Chapter 3, Part I
Vectors (Tools for Physics)

3-1 VECTORS AND THEIR COMPONENTS

A Scalar quantity: has only magnitude and is completely specified by a number and a unit, e.g. time (t = 3 s), volume (V = 4 m³), mass (m = 2 kg), density $\left(\rho = 5.3\,\text{kg/m}^3\right)$, distance, speed, amount of money. Scalars quantities of the same kind (units) are added using ordinary arithmetic, e.g. 2 s + 5 s = 7 s.

A Vector quantity: has both magnitude ("how much") and direction, e.g. displacement, velocity, acceleration, force. A vector quantity is represented by an arrow drawn to scale. The length of the arrow represents the magnitude of the quantity. The direction of the arrow represents the direction of the quantity.

Displacement	:	a train has moved 200 km to the north.
Velocity	:	a car is moving at 60 km/hr to the south.
Force	:	a man applied an upward force of 15 N to lift a package.

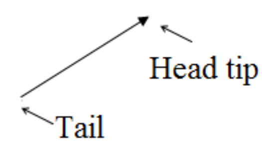
Symbols of vector quantities: are printed in bold face type (**v** = velocity, ***F*** = force), and expressed in handwriting by arrows over the letters, e.g. $\vec{\text{v}}$ and $\vec{F}$. When vector quantities are added, their directions must be taken into account.

Graphical representation of a vector:

To draw a vector, you have to draw a line with its magnitude represented by a convenient scale. The line will have tail and head.

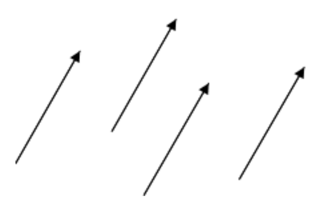
Arrows are used to represent vectors.

- The length of the arrow signifies magnitude
- The head of the arrow signifies direction



Some properties of vectors:

1. *Equality of two vectors*: Two vectors **A** and **B** are defined to be equal if they have the same magnitude and point in the same direction. Note that, any true vector can be moved parallel to itself without affecting the vector.



2. *Addition*: When two or more vectors are added together, all vectors must have the same unit.

Examples:

displacement + distance $\Rightarrow$ wrong

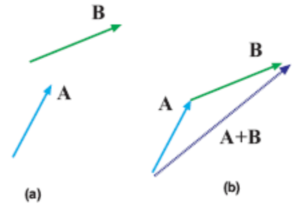
displacement + displacement $\Rightarrow$ correct

3-3 **Vectors; Vector Addition:** To add vector **B** to vector **A**, do the following steps (See figure 1.1):

1. Draw vector **A**, with its magnitude represented by a convenient scale, on graph paper.
2. Draw vector **B** to the same scale with its tail starting from the tip of **A**.
3. Connect the tail of **A** to the tip of **B**, which gives the sum **R**; **R = A + B.**
4. The sum of two (or more) vectors is often called the **resultant**.

Figure 1.1: Vector addition.

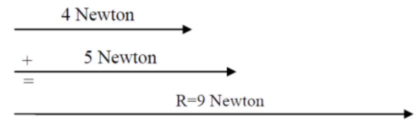
(a) shows the vectors A and B to be summed.



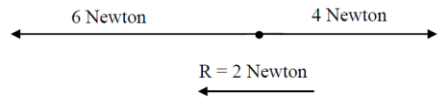
(b) shows how to perform the sum graphically.

<u>Combining Forces</u>: Two or more forces acting at a <u>point</u> can be combined into a single force known as the <u>resultant</u> force.

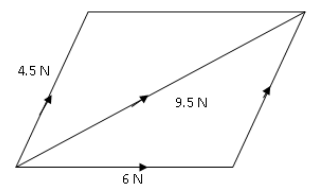
1. If the two vectors are in the same direction, the resultant is the sum of both, in the same direction. This value is the maximum we can get from these two vectors.



