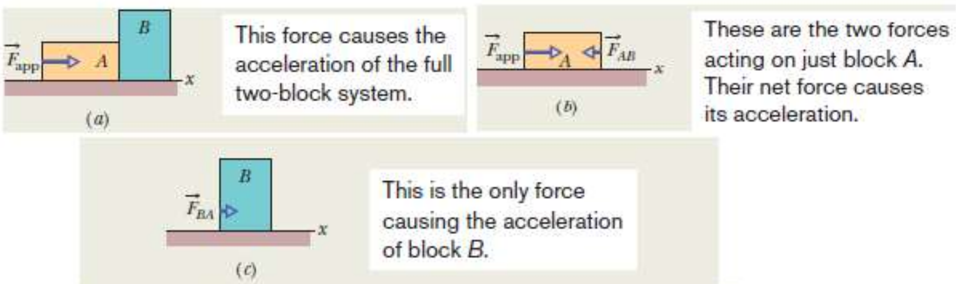
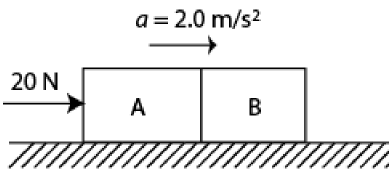
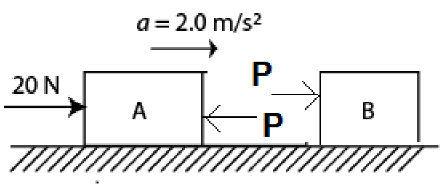


Figure 5-18 (*a*) A constant horizontal force $\vec{F}_{app}$ is applied to block *A*, which pushes against block *B*. (*b*) Two horizontal forces act on block *A*. (*c*) Only one horizontal force acts on block *B*.

Example: A constant force F of magnitude 20 N is applied to block A of mass m = 4.0 kg, which pushes block B as shown in the figure. The block slides over a frictionless flat surface with an acceleration of 2.0 m/s^2. What is the net force on block B?

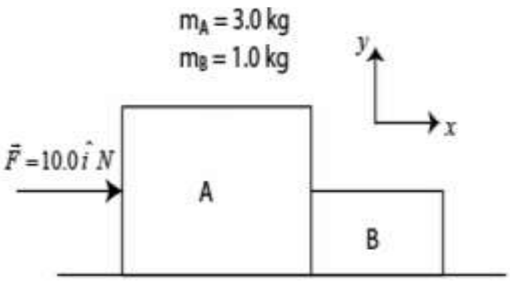


Answer: Consider the net force on block B is P is shown in the following figure.



Then applying the equation of motion on A will be:

$$m_A a_A = F - P \quad \Rightarrow \quad 4 \times 2 = 20 - P \quad \Rightarrow \quad P = \underline{12\ \text{N}}$$



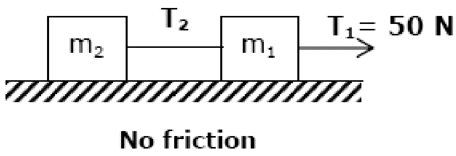
Example: Two boxes A and B ($m_A = 3.00$ kg and $m_B = 1.00$ kg) are in contact on a horizontal frictionless surface and move along the x-axis (shown in the figure). A horizontal force $\vec{F} = 10.0\hat{\text{i}}$ N is applied on Box A. The net force acting on A is $\vec{F}_1$ and on B is $\vec{F}_2$. Calculate $|\vec{F}_1|$ and $|\vec{F}_2|$.

Answer: The equation of motion for the combined masses is:

$$(m_A + m_B)a = F; \quad \Rightarrow \quad a = \frac{10}{3+1} = 2.5 \text{ m/s}^2$$

For the large mass: $3a = 10 - P \quad \Rightarrow \quad P = 2.5$ N; $\quad \Rightarrow \quad F_1 = 10 - P = \underline{7.5 \text{ N}}$

For the small mass: $1a = P \quad \Rightarrow \quad P = 2.5$ N; $\quad \Rightarrow F_2 = P = \underline{2.5 \text{ N}}$



Example: Two masses m_1 (= 2.0 kg) and m_2 (= 3.0 kg) are connected as shown in the figure. Find the tension T_2 if the tension T_1 = 50.0 N.

