Chapter 4 –I

Motion in Two and Three Dimensions

Please review the following sections:

4-1 POSITION AND DISPLACEMENT
4-2 AVERAGE VELOCITY AND INSTANTANEOUS VELOCITY
4-3 AVERAGE ACCELERATION AND INSTANTANEOUS ACCELERATION

4-4 PROJECTILE MOTION

KINEMATICS IN TWO DIMENSIONS
PREVIEW

Two-dimensional motion includes objects which are moving in two directions at the same time, such as a *projectile*, which has both horizontal and vertical motion. These two motions of a projectile are completely independent of one another, and can be described by *constant velocity* in the horizontal direction, and *free fall* in the vertical direction. Since the two-dimensional motion described in this chapter involves only constant accelerations, we may use the *kinematic equations*.

Equations of Kinematics in Two Dimensions

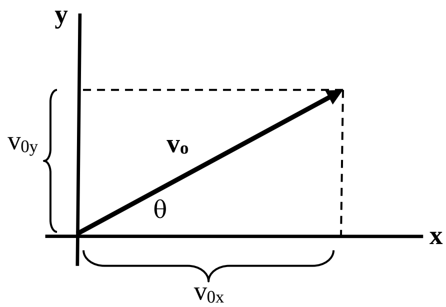
Chapter 2 dealt with displacement, velocity, and acceleration in *one dimension*. But if an object moves in the horizontal and vertical direction at the same time, we say that the object is is moving in *two dimensions*. We define any quantity which is horizontal with an x (such as v_x and a_x), and we define any quantity which is vertical with a y (such as v_y and a_y.)

Important Terms

Projectile: any object that is projected by a force and continues to move by its own inertia
Range of a projectile: the horizontal distance between the launch point of a projectile and where it returns to its launch height
Trajectory: the path followed by a projectile
Time of flight: The total time taken by the projectile to go up and come down to the same level from which it was projected is called time of flight.

Review of 1D motion

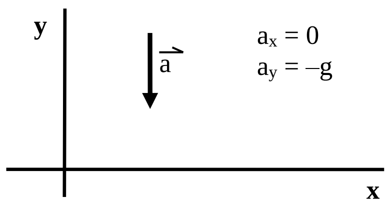
Consider a projectile fired from a cannon, with an initial velocity $\vec{v}_0$ with a direction of θ above the horizontal.



$$v_{0x} = v_0 \cos\theta$$

$$v_{0y} = v_0 \sin\theta$$

Acceleration is a vector, and can have any direction. But in the special case of acceleration due solely to gravity, the acceleration is always straight down.



Review of 1D motion:

$$a = \frac{dv}{dt}, \quad v = \frac{dx}{dt},$$

From these two equations, we can derive, for the special case a = constant,

(a) $v = v_o + at$

(b) $x = x_o + v_o t + (1/2)at^2$

(c) $v^2 = v_o^2 + 2a(x - x_o)$

(d) $\bar{v} = \dfrac{v_o + v}{2}$

x_o , v_o = initial position, initial velocity x, v = position, velocity at time t

Suppose that a = 0 . In this case v = constant, and $v = \bar{v} = \dfrac{\Delta x}{\Delta t} = v_0 = \text{constant}$

$$\Rightarrow \quad v_0 = \frac{x - x_0}{t}, \qquad v_0 t = x - x_0, \qquad x = x_0 + v_0 t$$

If a ≠ 0 then $x = \underbrace{x_0 + v_0 t}_{\text{position if } a = 0} + \underbrace{\tfrac{1}{2}at^2}_{\text{how much more (a > 0) or less (a < 0) distance you go if } a \neq 0}$

End of 1D motion review.

Now, 2D Motion

$$\vec{a} = \frac{d\vec{v}}{dt} \quad \Leftrightarrow \quad a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}$$

and $$\vec{v} = \frac{d\vec{r}}{dt} \quad \Leftrightarrow \quad v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}$$

Special case: $\vec{a} = \text{constant} \quad \Rightarrow \quad a_x = \text{constant}, \quad a_y = \text{constant}$

This is exactly like the 1D motion case, except now we have separate equations for x-motion and y-motion. We can treat the x-motion and y-motion <u>separately</u>:

$$\left.\begin{array}{l} x = x_o + v_{ox}\,t + \tfrac{1}{2}a_x\,t^2 \\ v_x = v_{ox} + a_x\,t \end{array}\right\} \; X$$

$$\left.\begin{array}{l} y = y_o + v_{oy}\,t + \tfrac{1}{2}a_y\,t^2 \\ v_y = v_{oy} + a_y\,t \end{array}\right\} \; Y$$

These are the x- and y-components of the vector equations

$$\vec{r} = \vec{r}_o + \vec{v}_o\,t + \tfrac{1}{2}\vec{a}\,t^2$$

$$\vec{v} = \vec{v}_o + \vec{a}\,t$$

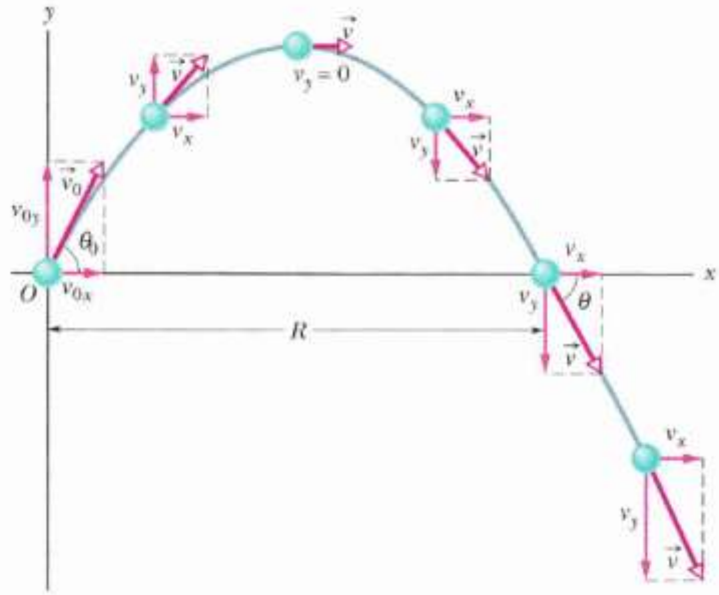
Kinematic Equations and Symbols in two dimensions

Horizontal direction	Vertical direction
$v_x = v_{ox} + a_x t$	$v_y = v_{oy} + a_y t$
$x = \frac{1}{2}(v_{ox} + v_x)t$	$y = \frac{1}{2}(v_{oy} + v_y)t$
$x = v_{ox}t + \frac{1}{2}a_x t^2$	$y = v_{oy}t + \frac{1}{2}a_y t^2$
$v_x^{\;2} = v_{ox}^{\;2} + 2a_x x$	$v_y^{\;2} = v_{oy}^{\;2} + 2a_y y$

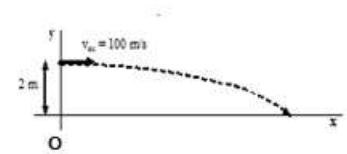
For a projectile near the surface of the earth: $a_x = 0$, v_x is constant, and $a_y = g = 10$ m/s^2.

➢ In projectile motion, the horizontal motion and the vertical motion are independent each other; that is, neither motion affects the other.

FIG. 4-10 The path of a projectile that is launched at $x_0 = 0$ and $y_0 = 0$, with an initial velocity $\vec{v}_0$. The initial velocity and the velocities at various points along its path are shown, along with their components. Note that the horizontal velocity component remains constant but the vertical velocity component changes continuously. The *range R* is the horizontal distance the projectile has traveled *when it returns to its launch height.*



Example: Horizontal Rifle. A rifle bullet is fired horizontally with v_{ox} = 100 m/s from an initial height of y_0 = 2.0 m. **Assume no air resistance.** How long (much time) is the bullet in flight? How far does the bullet go before it hits the ground?



Key idea in all projectile motion problems:

1- Treat x- and y-motions separately! The motion along the y-direction (vertical motion) is completely independent of the motion along the x-direction (horizontal motion).
2- Choose the coordinate! Here we are going to choose point O as the origin and the positive direction is upward.

Physical quantity	x-direction(horizontal motion)	y-direction (vertical motion)
Initial position	$x_0 = 0$	$y_0 = 2\,m,$
Final position	$x(\text{final}) = ?$	$y(\text{final}) = 0$
Initial velocity	$v_{ox} = 100\,m/s$	$v_{oy} = 0$
acceleration	$a_x = 0$	$a_y = -g = -9.8\,m/s^2$

The time to hit the ground is entirely controlled by the y-motion:

$$\underset{0}{y} = y_o + \underset{0}{v_{oy}}\, t - \tfrac{1}{2} g\, t^2 \quad \Rightarrow \quad 0 = y_0 - \tfrac{1}{2} g\, t^2 \;, \quad y_0 = \tfrac{1}{2} g\, t^2$$

$$2y_0 = g\, t^2 , \quad t^2 = \frac{2y_0}{g} , \quad t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2(2)}{9.8}} = \underline{0.64 \text{ s}}$$

Now we look at the x-equations to see how far along the x-direction the bullet traveled in 0.64 s.

$$a_x = 0 \quad \Rightarrow \quad v_x = \text{constant} = v_{ox} = 100 \text{ m/s}$$

$$x = \underset{0}{x_o} + v_{ox}\, t + \tfrac{1}{2} \underset{0}{a_x}\, t^2 = v_{ox}\, t = 100(0.64) = \underline{64 \text{ m}}$$

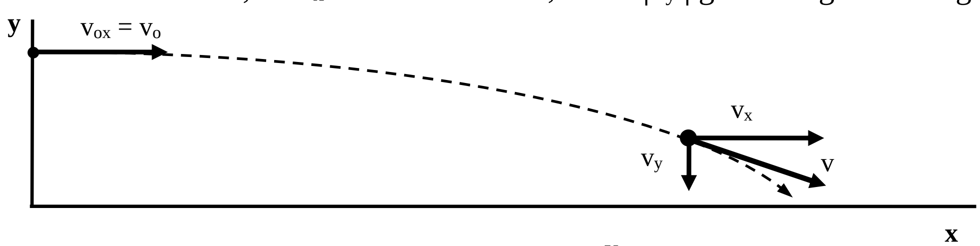
Why v_x = constant ? The force of gravity is straight down. There is no sideways force to change v_x (assuming no air resistance).