2. If the two vectors are in opposite directions, the resultant is the difference and acts in the direction of the larger vector. This value is the minimum we can get from these vectors.



3. Parallelogram rule for two vectors: Let two vectors be adjusted sides of a parallelogram the resultant, R, is the diagonal (magnitude and direction).

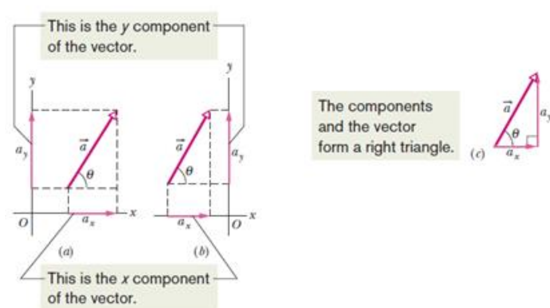


Summary of vector addition

- $\vec{B}+\vec{A}=\vec{A}+\vec{B}$ **(commutative law)**
- $\left(\vec{B}+\vec{A}\right)+\vec{C}=\vec{A}+\left(\vec{B}+\vec{C}\right)$ **(associative law)**
- $\vec{B}-\vec{A}=\vec{B}+(-\vec{A})$ **(vector subtraction)**
- $m\left(\vec{B}+\vec{A}\right)=m\vec{A}+m\vec{B}$ **(distribution law, multiplying by a constant)**

Components of Vectors

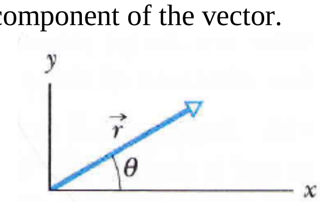
i- A **component** of a vector is the projection of the vector on an axis. The process of finding the components of a vector is called **resolving the vector.**



$$a_x = a\cos\theta \quad \text{and} \quad a_y = a\sin\theta, \qquad (3\text{-}5)$$

$$a = \sqrt{a_x^2 + a_y^2} \quad \text{and} \quad \tan\theta = \frac{a_y}{a_x} \qquad (3\text{-}6)$$

Example: A displacement vector, $\vec{r}$, in the *xy* plane is 15 m long and directed at angle $\theta = 30°$ as shown in the figure. Determine (a) the *x* component and (b) the *y* component of the vector.



Answer:

(a) With $|\vec{r}| = r = 15\text{ m}$ and $\theta = 30°$, the x component of $\vec{r}$ is given by

$$r_x = r\cos\theta = (15\text{ m})\cos 30° = 13\text{ m}.$$

(b) Similarly, the y component is given by:

$$r_y = r\sin\theta = (15\text{ m})\sin 30° = 7.5\text{ m}.$$

3-2 UNIT VECTORS, ADDING VECTORS BY COMPONENTS

A **unit vector** is a vector that has a magnitude of exactly 1 and points in a particular direction. It lacks both dimension and unit.

$$\vec{a} = a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} \qquad (3\text{-}7)$$

$$\vec{b} = b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}}. \qquad (3\text{-}8)$$

$$\Rightarrow \quad \vec{a}+\vec{b} = (a_x+b_x)\,\hat{\mathrm{i}} + (a_y+b_y)\,\hat{\mathrm{j}}$$

The unit vectors point along axes.

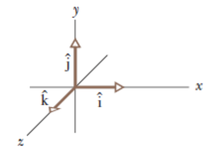


Figure 3.13 Unit vectors $\hat{\mathrm{i}}$, $\hat{\mathrm{j}}$, and $\hat{\mathrm{k}}$ define the directions of a right-handed coordinate system.

Adding Vectors by Components

Example: (a) What is the sum in unit–vector notation of the two vectors $\vec{a} = (4.0\,\hat{\mathrm{i}} + 3.0\,\hat{\mathrm{j}})$ m and $\vec{b} = (-13.0\,\hat{\mathrm{i}} + 7.0\,\hat{\mathrm{j}})$ m? (b) What are the magnitude and direction of a + b?