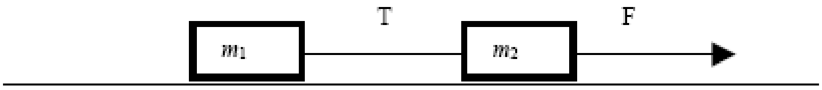
Answer: The equation of motion for whole masses is:

$$(m_1 + m_2)a = F = T_1; \quad \Rightarrow \quad a = \frac{T_1}{(2+3)} = \frac{50}{5} = 10 \text{ m/s}^2\,;$$

Then for m_1 alone,

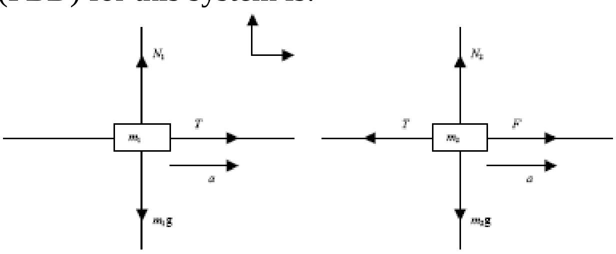
$$\Rightarrow \quad (m_1)a = T_1 - T_2 \quad \Rightarrow \quad T_2 = T_1 - m_1 a = 50 - 2\times 10 = \underline{30 \text{ N}}$$

Example 1 Consider a system consisting of two blocks (masses = m_1 & m_2) attached by a light cord on a smooth, flat table pulled to the right by a force F. Find the tension in the connecting cord and the acceleration of the system. Consider a) frictionless table b) table is rough with coefficient of kinetic friction μ_k .



Answer:

a) The free body diagram (FBD) for this system is:



FBD₁ $\sum F_x = T = m_1 a \quad (1)$; $\sum F_y = N_1 - m_1 g = 0$; $\therefore N_1 = m_1 g$

FBD₂ $\sum F_x = F - T = m_2 a \quad (2)$; $\sum F_y = N_2 - m_2 g = 0$; $\therefore N_2 = m_2 g$

Now if we combine equations (1) and (2):

$$m_1 a = T = F - m_2 a \therefore m_1 a = F - m_2 a$$

$$(m_1 + m_2)a = F$$

$$a = \frac{F}{m_1 + m_2}$$

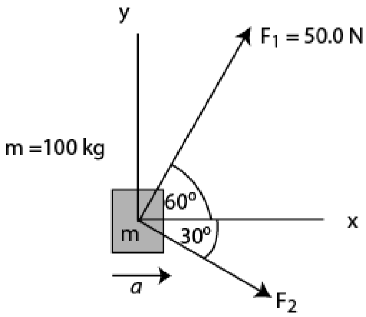
Note that this is in the form: $a = \frac{F}{\sum m}$

Plugging this value for acceleration back into equation (1) yields:

$$T = m_1\left(\frac{F}{m_1 + m_2}\right)$$

Note that this is in the form: $F = ma$

Example: Two students are dragging a box (m=100 kg) across a horizontal frozen Lake (i.e. assume no friction). The first student pulls with force F_1=50.0 N, while the second pulls with force F_2. The box is moving in the x-direction with acceleration a (see Figure). Assuming that friction is negligible, what is F_2?



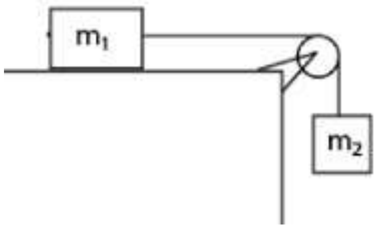
Answer: On x-axis, one finds

$Ma_x = F_1 \cos 60^\circ + F_2 \cos 30^\circ$; (not good to solve for F_2)

On y-axis, one finds:

$$Ma_y = 0 = F_1 \sin 60^\circ + F_2 \sin 30^\circ \quad \Rightarrow \quad F_2 = F_1\left(\frac{\sin 60^\circ}{\sin 30^\circ}\right) = \underline{86.6 \text{ N}};$$

Example: The figure shows a block of mass m_1 = 3.00 kg connected to a block of mass m_2 by a string passing over a frictionless pulley and placed on a frictionless horizontal table. The string and pulley have negligible mass. When m_1 is released from rest and accelerate at 1 m/s^2 across the table. What are



i- the tension in the string

ii- mass of m_2.

Answer: Using the usual coordinate system (*right* = +*x* and *up* = +*y*) for both blocks has the important consequence that for the m_1 = 3.0 kg block to have a positive acceleration ($a > 0$), block m_2 must have a negative acceleration of the same magnitude ($-a$). Thus, applying Newton's second law to the two blocks, we have

$$T = m_1 a = (3.0 \text{ kg})(1.0 \text{ m/s}^2) \quad \text{along x-axis}$$

$$T - m_2 g = m_2(-1.0 \text{ m/s}^2) \quad \text{along y-axis}$$

i- The first equation yields the tension T = 3.0 N.

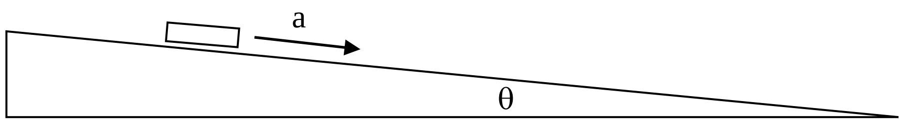
ii- The second equation yields the mass m_2 = 3.0/8.8 = 0.34 kg.