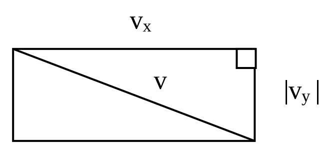
- Another question: What is the speed of the bullet as it falls?

$$v_x = \text{constant} = v_{ox} = v_o$$

$$v_y = \underset{0}{v_{oy}} + \underset{-g}{a_y}\, t = -g\, t$$

As the bullet travels, its v_x remains constant, while $|v_y|$ grows larger and larger.



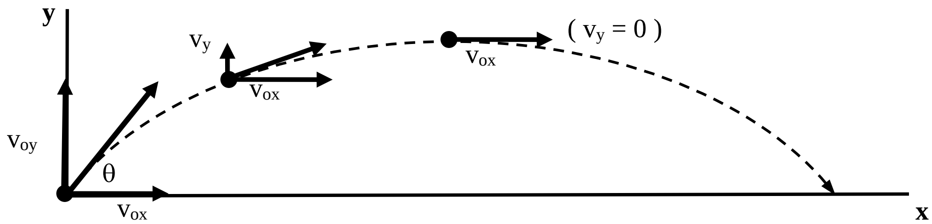


$$\text{speed} = \text{magnitude of velocity} = v = \sqrt{v_x^{\,2} + v_y^{\,2}} = \sqrt{v_{ox}^{\,2} + (g\,t)^2}$$

The speed is a minimum at t = 0 when v_y = 0 (the moment when the bullet leaves the gun). The speed is maximum when v_y is maximum, just before the bullet reaches the ground. Don't forget that we are assuming no air resistance. For a real rifle fired in real air, the bullet's speed is usually maximum when leaving the barrel, and then air resistance slows the bullet down as it travels.

Example: A projectile is fired on an airless world with initial speed v_0 at an angle θ above the horizontal. What is the minimum speed of the projectile? Answer: $v_{0x} = v_0 \cos\theta$.

Proof:
$$a_x = 0 \;\Rightarrow\; v_x = \text{constant} = v_{ox} = v_o \cos\theta$$
$$a_y = -g \;\Rightarrow\; v_y = v_{oy} - g\,t$$

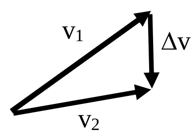


Here, the speed is minimum at the top of the trajectory, where $v_y = 0$.

Review of acceleration:

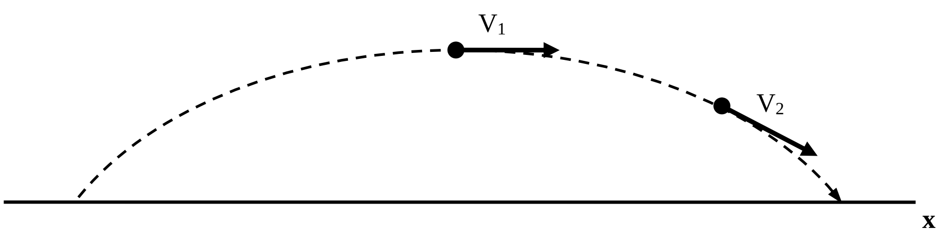
1D: $a = \dfrac{d\,v}{d\,t}$ **2D:** $\vec{a} = \dfrac{d\,\vec{v}}{d\,t} \approx \dfrac{\Delta\vec{v}}{\Delta t} = \dfrac{\vec{v}_2 - \vec{v}_1}{\Delta t}$

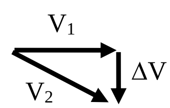
$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ means $\vec{v}_2 = \vec{v}_1 + \Delta\vec{v} \;\Rightarrow$



$\Delta\vec{v}$ is the vector you add to $\vec{v}_1$ to get $\vec{v}_2$.

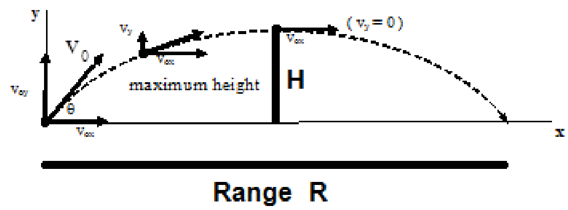
The direction of $\vec{a}$ is the same as the direction of $\Delta\vec{v}$
(since $\vec{a} = \Delta\vec{v} \times$ positive number $(1/\Delta t)$





The direction of the acceleration of gravity is the direction of Δ**V**: straight down!

The Vertical Motion



In the vertical motion, where the acceleration is constant, we have the equations:

$$y - y_0 = v_{0y}t - \tfrac{1}{2}gt^2$$
$$= (v_0 \sin\theta_0)t - \tfrac{1}{2}gt^2, \qquad (4\text{-}22)$$

where we used for the initial vertical velocity component $v_{0y} = v_0 \sin\theta$. Similarly,

$$v_y = v_0 \sin\theta_0 - gt \qquad (4\text{-}23)$$

and

$$v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0). \qquad (4\text{-}24)$$

Note that, the vertical velocity component behaves just as for a ball thrown vertically upward. It is directed upward initially, and its magnitude steadily decreases to zero, *which marks the maximum height of the path.* The vertical velocity component then reverses direction, and its magnitude becomes larger with time.

At maximum height, define $y - y_0 = H$, and use the condition $v_y^2 = 0$ at maximum height, we have the:

$$\boxed{H = \frac{v_o^2 \sin^2\theta_o}{2g}}$$

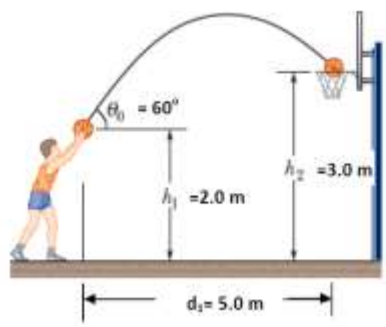
The Equation of the Path (Trajectory)

One can solve the 2-equation $x - x_0 = v_0 \cos\theta_o\, t$ and $y - y_0 = v_0 \sin\theta_o\, t - \frac{1}{2}gt^2$, to obtain:

$$y = (\tan\theta_0)x - \frac{gx^2}{2(v_0\cos\theta_0)^2} \quad \text{(trajectory).} \qquad (4\text{-}25)$$

which is similar to the equation of a parabola, so the path is *parabolic*. Note that: in driving (4-25) we used the values $y_0 = x_0 = 0$.

Example: A basketball player would like to throw a ball at an angle of $\theta_0 = 60^\circ$ above the horizontal such that the ball just goes through the center of the rim of the basket that is $h_2 = 3.0$ m high from the floor and it is at a horizontal distance of $d_1 = 5.0$ m from the player's hand (see the following figure). At the instant the ball leaves the player's hand, his hand is $h_1 = 2.0$ m above the floor. Find the magnitude of the initial velocity of the ball.



Answer:

Using the same coordinate system assumed in Eq. 4-25,

$$y = (\tan \theta_0)x - \frac{gx^2}{2(v_0 \cos \theta_0)^2} \quad \text{(trajectory).} \qquad (4\text{-}25)$$

we rearrange that equation to solve for the initial speed:

$$v_0 = \frac{x}{\cos \theta_0} \sqrt{\frac{g}{2\,(x \tan \theta_0 - y)}}$$

which yields $v_0 = 8.0$ m/s for $g = 9.8$ m/s^2, $x = 5.0$ m, $y = (3\text{-}2)$ m and $\theta_0 = 60°$.

The Horizontal Range

The *horizontal range "R"* of the projectile is the *horizontal* distance the projectile has traveled when it returns to its initial height (the height at which it is launched). To find range R, let us put $x - x_o = R$ in $x - x_0 = v_0 \cos\theta_o\ t$ and $y - y_o = 0$ in $y - y_0 = v_0 \sin\theta_o\ t - \frac{1}{2} g t^2$, obtaining

$$R = (v_0 \cos \theta_0)t$$

and

$$0 = (v_0 \sin \theta_0)t - \tfrac{1}{2}gt^2.$$

Eliminating t between these two equations yields

$$R = \frac{2v_0^2}{g} \sin \theta_0 \cos \theta_0.$$

Using the identity $\sin 2\theta_0 = 2 \sin \theta_0 \cos \theta_0$ (see Appendix E), we obtain

$$\boxed{R = \frac{v_o^2}{g}\sin(2\theta_o)} \qquad (4\text{-}26)$$

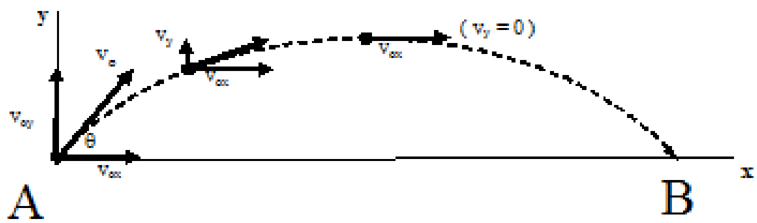
Notes:

1- The last equation does *not* give the horizontal distance traveled by a projectile when the final height is not the launch height.