Answer:

(a) Summing the corresponding components of vectors **a** and **b** we find:

$$\vec{a}+\vec{b} = (4.0 - 13.0)\,\mathbf{i} + (3.0 + 7.0)\,\mathbf{j} = -9.0\,\mathbf{i} + 10.0\,\mathbf{j}$$

This is the sum of the two vectors is unit–vector form.

(b) Using our results from (a), the magnitude of **a** + **b** is

$$|\mathbf{a}+\mathbf{b}| = \sqrt{(-9.0)^2 + (10.0)^2} = 13.4$$

and if **c** = **a** + **b** points in a direction _ as measured from the positive x axis, then the tangent of θ is found from

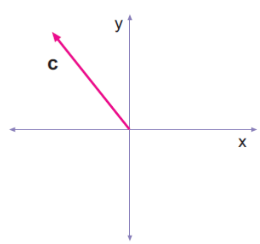
$$\tan\theta = \left(\frac{c_y}{c_x}\right) = -1.11$$

If we naively take the arctangent using a calculator, we are told:

$$\theta = \tan^{-1}(-1.11) = -48.0°$$

which is not correct because (as shown in the following figure), with c_x negative, and c_y positive, the correct angle must be in the second quadrant. The calculator was fooled because angles which differ by multiples of $180°$ have the same tangent. The direction we really want is

$$\theta = -48° + 180° = 132.0°$$



Vector **c**, With $c_x = -9.0$ and $c_y = +10.0$, the direction of **c** is in the second quadrant.

Example: A vector $\vec{A}$ is added to the sum of two vectors $\vec{B} = 3.0\hat{\mathrm{i}} - 2.0\hat{\mathrm{j}} - 2.0\hat{\mathrm{k}}$ and $\vec{C} = 2.0\,\hat{\mathrm{i}} - \hat{\mathrm{j}} + 3.0\hat{\mathrm{k}}$ such that $\vec{A}+\vec{B}+\vec{C} = \hat{\mathrm{k}}$. The vector $\vec{A}$ is:

Answer:

$$\vec{A}+\vec{B}+\vec{C} = \hat{\mathrm{k}} \quad \Rightarrow \quad \vec{A} + 3.0\hat{\mathrm{i}} - 2.0\hat{\mathrm{j}} - 2.0\hat{\mathrm{k}} + 2.0\,\hat{\mathrm{i}} - \hat{\mathrm{j}} + 3.0\hat{\mathrm{k}} = \hat{\mathrm{k}}$$

$$\Rightarrow \quad \vec{A} + 5.0\hat{\mathrm{i}} - 3\hat{\mathrm{j}} = 0$$

Example: Two vectors are given by: $\vec{a} = (4.0\hat{i} - 3.0\hat{j} + 1.0\hat{k})$ m $\vec{b} = (-1.0\hat{i} + 1.0\hat{j} + 4.0\hat{k})$ m

In unit-vector notation find (a) $\vec{a}+\vec{b}$, (b) $\vec{a}-\vec{b}$, and (c) a third vector $\vec{c}$ such that $\vec{a}-\vec{b}+\vec{c}=0$.

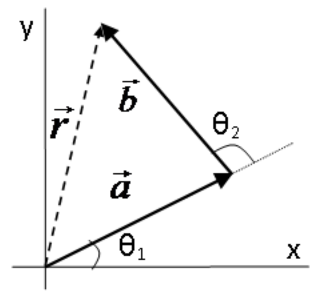
Answer:

(a) $\vec{a}+\vec{b} = [4.0+(-1.0)]\hat{i} + [(-3.0)+1.0]\hat{j} + (1.0+4.0)\hat{k} = (3.0\hat{i} - 2.0\hat{j} + 5.0\hat{k})$ m.

(b) $\vec{a}-\vec{b} = [4.0-(-1.0)]\hat{i} + [(-3.0)-1.0]\hat{j} + (1.0-4.0)\hat{k} = (5.0\hat{i} - 4.0\hat{j} - 3.0\hat{k})$ m.