Example: Drawing a free-body diagram and applying $\vec{F}_{net} = m\,\vec{a}$

A glider on an air track tilted an angle θ to the horizontal. <u>No friction</u>.

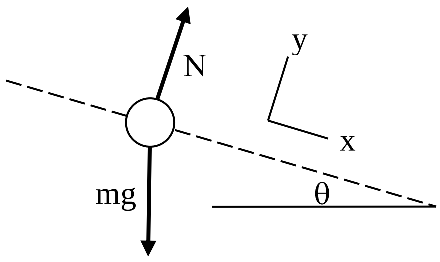
i- What is the magnitude of acceleration: a = ?

ii- What is the magnitude of the normal force: N = ?



Step 1: Draw free-body diagram.

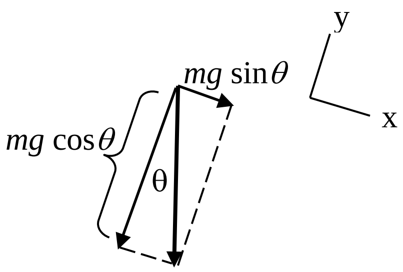
(only 2 forces here: normal force N and weight W = mg)



Step 2: Choose coordinate system. Here I chose tilted xy coordinates because I want the +x direction to be along the direction of the acceleration.

Step 3: Write down equations $\sum F_x = m\,a_x$, $\sum F_y = m\,a_y$

Notice that :
x-component of weight = +mg sin θ
y-component of weight = –mg cos θ (minus sign!)



$a_x = a$, $a_y = 0$ (acceleration is along +x-axis)

$\sum F_y = m\,a_y \Rightarrow \quad +N - mg\cos\theta = 0$

$N = mg\cos\theta$

$\sum F_x = m\,a_x \Rightarrow \quad +mg\sin\theta = m\,a$, m's cancel so $a = g\sin\theta$

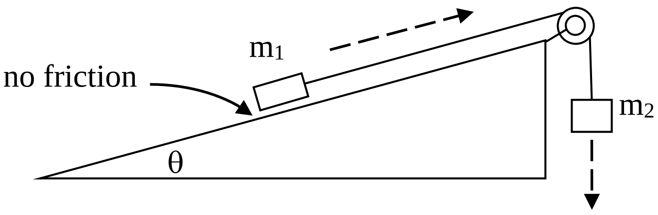
Notice that as $\theta \to 0$ (track becomes horizontal), $\cos\theta \to 1$, $\sin\theta \to 0$, $N \to mg$, $a \to 0$, as expected.

NOTICE that N is NOT equal to mg. The equation N = mg is only true in a very special situation: when the mass m is NOT accelerating and is sitting on a horizontal surface.

Dubson's Law: In general, $N \neq mg$. The statement N = mg is NOT a law. It is only true in special cases.

Solving F_{net} = m a problems with multiple bodies

Problem: A mass m_1 is pulled up a frictionless incline by a string over a pulley and a hanging mass m_2.

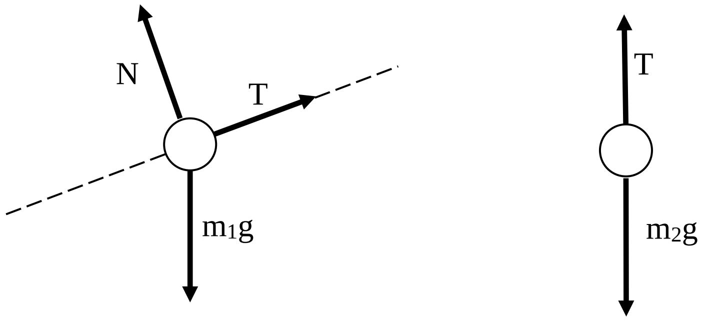


We <u>know</u> m_1, m_2, and the angle θ.

We <u>seek</u> :
T = tension in the cord,
a = acceleration of the mass,
N = normal force on m_1

Step 1

Draw a free-body diagram for each moving object.



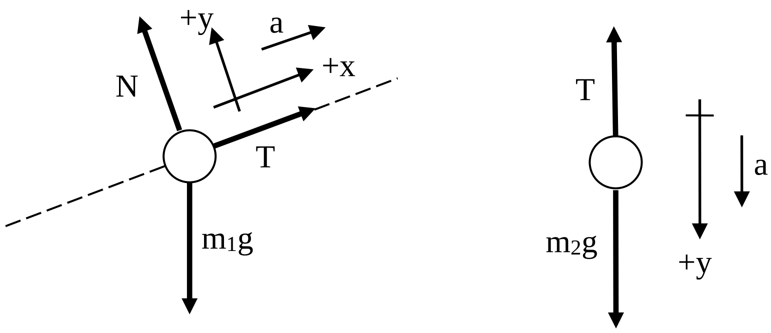
Label the force arrows with the <u>magnitudes</u> of the forces.