2- R in Eq. 4-26 has its maximum value when sin $\sin\ 2\theta_\circ = 1$, which corresponds to $2\theta^\circ = 90^\circ$ or $\theta^\circ = 45^\circ$.

3- However, when the launch and landing heights differ, as in many sports, a launch angle of 45° does not yield the maximum horizontal distance.

Summary of the projectile motion



If a projectile is launched at point A and landed at point B, then we have:

i- Kinematic equations

$$x - x_0 = (v_0 \cos \theta_0)t, \qquad (4\text{-}21)$$

$$y - y_0 = (v_0 \sin \theta_0)t - \tfrac{1}{2}gt^2, \qquad (4\text{-}22)$$

$$v_y = v_0 \sin \theta_0 - gt, \qquad (4\text{-}23)$$

$$v_y^2 = (v_0 \sin \theta_0)^2 - 2g(y - y_0). \qquad (4\text{-}24)$$

ii- Range $\boxed{R = \frac{v_o^2}{g}\sin(2\theta_o)}$

iii- Maximum height $\boxed{H = \frac{v_o^2 \sin^2\theta_o}{2g}}$

iv- Flight time $\boxed{T = 2t_H = 2\frac{v_{oy}}{g} = 2\frac{v_o \sin\ \theta_o}{g}}$, t_H = time to reach the maximum height $(v_y = 0)$.

Example: A projectile is fired over a flat horizontal land. It takes 10 s to reach its range of $R = 100$ m. What is the speed of the projectile at the highest point of its trajectory?

Answer:

At the highest point: $x = R/2 = 50$ m, the time $t = 10/2 = 5$ s; and $v_y = 0$;

$$\therefore v_{ox} = \frac{x}{t} = \frac{50}{5} = 10 \text{ m/s}$$

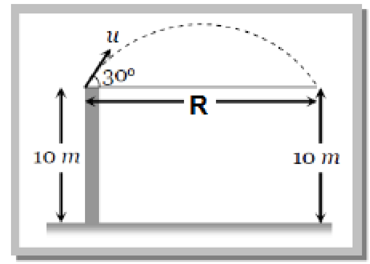
Example: The minimum speed of a projectile during the whole flight is 5.0 m/s. It takes 4.0 s to reach its horizontal range. What is the horizontal range of the projectile?

Answer:

The minimum speed of the projectile motion is at the highest point, when $v_y = 0$. So minimum speed = V_x

Horizontal range = $V_x \times t$ = 5 m/s × 4 s = 20 m

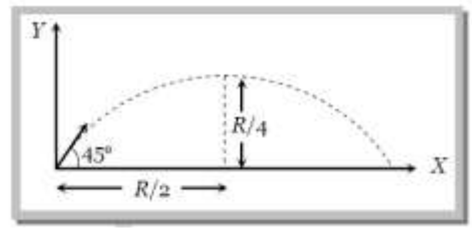
Example: A boy playing on the roof of a 10 m high building throws a ball with a speed of 10 m/s at an angle of 30° with the horizontal. How far from the throwing point will the ball be at the height of 10.0 m from the ground? (take g = 10 m/s^2)



Answer: Simply we have to calculate the range of projectile

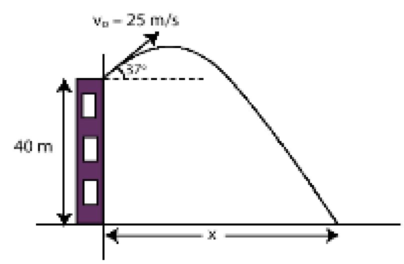
$$R = \frac{v_o^2}{g}\sin(2\theta) = \frac{(10)^2 \sin(2\times 30^o)}{10} = 5\sqrt{3} = \underline{8.66\text{ m}}.$$

Example: A projectile is thrown into space so as to have maximum horizontal range R. Taking the point of projection as origin, the co-ordinates of the point where the speed of the particle is minimum are.



Answer: For maximum horizontal Range $\theta = 45^\circ$. From $R = 4\text{H}\cot(2\theta) = 4\text{H}$. [As $\theta = 45^\circ$, for maximum range.] Speed of the particle will be minimum at the highest point of parabola. So the co-ordinate of the highest point will be $(\text{R}/2, \text{R}/4)$.

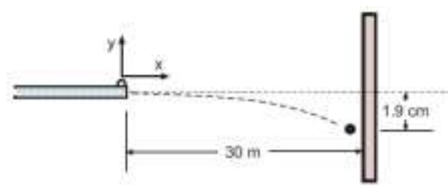
Example: A ball is kicked from the roof of a building, of height 40 m, with an initial velocity of 25 m/s at an angle of 37° to the horizontal (see Fig). How far from the base of the building will the ball land?



Answer: $v_{oy} = v_o \sin 37^\circ = 15$ m/s; and $v_{ox} = v_o \cos 37^\circ = 20$ m/s .

At the lowest point: $y = -40 \Rightarrow -40 = 15t - \frac{1}{2}(9.8)t^2 \Rightarrow t = 4.77, -1.71$

$$\therefore x = v_{ox}t = 20\times 4.77 = 95\text{ m}$$



Example: A rifle is aimed horizontally at a target 30 m away. The bullet hits the target 1.9 cm below the aiming point, as shown in the figure. What is the bullet's time of flight?

Answer: We have $x_0 = 0$ and $y_0 = 0$. We also know the acceleration:

$$a_x = 0 \text{ and } a_y = -9.80\text{ m/s}^2 = -g$$

The gun is fired horizontally so that $v_{0y} = 0$, but we don't know v_{0x}. We don't know the time of flight but we do know that when x has the value 30 m, then y has the value -1.9×10^{-2} m. With the equation:

$$y = y_o + v_{oy}t + \frac{1}{2}a_y t^2 = 0 + 0 - \frac{1}{2}gt^2 \quad \Rightarrow \quad t^2 = \frac{-2y}{g} = -\frac{2(-1.9\times 10^{-2})}{9.8} = 3.9\times 10^{-3}\text{s}^2$$

$$t = 6.2\times 10^{-2}\text{s}$$

Chapter 4 –II

4-5 UNIFORM CIRCULAR MOTION

Circular motion is another example of motion in two dimensions. To create circular motion in a body it must be given some initial velocity and a force must then act on the body which is always directed at right angles to instantaneous velocity.

A particle is in **uniform circular motion** if it travels around a circle or a circular arc at constant (*uniform*) speed. Although the speed does not vary, *the particle is accelerating* because the velocity changes in direction.

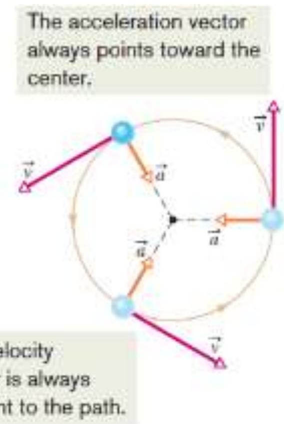


Figure 4-16 Velocity and acceleration vectors for uniform circular motion.

Notes:

1- There is a relationship between the velocity and acceleration vectors at various stages during uniform circular motion.
2- Both vectors have constant magnitude, but their directions change continuously.
3- The velocity is always directed tangent to the circle in the direction of motion.
4- The acceleration is always directed *radially inward*. Because of this, the acceleration associated with uniform circular motion is called a centripetal (meaning "center seeking") acceleration. The magnitude of this acceleration $\vec{a}$ is

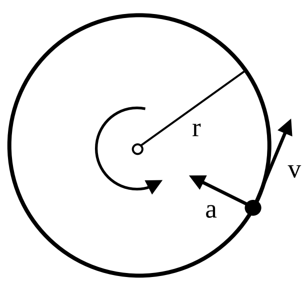
$$a = |\vec{a}| = \frac{v^2}{\mathrm{r}}$$

where r is the radius of the circle and v is the speed of the particle.
5- During this acceleration at constant speed, the particle travels the circumference of the circle (a distance of $2\pi r$) in time

$$\text{speed } v = |\vec{\mathrm{v}}| = \frac{\text{distance}}{\text{time}} = \frac{2\pi\ \mathrm{r}}{T} \qquad T = \frac{2\pi r}{v} \qquad \Rightarrow \quad a = \frac{2\pi v}{T}$$

6- T is called the *period of revolution*, or simply the *period*, of the motion. It is, in general, the time for a particle to go around a closed path exactly once. In other words T = period = time for 1 complete revolution, 1 cycle.

Example: acceleration on a merry-go-round. Radius r = 5 m , period T = 3 s



$$\mathrm{v} = \frac{2\pi\,\mathrm{r}}{T} = \frac{2\pi(5)}{3} = 10.5\ \mathrm{m/s}$$

$$\mathrm{a} = \frac{\mathrm{v}^2}{\mathrm{r}} = \frac{(10.5)^2}{5} = 22\ \mathrm{m/s^2} \times \frac{1\mathrm{g}}{9.8\,\mathrm{m/s^2}} = 2.3\ \mathrm{g's}\ !$$

A human can withstand an acceleration of about 5 g's for a few minutes or ~10 g's for a few seconds without losing consciousness.