(c) The requirement $\vec{a}-\vec{b}+\vec{c}=0$ leads to $\vec{c} = \vec{b} - \vec{a}$, which is the opposite of what was found in part (b). Thus,

$$\vec{c} = (-5.0\hat{i} + 4.0\hat{j} + 3.0\hat{k}) \text{ m.}$$



Example: The two vectors $\vec{a}$ and $\vec{b}$ in the Figure have equal magnitude of 10.0 m and the angles are $\theta_1 = 30°$ and $\theta_2 = 105°$. Find the (a) x and (b) y components of their vector sum $\vec{r}$, (c) the magnitude of $\vec{r}$, and (d) the angle $\vec{r}$ makes with the positive direction of the x-axis.

Answer: Using the cosine law for general triangles (and since $\vec{a}$, $\vec{b}$ and $\vec{r}$ form an isosceles triangle, the angles are easy to figure:

$$r^2 = a^2 + b^2 - 2\, a\, b \cos(180 - \theta_2).$$

Note the angle $\vec{b}$ makes with the $+x$ axis is 30° + 105° = 135° and apply the following relations:

$$a_x = a\cos(\theta),\quad a_y = a\sin(\theta),\quad |a| = \sqrt{a_x^2 + a_y^2},\quad \text{and } \vec{r} = \vec{a} + \vec{b} = (a_x + b_x)\hat{i} + (a_y + b_y)\hat{j}$$

First, find the x and y components of the vectors **a** and **b**. The vector **a** makes an angle of 30° with the $+x$ axis, so its components are:

$a_x = a \cos 30° = (10.0 \text{ m}) \cos 30° = 8.66$ m

$a_y = a \sin 30° = (10.0 \text{ m}) \sin 30° = 5.00$ m

The vector **b** makes an angle of 135° with the $+x$ axis (30° plus 105° more) so its components are:

$b_x = b \cos 135° = (10.0 \text{ m}) \cos 135° = -7.07$ m

$b_y = b \sin 135° = (10.0 \text{ m}) \sin 135° = 7.07$ m

(a) The x component of $\vec{r}$ is $r_x = (10.0 \text{ m}) \cos 30° + (10.0 \text{ m}) \cos 135° \Rightarrow r_x = 1.59$ m.

(b) The y component of $\vec{r}$ is $r_y = (10.0 \text{ m}) \sin 30° + (10.0 \text{ m}) \sin 135° \Rightarrow r_y = 12.1$ m.

(c) The magnitude of $\vec{r}$ is

$$r = |\vec{r}| = \sqrt{(1.59 \text{ m})^2 + (12.1 \text{ m})^2} = 12.2 \text{ m.}$$

(d) The angle between $\vec{r}$ and the $+x$ direction is

$$\text{Arctan } (r_y/r_x)\text{: } \tan^{-1}(r_y/r_x) = \tan^{-1}(12.1 / 1.59) = 82.5°$$

Since the components of **r** are both positive, the vector does lie in the first quadrant so that the inverse tangent operation has (this time) given the correct answer. So the direction of **r** is given by $\theta = 82.5°$.

Vectors Addition by components (in two dimensions) See Figure 1.2

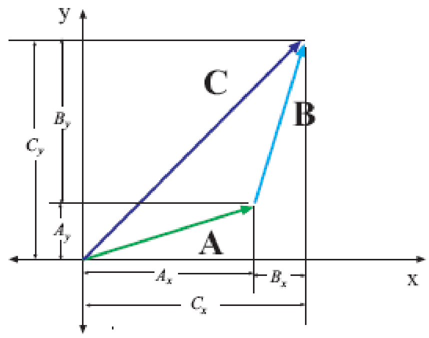


Figure 1.2: Addition of vectors by components (in two dimensions).