Notice that T is the same for both objects (by NIII).

Use m_1, m_2, in the diagrams, not m.

Step 2

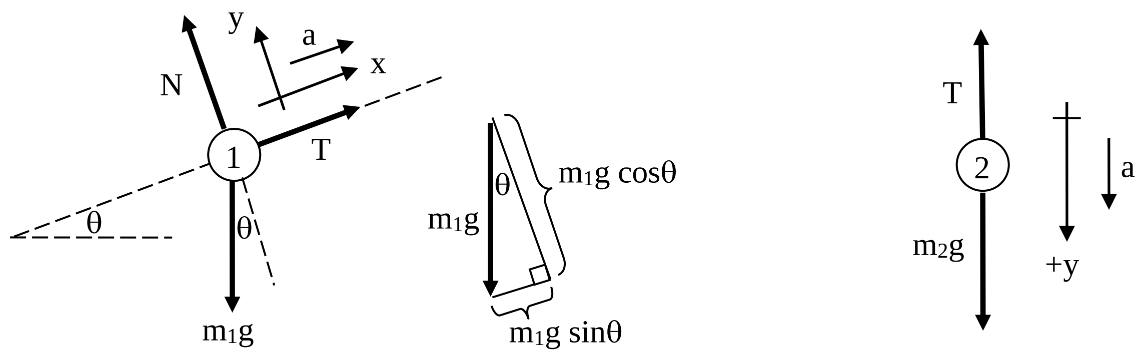
For each object, choose x·y axes so that the acceleration vector **a** is in the (+) direction.



Notice that we can choose different axes for the different objects. And we can tilt the axes if necessary.

Step 3

For each object, write the equations $\sum F_x = m\,a_x$, $\sum F_y = m\,a_y$



m_1 : x-eqn (1) $+T - m_1 g \sin\theta = m_1 a$

y-eqn (2) $+N - m_1 g \cos\theta = 0$

m_2 : (3) $+m_2 g - T = m_2 a$

Notice that $|\vec{a}| = a$ is the same for both m_1 and m_2 since they are connected by a string that doesn't stretch.

Now we have a messy algebra problem with 3 equations in 3 unknowns:

(1) $+T - m_1 g \sin\theta = m_1 a$

(2) $+N - m_1 g \cos\theta = 0$ The unknowns are T , N , and a.

(3) $+m_2 g - T = m_2 a$

We can solve for N right away. Eqn (2) $\Rightarrow$ $\boxed{N = m_1 g \cos\theta}$

Now we have 2 equations [(1) & (3)] in 2 unknowns (a & T).

Solve (1) for T and plug into (3) to get an equation without an T :

(1) $\Rightarrow$ $T = m_1 g \sin\theta + m_1 a = m_1(g \sin\theta + a)$

(3) $\Rightarrow$ $m_2 g - [m_1(g \sin\theta + a)] = m_2 a$

Now solve this last equation for a:

$$m_2 g - m_1 g \sin\theta = +m_1 a + m_2 a = a(m_1 + m_2)$$

$$a = \frac{m_2 g - m_1 g \sin\theta}{(m_1 + m_2)}$$

Finally, if you have any strength left, we can solve for tension T by plugging our expression for acceleration a back into either (1) or (3) and solving for T. From (3), we have

$T = m_2 g - m_2 a = m_2(g - a)$.

Plugging in our big expression for *a*, we get

$$T = m_2(g-a) = m_2\left(g - \frac{m_2 g - m_1 g \sin\theta}{(m_1+m_2)}\right)$$

We can simplify:

$$T = m_2\left(\frac{g(m_1+m_2)}{(m_1+m_2)} - \frac{m_2 g - m_1 g \sin\theta}{(m_1+m_2)}\right) = m_2\left(\frac{m_1 g + m_2 g - m_2 g + m_1 g \sin\theta}{(m_1+m_2)}\right)$$

$$= m_2\left(\frac{m_1 g + m_1 g \sin\theta}{(m_1+m_2)}\right) = \frac{m_2 m_1 g}{(m_1+m_2)}(1 + \sin\theta)$$

Do these expressions make sense? Let's check some *limits*.

If $m_1 = 0$, then m_2 should be freely falling with $a = g$ and the tension T should be zero. Check that this is so.

If $m_2 = 0$, then m_1 should slide down the incline with acceleration $a = -g \sin\theta$ (since it would be accelerating in the negative direction). Check that this is so.