Example: A stone is tied to a string and rotated in a circle of radius 4 m at a constant speed. If the magnitude of its acceleration is 16 m/s^2, what is the period of the motion?
Ans:

$$v = \sqrt{r.a} = \sqrt{(4.0m \times 16\, m/s2)} = 8.0\, m/s$$
$$T = 2\pi r/v = (2\pi \times 4.0m)/(\,8.0\, m/s) = \pi\ s$$

Example: A stone is tied to one end of a string 50 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 10 revolutions in 20 s, what is the magnitude of acceleration of the stone?
Answer:

$$\text{Time period} = \frac{\text{Total time}}{\text{No. of revolution}} = \frac{20}{10} = 2\,sec$$

$$\therefore a_c = \frac{4\pi^2}{T^2}.r = \frac{4\pi^2}{(2)^2}\times(1/2)m/s^2 = 4.93\,m/s^2 = 493\,cm/s^2$$

Example: An athlete completes one round of a circular track of radius 10 m in 40 sec. The distance covered by him in 2 min 20 sec is

$$\text{No. of revolution (n)} = \frac{\text{Total time of motion}}{\text{Time period}} = \frac{140\text{ s}}{40\text{ s}} = 3.5$$

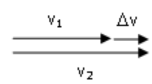
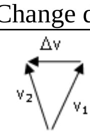
$$\text{Distance covered by an athlete in n-revolution} = n(2\pi r) = 3.5(2\times\frac{22}{7}\times 10) = 220\text{ m}$$

Example: Two cars going round curve with speeds one at 90 km/h and other at 15 km/h. Each car experiences same acceleration. The radii of curves are in the ratio of
Answer:

$$\text{Centripetal acceleration} = \frac{v_1^2}{r_1} = \frac{v_2^2}{r_2}\ \text{(given)} \qquad \therefore \frac{r_1}{r_2} = \frac{v_1^2}{v_2^2} = \left(\frac{90}{15}\right)^2 = \frac{36}{1}$$

Comments (for your information) An object moving in a circle is *accelerating*, because its velocity is *changing* -- changing direction. Recall the definition of acceleration:

$$\vec{a} \equiv \frac{d\vec{v}}{dt} \approx \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$, velocity **v** can change is two ways:

change magnitude	Change direction
	

Example: A stone is tied to a 0.50 m string and rotated at a constant speed of 2.0 m/s in a vertical circle. Its acceleration at the bottom of the circle is:

Answer: In general

$$a = \frac{v^2}{r} = \frac{2^2}{0.5} = 8 \text{ m/s}^2$$

At the bottom of the circle the acceleration will be pointed to the center, i.e. up.

Example: An object is moving in a circular path of radius π meters at constant speed of 4.0 m/s. The time required for one revolution (periodic time) is:

Answer:

$$T = \frac{\text{circumference}}{\text{speed}} = \frac{2\pi \times r}{v} = \frac{2\pi \times \pi}{4} = \frac{\pi^2}{2} \text{ s}$$

Example: If the moon makes a complete circle around the earth in 29 *days* (= 2.5 × 10^6 s) and the distance between the center of earth and the center of the moon is 3.8 x10^8 *m*, then the magnitude of centripetal acceleration on the moon is:

Answer:

$$T = \frac{2\pi r}{v} \quad \Rightarrow \quad v = \frac{2\pi r}{T}$$

$$a = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2 \times 3.8 \times 10^8}{2.5 \times 10^6}$$

$$= 2.4 \times 10^{-3} \text{ m/s}^2$$

In one model of the hydrogen atom, an electron orbits a proton in a circle of radius 5.28×10^{-11} m with a speed of $2.18 \times 10^{6}\,\frac{\mathrm{m}}{\mathrm{s}}$. (a) What is the acceleration of the electron in this model? (b) What is the period of the motion?

(a) The electron moves in a circle with constant speed. It is accelerating *toward the center of the circle* and the acceleration has magnitude $a_{\mathrm{cent}} = \frac{v^2}{r}$. Substituting the given values, we have:

$$a_{\mathrm{cent}} = \frac{v^2}{r} = \frac{(2.18 \times 10^{6}\,\frac{\mathrm{m}}{\mathrm{s}})^2}{(5.28 \times 10^{-11}\,\mathrm{m})} = 9.00 \times 10^{22}\,\frac{\mathrm{m}}{\mathrm{s}^2}$$

The acceleration has magnitude $9.00 \times 10^{22}\,\frac{\mathrm{m}}{\mathrm{s}^2}$.

(b) As the electron makes one trip around the circle of radius r, it moves a distance $2\pi r$ (the circumference of the circle). If T is the period of the motion, then the speed of the electron is given by the ratio of distance to time,

$$v = \frac{2\pi r}{T} \qquad \text{which gives...} \qquad T = \frac{2\pi r}{v}$$

which shows why Eq. 3.22 is true. Substituting the given values, we get:

$$T = \frac{2\pi(5.28 \times 10^{-11}\,\mathrm{m})}{(2.18 \times 10^{6}\,\frac{\mathrm{m}}{\mathrm{s}})} = 1.52 \times 10^{-16}\,\mathrm{s}$$

The period of the electron's motion is 1.52×10^{-16} s.

A rotating fan completes 1200 revolutions every minute. Consider a point on the tip of a blade, at a radius of 0.15 m. (a) Through what distance does the point move in one revolution? (b) What is the speed of the point? (c) What is its acceleration? (d) What is the period of the motion?

(a) As the fan makes one revolution, the point in question moves through a circle of radius 0.15 m so the distance it travels is the circumference of that circle, i.e.

$$d = 2\pi r = 2\pi(0.15\,\mathrm{m}) = 0.94\,\mathrm{m}$$

The point travels 0.94 m.

(b) If in one minute (60 s) the fan makes 1200 revolutions, the time to make *one* revolution must be

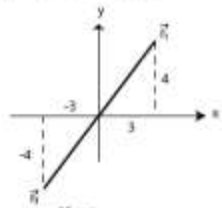
$$\text{Time for one rev} = T = \frac{1}{1200} \cdot (1.00\,\mathrm{min}) = \frac{1}{1200} \cdot (60.0\,\mathrm{s}) = 5.00 \times 10^{-2}\,\mathrm{s}$$

Using our answer from part (a), we know that the point travels 0.94 m in 5.000×10^{-2} s, moving at constant speed. Therefore that speed is:

$$v = \frac{d}{T} = \frac{0.94\,\mathrm{m}}{5.000 \times 10^{-2}\,\mathrm{s}} = 19\,\frac{\mathrm{m}}{\mathrm{s}}$$

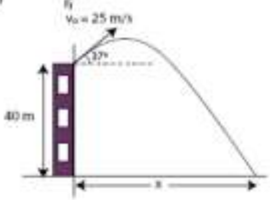
PHYS101 – Chapter 4 (Instructor: Dr. Al–Shukri)

Q1. The position of a particle is initially $\vec{r}_i = (3.0\hat{i} + 4.0\hat{j})$ m. and 10 s later it is $\vec{r}_f = (-3.0\hat{i} - 4.0\hat{j})$ m (see the Figure). What is its average velocity during this time interval ?



a. $(-0.60\hat{i} - 0.80\hat{j})$ m/s
b. $(0.60\hat{i} + 0.80\hat{j})$ m/s
c. 1.00 m/s, at angle 45°
d. 1.00 m/s, at angle -45° e. zero

Q2. A ball is kicked from the roof of a building (height = 40 m) with an initial velocity of 25 m/s at an angle of 37° to the horizontal (see the Figure). How far from the base of the building will the ball land?



a. 95 m b. 66 m c. 34 m d. 48 m e. 133 m

Q3. A satellite is placed in a circular orbit 8.0×10^3 km from the center of the earth. If it takes the satellite 2.0 hours to complete one revolution, what is its centripetal acceleration?

a. 6.1 m/s^2 towards the center of the earth
b. 6.1 m/s^2 away from the center of the earth
c. 2.4 m/s^2 toward the center of the earth
d. 2.4 m/s^2 away from the center of the earth
e. almost zero

Q4. A boat is sailing due North at a speed of 4.0 m/s with respect to the water of a river. If the water is moving due East at a speed of 3.0 m/s relative to the ground, what is the velocity of the boat relative to the ground?

a. 5.0 m/s making an angle 37° east of north
b. 5.0 m/s making an angle 53° east of north
c. 5.0 m/s east of north d. 1.0 m/s west of south

Q5. At $t = 0.0$, a particle leaves the origin with a velocity $\vec{v}_o = 12\hat{j}$ m/s. Its acceleration is given by $\vec{a} = (1.0\hat{i} - 4.0\hat{j})$ m/s^2. When the particle reaches its maximum y coordinate, the y component of its velocity is zero.

(a) Find the time the particle takes to reach the maximum y coordinate (3 s)
(b) Find the coordinates of the particle at that time. (4.5 , 18) m

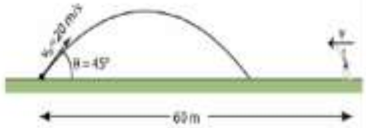
Q6. A cannon on the ground shoots out a ball at 60° to the horizontal with an initial speed of 30.0 m/s. The ball strikes a target located at a horizontal distance of 75.0 m from the cannon and at a height h above the ground.

(a) How long is the ball in the air ? (5.00 s)
(b) What is the value of h? (7.40 m)

Q7. A spacecraft is circling the moon in an orbit of radius 1800 km. The spacecraft takes 19.0 h to complete one revolution about the moon.