Any vector can be expressed as a sum of multiples of these basic vectors; for example, for the vector $\mathbf{A}$ we would write:

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}\ .$$

Here we would say that A_x is the x component of the vector $\mathbf{A}$; likewise for y and z.

In Fig. 1.2 we illustrate how we get the components for a vector which is the *sum* of two other vectors. If

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} \qquad \text{and} \qquad \mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$$

then

$$\mathbf{A} + \mathbf{B} = (A_x + B_x)\mathbf{i} + (A_y + B_y)\mathbf{j} + (A_z + B_z)\mathbf{k} \tag{1.2}$$

Once we have found the (Cartesian) component of two vectors, addition is simple; just add the *corresponding components* of the two vectors to get the components of the resultant vector.

When we multiply a vector by a scalar, the scalar multiplies each component; If $\mathbf{A}$ is a vector and n is a scalar, then

$$c\mathbf{A} = cA_x\mathbf{i} + cA_y\mathbf{j} + cA_z\mathbf{k} \tag{1.3}$$

In terms of its components, the magnitude ("length") of a vector $\mathbf{A}$ (which we write as A) is given by:

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2} \tag{1.4}$$

Many of our physics problems will be in two dimensions (x and y) and then we can also represent it in **polar** form. If $\mathbf{A}$ is a two–dimensional vector and θ as the angle that $\mathbf{A}$ makes with the $+x$ axis *measured counter-clockwise* then we can express this vector in terms of components A_x and A_y or in terms of its magnitude A and the angle θ. These descriptions are related by:

$$A_x = A\cos\theta \qquad\qquad A_y = A\sin\theta \tag{1.5}$$

$$A = \sqrt{A_x^2 + A_y^2} \qquad\qquad \tan\theta = \frac{A_y}{A_x} \tag{1.6}$$

When we use Eq. 1.6 to find θ from A_x and A_y we need to be careful because the inverse tangent operation (as done on a calculator) might give an angle in the wrong quadrant; one must think about the signs of A_x and A_y.

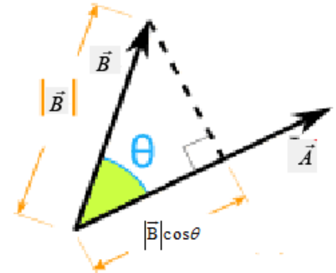
Chapter 3, Part II
Vectors (Tools for Physics)

3-3 MULTIPLYING VECTORS

Multiplying a Vector by a Vector

There are two ways to multiply a vector by a vector: one way produces a scalar (called the *scalar product*), and the other produces a new vector (called the *vector product*).

i- **Dot product** — also known as the "**scalar product**", an operation that takes two vectors and returns a scalar quantity. The dot product of two vectors, $\vec{A}$ and $\vec{B}$, can be defined as the product of the magnitudes of the two vectors and the cosine of the angle between the two vectors. Alternatively, it is defined as the product of the projection of the first vector onto the second vector and the magnitude of the second vector. Thus,



$$\vec{A}\cdot\vec{B}=|\vec{A}|\underbrace{|\vec{B}|\cos\theta}_{\text{component of }\vec{B}\text{ along }\vec{A}}$$

in which θ is the angle between the directions of $\vec{A}$ and $\vec{B}$.
In unit-vector notation and in 2-dimenstion, one finds

$$\vec{A}\cdot\vec{B}=\left(A_x\hat{i}+A_y\hat{j}\right)\cdot\left(B_x\hat{i}+B_y\hat{j}\right)=A_x\hat{i}\cdot\left(B_x\hat{i}+B_y\hat{j}\right)+A_y\hat{j}\cdot\left(B_x\hat{i}+B_y\hat{j}\right)$$
$$=A_xB_x\hat{i}\cdot\hat{i}+\cancel{A_xB_y\hat{i}\cdot\hat{j}}+\cancel{A_yB_x\hat{j}\cdot\hat{i}}+A_yB_y\hat{j}\cdot\hat{j}$$
$$=A_xB_x+A_yB_y$$

which it is expanded according to the distributive law. It is easy to generalize the above expression to 3 or more dimensions. Note that: We have been used the following identities:

$$\hat{i}\cdot\hat{i}=\hat{j}\cdot\hat{j}=\cdots=1,$$
$$\hat{i}\cdot\hat{j}=\hat{j}\cdot\hat{i}=\cdots=0$$

Dot-product facts:

$\vec{A}\cdot\vec{B}=\vec{B}\cdot\vec{A}$.