(a) Find the speed of the spacecraft. [595 km/h = 165 m/s]
(b) Find the centripetal acceleration of the spacecraft. [197 km/h^2 = 0.0152 m/s^2]

Q8. A ball is kicked from the ground with an initial speed of 20 m/s at an angle of 45°. A player 60 m away starts running to catch the ball at that instant (see the Figure). What must be his average speed (s) if he has to catch the ball just before it hits the ground?



a. 6.6 m/s b. 10 m/s
c. 20 m/s d. 2.0 m/s

Q9. A ball is thrown with a velocity $\vec{v}_0 = (3\hat{i} + 5\hat{j})$ m/s from the ground. Its velocity just before it strikes the ground is:

a. $\vec{v}_0 = (3\hat{i} - 5\hat{j})$ m/s b. $\vec{v}_0 = (3\hat{i} + 5\hat{j})$ m/s
c. $\vec{v}_0 = 3\hat{i}$ m/s d. $\vec{v}_0 = 5\hat{j}$ m/s
e. $\vec{v}_0 = -5\hat{j}$ m/s

Q10. The position of a particle as a function of time is given by $\vec{r} = 3.0t\hat{i} + 2.0t^2\hat{j}$. Find the angle between the velocity and acceleration of the particle at t =5.0 s.

a. 8.5° b. 0° c. 90° d. 45° e. 78°

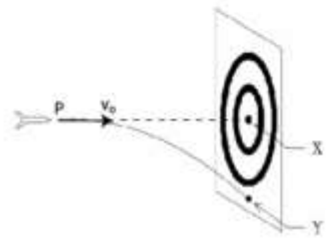
Q11. Car A is moving towards East with speed 15.0 m/s and car B is moving towards West with speed 25.0 m/s, both relative to the ground. Find the velocity of car B relative to car A.

a. 40.0 m/s towards West b. 5.00 m/s towards West
c. 5.00 m/s towards East d. 40.0 m/s towards East

Q12. A train traveling north at 20 m/s turns and then travels south at 20 m/s. The change in its velocity is:

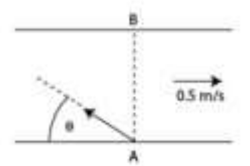
a. 40 m/s south b. 20 m/s north c. 20 m/s south
d. 40 m/s north e. 0 m/s

Q13. An arrow is shot horizontally from a point P toward X as shown in the Fig. It hits at a point Y, 0.20 s later. If the speed of the arrow at P is v_o= 11 m/s, find the distance PX.



a. 2.2 m b. 1.0 m
c. 1.8 m d. 0.3 m

Q14. A boy wishes to swim across a river from A to B. He can swim at 1.0 m/s in still water and the river is flowing at 0.50 m/s (see the Figure). At what angle θ should he be heading?

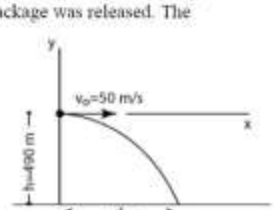


a. 60° b. 30° c. 45° d. 20°

Q15. A stone is tied to a 0.50 m string and rotated at a constant speed of 2.0 m/s in a vertical circle. Its acceleration at the bottom of the circle is:

a. 8.0 m/s2, up b. 9.8m/s2, down c. 8.0m/s2, down
d. 32 m/s2, up e. 9.8 m/s2, up

Q16. The plane shown in the Figure, is in a level flight at a height of 490 m and a speed of 50 m/s when a package was released. The horizontal distance between the release point and the point where the package strikes the ground is:



a. 500 m b. 300 m
c. 980 m d. 150 m
e. 100 m

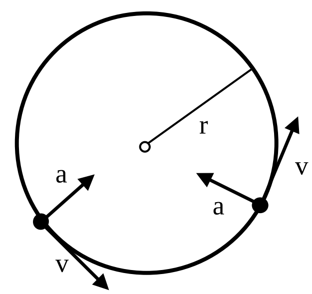
Q17. An object moves with a constant acceleration $\vec{a} = (-8.0\hat{i} + 7.0\hat{j})$ m/s^2. At $t = 0$ the velocity $\vec{v}_0 = 40\hat{i}$ m/s. The velocity at time $t = 5.0$ s is:

a. $35\hat{j}$ m/s b. $(-40\hat{i} + 35\hat{j})$ m/s c. $(40\hat{i} + 35\hat{j})$ m/s
d. $-35\hat{i}$ m/s e. $(-40\hat{i} - 35\hat{j})$ m/s f. $(40\hat{i} - 35\hat{j})$ m/s

Extra (not needed for the exam)

For circular motion with constant speed, we will show that

1) the magnitude of the acceleration is $a = |\vec{a}| \equiv \frac{v^2}{r}$



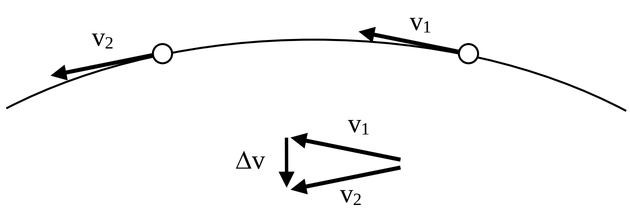
2) the direction of the acceleration is always towards the center of motion. This is centripetal acceleration. "centripetal" = "toward center" Notice that the direction of acceleration vector is always changing, therefore this is not a case of constant acceleration (so we cannot use the "constant acceleration formulas")

Is claim (1) sensible?

Check units: $\left[\frac{v^2}{r}\right] = \frac{(m/s)^2}{m} = \frac{\left(\frac{m^2}{s^2}\right)}{m} = \frac{m}{s^2} = [a]$ Yep.

Think: to get a big *a*, we must have a rapidly changing velocity. Here, we need to rapidly change the direction of vector **v** $\Rightarrow$ need to get around circle quickly $\Rightarrow$ need either large speed v or a small radius r. $\Rightarrow$ $a = v^2 / r$ makes sense. (Proof is given below.)

Is claim (2) sensible? Observe that vector $\Delta\mathbf{v}$ is toward center of circle.

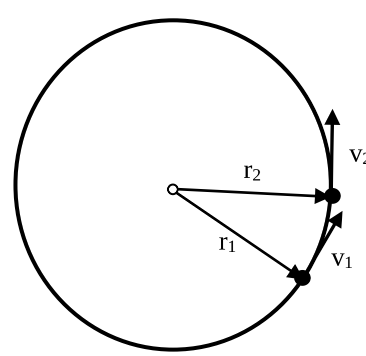


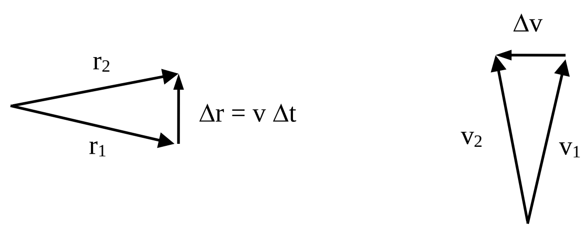
Direction of **a** = direction toward which velocity is changing

Proof of $a = v^2 / r$ for circular motion with constant speed (not required)

The proof involves geometry (similar triangles). It is mathematically simple, but subtle.

Consider the motion of a particle on a circle of radius r with constant speed v. And consider the position of the particle at two times separately by a short time interval Δt. (In the end we will take the limit as $\Delta t \to 0$.) We can draw a vector diagrams representing $\vec{r}_1 + \Delta\vec{r} = \vec{r}_2$ and $\vec{v}_1 + \Delta\vec{v} = \vec{v}_2$:





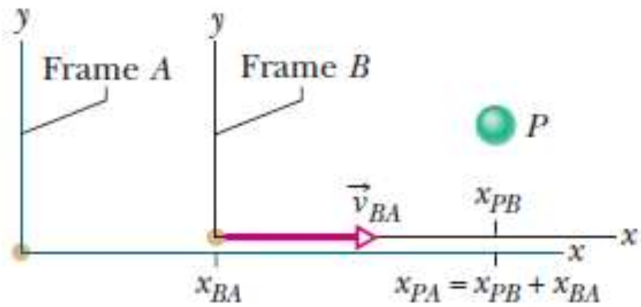
Notice that these are *similar* triangles (same angles, same length ratios). Also, note that $|\vec{r}_1| = |\vec{r}_2| = r$ and $|\vec{v}_1| = |\vec{v}_2| = v$.

Because the triangles are similar, we can write $\frac{\Delta v}{v} = \frac{\Delta r}{r} = \frac{v\,\Delta t}{r}$. A little algebra gives $\frac{\Delta v}{\Delta t} = \frac{v \cdot v}{r}$. Finally, we take the limit $\Delta t \rightarrow 0$ and get acceleration $a = \frac{v^2}{r}$.

Chapter 4 –III

4-6 RELATIVE MOTION IN ONE DIMENSION

When two frames of reference *A* and *B* are moving relative to each other at *constant velocity*, the velocity of a particle *P* as measured by an observer in frame *A* usually differs from that measured from frame *B*.



Define "**Note** how this reading is supported by the **sequence of the subscripts**":

$\vec{x}_{PA} \equiv$ the coordinate of *P* as measured by *A*,

$\vec{x}_{PB} \equiv$ the coordinate of *P* as measured by *B*,

$\vec{x}_{BA} \equiv$ the coordinate of *B* as measured by *A*,

Then

$$\vec{x}_{PA} = \vec{x}_{PB} + \vec{x}_{BA}$$

and the velocity

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$$

where

$\vec{v}_{PA} \equiv$ the velocity of *P* with respect to *A*,

$\vec{v}_{PB} \equiv$ the velocity of *P* with respect to *B*,

$\vec{v}_{BA} \equiv$ the velocity of *B* with respect to *A*,

and the acceleration

$$\vec{a}_{PA} = \vec{a}_{PB} + \vec{a}_{BA}$$

Note: if $\vec{v}_{BA}$ = constant, then a_{BA} will be zero and

$$\vec{a}_{PA} = \vec{a}_{PB}$$

Notes:

1- We will consider the frame A is stationary (not moving). So, in the problem we consider it as ground or earth.

2- Using $\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$, then the relative velocity as $\vec{v}_{BA} = \vec{v}_{PA} - \vec{v}_{PB} = \vec{v}_{A} - \vec{v}_{B}$.

For a pair of two moving objects moving uniformly, there are two values of relative velocity corresponding to two reference frames. The values differ only in sign – not in magnitude. This is clear from the example here.

Example: Two cars, standing a distance apart, start moving towards each other with speeds 1 m/s and 2 m/s along a straight road. What is the speed with which they approach each other?

Solution: Let us consider that "A" denotes Earth, "B" denotes first car and "C" denotes second car. The equation of relative velocity for this case is :

$$\Rightarrow \quad v_{CA} = v_{BA} + v_{CB}$$

Here, we need to fix a reference direction to assign sign to the velocities as they are moving opposite to each other and should have opposite signs. Let us consider that the direction of the velocity of B is in the reference direction, then

$$v_{BA} = 1\,m/s \text{ and } v_{CA} = -2\,m/s\,.$$

Now

$$v_{CA} = v_{BA} + v_{CB}$$
$$\Rightarrow \quad -2 = 1 + v_{CB}$$
$$\Rightarrow v_{CB} = -2 - 1 = -3\,m/s$$

This means that the car "C" is approaching "B" with a speed of -3 m/s along the straight road. Equivalently, it means that the car "B" is approaching "C" with a speed of 3 m/s along the straight road. We, therefore, say that the two cars approach each other with a relative speed of 3 m/s.

Example: Two cars start moving away from each other with speeds 1 m/s and 2 m/s along a straight road. What are relative velocities? Discuss the significance of their sign.

Solution: Let the cars be denoted by subscripts “1” and “2”. Let us also consider that the direction v2v2 is the positive reference direction, then relative velocities are :

Relative velocity

$$\Rightarrow \; v_{12} = v_1 - v_2 = -1 - 2 \quad = -3 m/s$$
$$\Rightarrow \; v_{21} = v_2 - v_1 = 2 - (-1) = 3 m/s$$

The sign attached to relative velocity indicates the direction of relative velocity with respect to reference direction. The directions of relative velocity are different, depending on the reference object.

However, two relative velocities with different directions mean same physical situation. Let us read the negative value first. It means that car 1 moves away from car 2 at a speed of 3 m/s in the direction opposite to that of car 2. This is exactly the physical situation. Now for positive value of relative velocity, the value reads as car 2 moves from car 1 in the direction of its own velocity. This also is exactly the physical situation. There is no contradiction as far as physical interpretation is concerned. Importantly, the magnitude of approach – whatever be the sign of relative velocity – is same.

Example: A jet cruising at a speed of 1000 km/hr ejects hot air, in the opposite direction. If the speed of hot air with respect to Jet is 800 km/hr, then find its speed with respect to ground.

Solution: Let us define: jet $\equiv B$, hot air $\equiv P$, the ground $\equiv A$, and the direction of Jet be in the + x – direction. Then, we can have the following equation:

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$$

Input: $\vec{v}_{BA} = 1000 \text{ km/hr}$, $\vec{v}_{PB} = -800 \text{ km/hr}$, $\vec{v}_{BA} = ??$,

$$\therefore \quad \vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA} = 1000 + (-800) = 200 \text{ km/hr}$$

So, the speed of the hot air with respect to ground is 200 km/hr.

Example: If two cars, at constant speeds, move towards each other, then the linear distance between them decreases at 6 km/hr. If the cars move in the same direction with same speeds, then the linear distance between them increases at 2 km/hr. Find the speeds of two cars.

Solution: Let the speeds of the cars are "u" and "v" respectively. When they move towards each other, the relative velocity between them is:

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = u - (-v) = u + v = 6 \text{ km/hr} .$$

When they are moving in the same direction, the relative velocity between them is:

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = u - v = 2 \text{ km/hr} .$$

Solving two linear equations, we have : $u = 4 \text{ km/hr}$, and $v = 2 \text{ km/hr}$

Example: Two cars *A* and *B* move along parallel paths from a common point in a given direction. If "u" and "v" be their speeds ($u > v$), then find the separation between them after time "*t*".

Solution: The relative velocity of the cars is :

$$v_{12} = v_1 - v_2 = u - v$$

The separation between the cars is:

$$x = v_{12} t = (u - v)t$$

4-7 RELATIVE MOTION IN TWO DIMENSIONS
Please read this section in your text

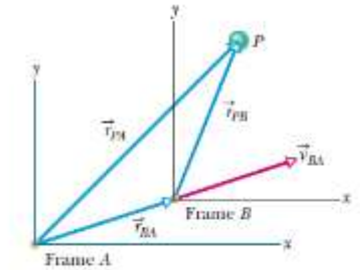
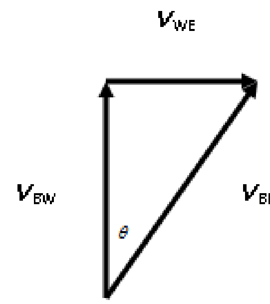


Figure 4-19 Frame B has the constant two-dimensional velocity $\vec{v}_{BA}$ relative to frame A. The position vector of B relative to A is $\vec{r}_{BA}$. The position vectors of particle P are $\vec{r}_{PA}$ relative to A and $\vec{r}_{PB}$ relative to B.

$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}. \tag{4-43}$$

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}. \tag{4-44}$$

$$\vec{a}_{PA} = \vec{a}_{PB}. \tag{4-45}$$



Example1: A boat moves at right angles to the motion of a stream at 4 m/s. If the stream is moving at 3 m/s , what is the magnitude and the direction of the boat's velocity in m/s as observed by someone standing on the shore?

Answer: This is a classic problem involving two-dimensional relative motion.

$$\vec{V}_{BE} = \vec{V}_{BW} + \vec{V}_{WE} = 4.0\hat{\text{j}} + 3\,\hat{\text{i}} \quad \Rightarrow \quad \left|\vec{V}_{BE}\right| = 5 \text{ m/s}$$

$$\theta = \tan^{-1}(\frac{3}{4}) \approx 37^{\circ}$$

Example3: An observer notes that a swimmer is swimming upstream at 0.90 m/s and a second swimmer is swimming downstream moving at 1.5 m/s . If each swimmer is actually moving at 1.2 m s with respect to the water, how fast is the stream flowing in m/s?

Answer:

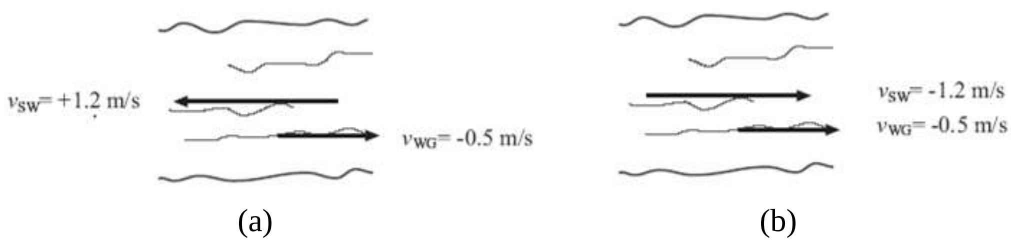
$$V_{ME} = V_{MW} + V_{WE} \quad \Rightarrow \quad 0.9\,\hat{\text{i}} = 1.2\,\hat{\text{i}} - V_{WE}\,\hat{\text{i}} \qquad (1)$$

$$1.5\,\hat{\text{i}} = 1.2\,\hat{\text{i}} + V_{WE}\,\hat{\text{i}} \qquad (2)$$

Subtracting (2) from (1) one gets:

$$0.6\,\hat{\text{i}} = 2.0 V_{WE}\,\hat{\text{i}} \quad \Rightarrow \quad \left|V_{WE}\right| = 0.3 \text{ m/s}$$

Example: A river has a steady speed of 0.500 m/s. A student swims upstream a distance of 1.00 km and returns to the starting point. If the student can swim at a speed of 1.20 m/s in still water, how long does the trip take? Compare this with the time the trip would take if the water were still.



(a) Velocities for case where swimmer swims upstream. (b) Velocities for case where swimmer swims downstream.

Answer:
What happens if the water is still? The student swims a distance of 1.00 km "upstream" at a speed of 1.20 m/s ; using the simple distance/time formula $d = vt$ the time for the trip is

$$t = \frac{d}{v} = \frac{1.0\times10^3 \text{ m}}{1.20 \text{ m/s}} = 833 \text{ s}$$

and the same is true for the trip back "downstream". So the total time for the trip is

$$833 \text{ s} + 833 \text{ s} = 1.67 \times 10^3 \text{ s} = 27.8 \text{ min}$$

Good enough, but what about the case where the water is not still? And what does that have to do with relative velocities? In the above Figure, the river is shown; it flows in the $-x$ direction. At all times, the velocity of the water with respect to the ground is

$$v_{WG} = -0.500 \text{ m/s}.$$

When the student swims upstream, as represented in the above Figure (a), his velocity with respect to the water is

$$v_{SW} = 1.20 \text{ m/s}.$$

We know this because we are given his swimming speed for still water.
Now we are interested in the student's velocity with respect to the ground, which we will call v_{SG} . It is given by the sum of his velocity with respect to the water and the water's velocity with respect to the ground:

$$v_{SG} = v_{SW} + v_{WG} = +1.20\,\tfrac{\text{m}}{\text{s}} - 0.500\,\tfrac{\text{m}}{\text{s}} = 0.70\,\tfrac{\text{m}}{\text{s}}$$

and so to cover a displacement of Δx = 1.00 km (measured along the ground!) requires a time

$$\Delta t = \frac{\Delta x}{v_{SG}} = \frac{1.00 \times 10^3 \text{ m}}{0.70\,\frac{\text{m}}{\text{s}}} = 1.43 \times 10^3 \text{ s}$$

Then the student swims downstream (Figure(b)) and his velocity with respect to the water is

$$v_{SW} = -1.20\,\tfrac{\text{m}}{\text{s}}$$

giving him a velocity with respect to the ground of

$$v_{SG} = v_{SW} + v_{WG} = -1.20\,\tfrac{\text{m}}{\text{s}} - 0.500\,\tfrac{\text{m}}{\text{s}} = 1.70\,\tfrac{\text{m}}{\text{s}}$$

so that the time to cover a displacement of $\Delta x = -1.00$km is

$$\Delta t = \frac{\Delta x}{v_{SG}} = \frac{(-1.00 \times 10^3 \text{ m})}{(-1.70\,\frac{\text{m}}{\text{s}})} = 5.88 \times 10^2 \text{ s}$$

The total time to swim upstream and then downstream is

$$\begin{aligned} t_{\text{Total}} &= t_{\text{up}} + t_{\text{down}} \\ &= 1.43 \times 10^3\,\text{s} + 5.88 \times 10^2\,\text{s} = 2.02 \times 10^3\,\text{s} = 33.6\,\text{min}\,. \end{aligned}$$

Q. A ship sails due north at 4.50 m/s relative to the ground while a boat heads northwest with a speed of 5.20 m/s relative to the ground. Find the speed of the ship relative to the boat.