$|\vec{A}\cdot\vec{B}|=|\vec{A}||\vec{B}|$ if $\vec{A}$ and $\vec{B}$ are *parallel*, because then θ = 0º or θ = 180º degrees. This gives the *maximum magnitude.*

$|\vec{A}\cdot\vec{B}|=0$ if $\vec{A}$ and $\vec{B}$ are *perpendicular*, because then θ = 90º or θ = 270º degrees. This gives the *minimum magnitude*.

Example: Consider the vector $\vec{A}=3.0\,\hat{i}+4.0\hat{j}$. Find the vector that is perpendicular to $\vec{A}$.

Answer: We are looking for the vector $\vec{B}$ in which $\vec{A}\cdot\vec{B}=0$

$$\vec{A}\cdot\vec{B}=0\quad\Rightarrow\quad A_xB_x+A_yB_y=0\quad\Rightarrow\quad\frac{B_x}{B_y}=-\frac{A_y}{A_x}=\frac{4}{-3}\text{or}\frac{-4}{3}.$$

$$\Rightarrow \quad \vec{B} = 4.0 \ \hat{i} - 3.0 \ \hat{j}$$

Example: Two vectors are given by:

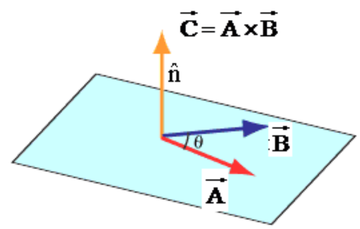
$$\vec{A} = 2.0\,\hat{i} - 4.0\,\hat{j}, \quad \text{and} \quad \vec{B} = 3.0\,\hat{i} + 4.0\,\hat{j}.$$

Find the component of $\vec{A}$ along the direction of $\vec{B}$.

Answer: **t**he component of $\vec{A}$ along the direction of $\vec{B}$ is:

$$\left|\vec{A}\right|\cos\theta = \frac{\vec{A}.\,\vec{B}}{\vec{B}} = \frac{(2)\times(3)+(-4)(4)}{\sqrt{3^2+4^2}} = -2$$

ii- Cross product — also known as the "vector product", a binary operation on two vectors that results in another vector. The cross product of two vectors in 3-space is defined as the vector perpendicular to the plane determined by the two vectors whose magnitude is the product of the magnitudes of the two vectors and the sine of the angle between the two vectors. So, if $\hat{n}$ is the unit vector perpendicular to the plane determined by vectors $\vec{A}$ and $\vec{B}$,



$$\vec{C} = \vec{A}\times\vec{B} = \left|\vec{A}\right|\times\left|\vec{B}\right|\sin\theta\ \hat{n},$$

in which θ is the angle between the vectors $\vec{A}$ and $\vec{B}$ when they are drawn tail-to-tail. $\hat{n}$ is the unit vector in the direction perpendicular to the plane that contains $\vec{A}$ and $\vec{B}$.

Magnitude: $\left|\vec{A}\times\vec{B}\right| = \left|\vec{A}\right|\left|\vec{B}\right|\sin\theta$.

Direction: The vector $\vec{C} = \vec{A}\times\vec{B}$ is perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$. Use the right-hand-rule (RHR) to find out whether it is pointing into or out of the plane.