A) 3.77 m/s
B) 2.39 m/s
C) 7.95 m/s
D) 1.25 m/s
E) 6.11 m/s

Ans:

$$\vec{v}_{so} = \vec{v}_{bo} + \vec{v}_{bs}$$

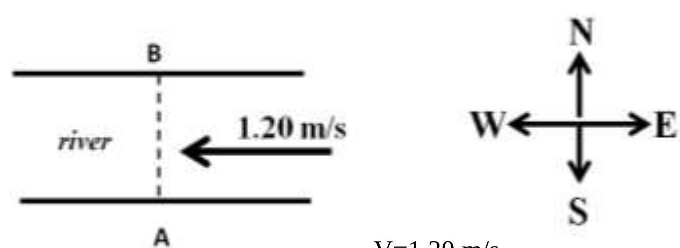
$$4.5\,\hat{j} = -3.68\,\hat{i} + 3.68\,\hat{j} + \vec{v}_{bs}$$

$$\vec{v}_{bs} = 3.68\,\hat{i} + 0.823\,\hat{j}$$

$$\vec{v}_{bs} = \sqrt{(3.68)^2 + (0.823)^2} = \mathbf{3.77\ m/s}$$

Q. A boat is to travel from point A to point B directly across a river. The water in the river flows with a velocity of 1.20 m/s toward the west, as shown in **Figure 3**. If the speed of the boat in still water is 1.85 m/s, at what angle from the north must the boat head?

Figure # 3

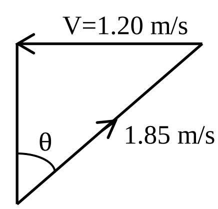


A) 40.4^0 east of north
B) 30.2^0 west of north
C) 10.5^0 east of north
D) 90.0^0 west of north
E) 55.0^0 west of north

Ans:

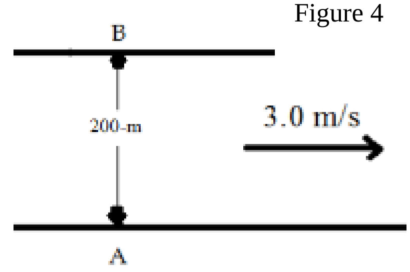
$$\theta = \sin^{-1}\left(\frac{1.2}{1.85}\right)$$

$$= 40.44° \text{ of north}$$



Q. Figure 4 shows a 200-m wide river which has a uniform flow speed of 3.0 m/s toward the east. A boat with a speed of 8.0 m/s relative to the water leaves the south bank at point A and crosses the river to point B directly north of its departure point. How long does it take the boat to cross the river?

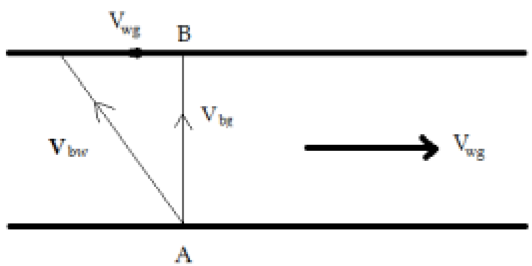
Figure 4



A) 27 s

B) 23 s
C) 25 s
D) 29 s
E) 17 s

Answer:



$$V_{bg} = \sqrt{V_{bw}^2 - V_{wg}^2} = \sqrt{8^2 - 3^2} = 7.4 \text{ m/s}$$

$$t = \frac{d}{V_{bg}} = \frac{200 \text{ m}}{7.4 \text{ m/s}} = 27 \text{ s}$$

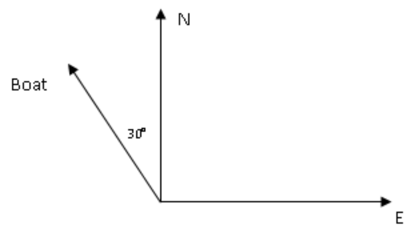
Extra Examples

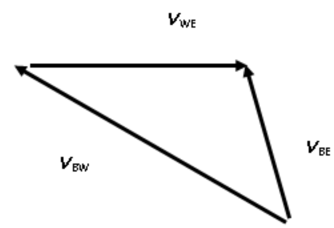
Example: A 200-m-wide river flows due east at a uniform speed of 2.0 m/s. A boat with a speed of 8.0 m/s relative to the water leaves the south bank pointed in a direction 30° west of north. What are the

a- Magnitude and

b- Direction of the boat's velocity relative to the ground?

c- How long does the boat take to cross the river?





Answer:

This is a classic problem involving two-dimensional relative motion. We align our coordinates so that *east* corresponds to +*x* and *north* corresponds to +*y*. We define the following:

$V_{BW} = -8.0\sin 30^{\circ}\,\hat{i} + 8.0\cos 30^{\circ}\,\hat{j} = -4.0\,\hat{i} + 6.9\,\hat{j}$ = speed of boat with respect to water.

$V_{WE} = 2\,\hat{i}$ = speed of water with respect to earth.

$V_{BE} = ?$ = speed of boat with respect to earth.

a- we can solve the vector addition problem for V_{BG}

$$V_{BE} = V_{BW} + V_{WE} = 2.0\,\hat{i} + \left(-4.0\,\hat{i} + 6.9\,\hat{j}\right) = -2.0\,\hat{i} + 6.9\,\hat{j}$$

Thus, we find $|V_{BE}| = 7.2$ m/s

b- The direction of V_{BE} is $\theta = \tan^{-1}\left(\frac{6.9}{-2.0}\right) = -7.38^{\circ}$ (measured clockwise from the +*x* axis), or $-7.38^{\circ} + 180^{\circ} = 106^{\circ}$ (measured counterclockwise from the +*x* axis) or 16° west of north.

c- The velocity is constant, and we apply $y - y_0 = v_y t$ in a reference frame. Thus, in the *ground* reference frame, we have 200 =7.2 sin(106°)$t \Rightarrow \; t = 29$ s . Note: if a student obtains "28 s", then the student has probably neglected to take the *y* component properly (a common mistake).

Example 3.14 Flying in a crosswind

The compass of an airplane indicates that it is headed due north, and its airspeed indicator shows that it is moving through the air at 240 km/h. If there is a wind of 100 km/h from west to east, what is the velocity of the airplane relative to the earth?

SOLUTION

IDENTIFY: This problem involves velocities in two dimensions (northward and eastward), so it is a relative velocity problem using vectors.

SET UP: We are given the magnitude and direction of the velocity of the plane (P) relative to the air (A). We are also given the magnitude and direction of the wind velocity, which is the velocity of the air (A) with respect to the earth (E):

$$\vec{v}_{P/A} = 240\ \text{km/h} \quad \text{due north}$$
$$\vec{v}_{A/E} = 100\ \text{km/h} \quad \text{due east}$$

Our target variables are the magnitude and direction of the velocity of the plane (P) relative to the earth (E), $\vec{v}_{P/E}$. We'll find these using Eq. (3.36).

EXECUTE: Using Eq. (3.36), we have

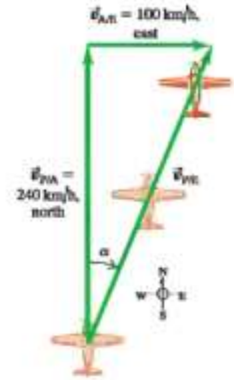
$$\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$$

Figure 3.35 shows the three relative velocities and their relationship; the unknowns are the speed $v_{P/E}$ and the angle α. From this diagram we find

$$v_{P/E} = \sqrt{(240\ \text{km/h})^2 + (100\ \text{km/h})^2} = 260\ \text{km/h}$$
$$\alpha = \arctan\left(\frac{100\ \text{km/h}}{240\ \text{km/h}}\right) = 23° \text{ E of N}$$

EVALUATE: The crosswind increases the speed of the airplane relative to the earth, but at the price of pushing the airplane off course.

3.35 The plane is pointed north, but the wind blows east, giving the resultant velocity $\vec{v}_{P/E}$ relative to the earth.



Example 3.15 Correcting for a crosswind

In Example 3.14, in what direction should the pilot head to travel due north? What will be her velocity relative to the earth? (Assume that her airspeed and the velocity of the wind are the same as in Example 3.14.)

SOLUTION

IDENTIFY: Like Example 3.14, this is a relative velocity problem with vectors.

SET UP: Figure 3.36 illustrates the situation. The vectors are arranged in accordance with the vector relative-velocity equation, Eq. (3.36):

$$\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$$

As Fig. 3.36 shows, the pilot points the nose of the airplane at an angle β into the wind to compensate for the crosswind. This angle, which tells us the direction of the vector $\vec{v}_{P/A}$ (the velocity of the airplane relative to the air), is one of our target variables. The other target variable is the speed of the airplane over the ground, which is the magnitude of the vector $\vec{v}_{P/E}$ (the velocity of the airplane relative to the earth). Here are the known and unknown quantities:

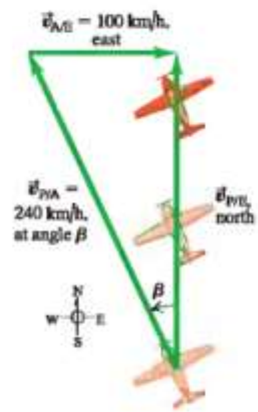
$$\vec{v}_{P/E} = \text{magnitude unknown} \quad \text{due north}$$
$$\vec{v}_{P/A} = 240\ \text{km/h} \quad \text{direction unknown}$$
$$\vec{v}_{A/E} = 100\ \text{km/h} \quad \text{due east}$$

We can solve for the unknown target variables using Fig. 3.36 and trigonometry.

EXECUTE: From the diagram, the speed $v_{P/E}$ and the angle β are given by

$$v_{P/E} = \sqrt{(240\ \text{km/h})^2 - (100\ \text{km/h})^2} = 218\ \text{km/h}^2$$
$$\beta = \arcsin\left(\frac{100\ \text{km/h}}{240\ \text{km/h}}\right) = 25°$$

3.36 The pilot must point the plane in the direction of the vector $\vec{v}_{P/A}$ to travel due north relative to the earth.



The pilot should point the airplane 25° west of north, and her ground speed is then 218 km/h.

EVALUATE: Note that there were two target variables—the magnitude of a vector and the direction of a vector—in both this example and Example 3.14. The difference is that in Example 3.14, the magnitude and direction referred to the *same* vector ($\vec{v}_{P/E}$), whereas in this example they referred to *different* vectors ($\vec{v}_{P/E}$ and $\vec{v}_{P/A}$).

It's no surprise that a headwind reduces an airplane's speed relative to the ground. This example shows that a *crosswind* also slows an airplane down—an unfortunate fact of aeronautical life.

Sample Problem 4.08 **Relative motion, two dimensional, airplanes**

In Fig. 4-20*a*, a plane moves due east while the pilot points the plane somewhat south of east, toward a steady wind that blows to the northeast. The plane has velocity $\vec{v}_{PW}$ relative to the wind, with an airspeed (speed relative to the wind) of 215 km/h, directed at angle θ south of east. The wind has velocity $\vec{v}_{WG}$ relative to the ground with speed 65.0 km/h, directed 20.0° east of north. What is the magnitude of the velocity $\vec{v}_{PG}$ of the plane relative to the ground, and what is θ?

KEY IDEAS

The situation is like the one in Fig. 4-19. Here the moving particle P is the plane, frame A is attached to the ground (call it G), and frame B is "attached" to the wind (call it W). We need a vector diagram like Fig. 4-19 but with three velocity vectors.

Calculations: First we construct a sentence that relates the three vectors shown in Fig. 4-20*b*:

$$\underset{(PG)}{\text{velocity of plane relative to ground}} = \underset{(PW)}{\text{velocity of plane relative to wind}} + \underset{(WG)}{\text{velocity of wind relative to ground.}}$$

This relation is written in vector notation as

$$\vec{v}_{PG} = \vec{v}_{PW} + \vec{v}_{WG}. \qquad (4\text{-}46)$$

We need to resolve the vectors into components on the coordinate system of Fig. 4-20*b* and then solve Eq. 4-46 axis by axis. For the y components, we find

$$v_{PG,y} = v_{PW,y} + v_{WG,y}$$

or $\quad 0 = -(215 \text{ km/h}) \sin\theta + (65.0 \text{ km/h})(\cos 20.0°).$

Solving for θ gives us

$$\theta = \sin^{-1}\frac{(65.0 \text{ km/h})(\cos 20.0°)}{215 \text{ km/h}} = 16.5°. \qquad \text{(Answer)}$$

Similarly, for the x components we find

$$v_{PG,x} = v_{PW,x} + v_{WG,x}.$$

Here, because $\vec{v}_{PG}$ is parallel to the x axis, the component $v_{PG,x}$ is equal to the magnitude v_{PG}. Substituting this notation and the value $\theta = 16.5°$, we find

$$v_{PG} = (215 \text{ km/h})(\cos 16.5°) + (65.0 \text{ km/h})(\sin 20.0°)$$
$$= 228 \text{ km/h}. \qquad \text{(Answer)}$$

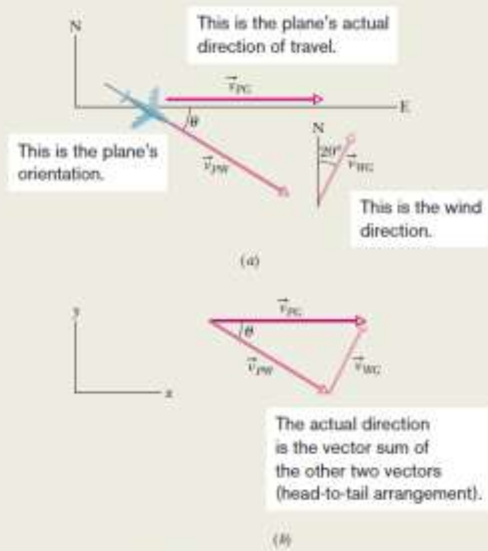


Figure 4-20 A plane flying in a wind.

Q: Wind blows with a speed of 60.0 km/h from the north towards south. A plane flies at 45.0° north of east at a speed of 200 km/h relative to the wind. The resultant speed of the plane relative to the ground is

A) 163 km/h
B) 176 km/h
C) 100 km/h
D) 143 km/h
E) 120 km/h

Ans:

$\mathrm{w} \longrightarrow \mathrm{wind};\ \ \mathrm{p} \longrightarrow \mathrm{plane}\ ;\mathrm{g}\ \longrightarrow \mathrm{ground}$

$$\vec{v}_{wg} = -6.00\,\hat{j}\ (k/h)$$

$$\vec{v}_{pw} = (200 \times \cos 45)\hat{\imath} + (200 \times \sin 45)\hat{j} = 141\,\hat{\imath} + 141\hat{j}\ (km/h)$$

$$\vec{v}_{pg} = \vec{v}_{pw} + \vec{v}_{wg} = 141\,\hat{\imath} + 81\,\hat{j}\ (km/h)$$

$$\therefore v_{pg} = [(141)^2 + (81)^2]^{\frac{1}{2}} = 163\ km/h$$

Q4. A boat takes 3 hours to travel 30 km along the river flow, then 5 hours to return to its starting point. How fast, in km/h, is the river flowing?

A) 2
B) 8
C) 6
D) 4
E) 9

Ans:

$Boat \rightarrow u; river \ \rightarrow v$

$$t_1 = \frac{d}{u+v}$$

$$u + v = \frac{d}{t_1} \rightarrow (1)$$

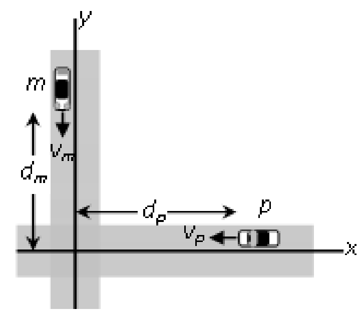
$$u - v = \frac{d}{t_2} \rightarrow -u + v = -\frac{d}{t_2} \rightarrow (2)$$

Adding (1) and (2)

$$v = \frac{d}{2}\left(\frac{1}{t_1} - \frac{1}{t_2}\right)$$

$$= \frac{30}{2}\left(\frac{1}{3} - \frac{1}{5}\right) = 2\ km/h$$

79. Two highways intersect as shown in the Figure. At the instant shown, a police car p is distance d_p = 800 m from the intersection and moving at speed v_p = 80 km/h. Motorist m is distance d_m = 600 m from the intersection and moving at speed v_m = 60 km/h.



(a) In unit-vector notation, what is the velocity of the motorist with respect to the police car?
(b) For instant shown in the Figure, what is the angle between the velocity found in (a) and the line of sight between the two cars?
(c) If the cars maintain their velocities, do the answer to (a) and (b) change as the cars move nearer the intersection?

We denote the police and the motorist with subscripts p and m, respectively. The coordinate system is indicated in the Figure.

Answer:

(a) The velocity of the motorist with respect to the police car is

$$\vec{v}_{m\,p} = \vec{v}_m - \vec{v}_p = (-60 \text{ km/h})\hat{\text{j}} - (-80 \text{ km/h})\hat{\text{i}}$$
$$= (80 \text{ km/h})\hat{\text{i}} - (60 \text{ km/h})\hat{\text{j}}.$$

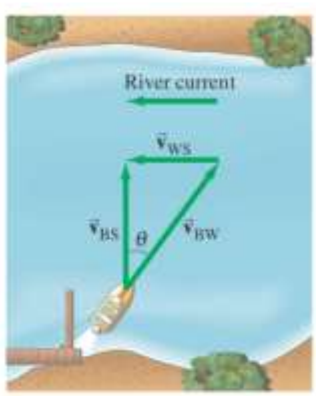
(b) $\vec{v}_{m\,p}$ does happen to be along the line of sight. Referring to the Figure, we find the vector pointing from one car to another is

$\vec{r} = (800 \text{ m})\hat{\text{i}} - (600 \text{ m})\hat{\text{j}}$ (from m to p). Since the ratio of components in $\vec{r}$ is the same as in $\vec{v}_{mp}$, they must point the same direction.

(c) No, they remain unchanged (The velocities are $\perp$ and constant).

1- A river is flowing 0.20 m/s east. A boat in this river has a speed of 0.40 m/s directed 60° south of east relative to the earth. Find the velocity of the boat relative to the river. (Ans: 0.35 m/s, south)

Answer:



$$\vec{v}_{BS} = \vec{v}_{BW} + \vec{v}_{WS}$$