In unit-vector:

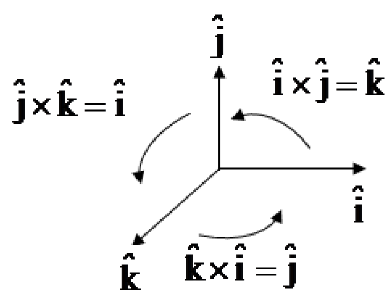
$$\vec{A}\times\vec{B} = \left(A_x\hat{i} + A_y\hat{j}\right)\times\left(B_x\hat{i} + B_y\hat{j}\right)$$

which we may expand with the distributive law.

Unit vectors multiplication:

$$\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = \hat{0},$$

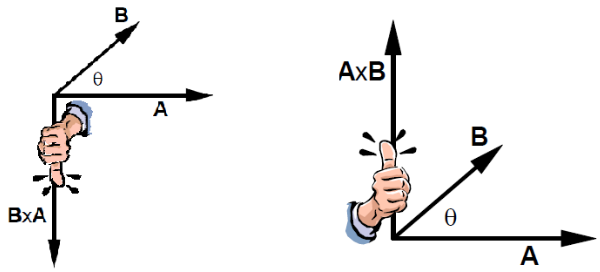
$$\hat{i}\times\hat{j} = \hat{k}, \quad \hat{j}\times\hat{k} = \hat{i}, \quad \hat{k}\times\hat{i} = \hat{j}$$



Note that:

$$\vec{A}\times\vec{B}=\left(A_x\hat{i}+A_y\hat{j}+A_z\hat{k}\right)\times\left(B_x\hat{i}+B_y\hat{j}+B_z\hat{k}\right)$$
$$=\left(A_yB_z-A_zB_y\right)\hat{i}-\left(A_xB_z-A_zB_x\right)\hat{j}+\left(A_xB_y-A_yB_x\right)\hat{k}$$
$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$

Right-hand-rule (RHR): Here's how it works. Imagine an axis going through the tails of **A** and **B**, perpendicular to the plane containing them. Grab the axis with your *right* hand so that your fingers sweep **A** into **B**. Your outstretched thumb points in the direction of **A**x**B**.



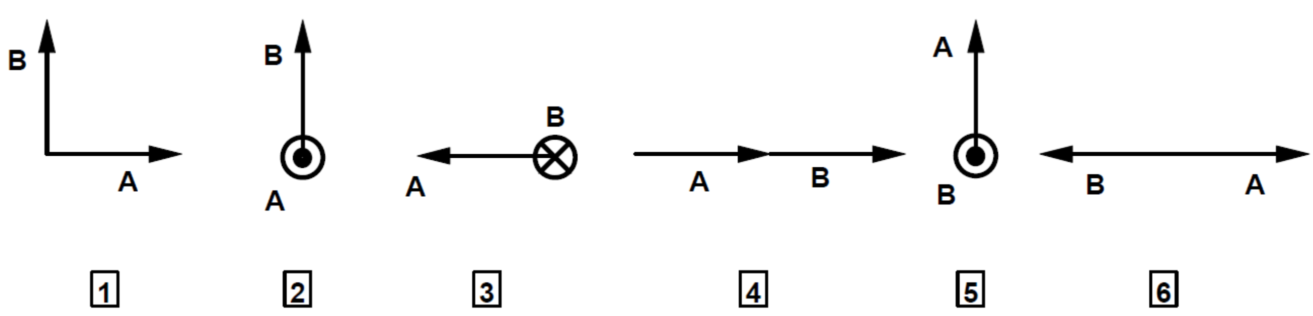
Cross-product facts:

$\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$.

$\left|\vec{A}\times\vec{B}\right|=0$ if $\vec{A}$ and $\vec{B}$ are *parallel*, because then θ = 0° or θ = 180° degrees. This gives the *minimum magnitude.*

$\left|\vec{A}\times\vec{B}\right|=\left|\vec{A}\right|\left|\vec{B}\right|$ if $\vec{A}$ and $\vec{B}$ are *perpendicular*, because then θ = 90° or θ = 270° degrees. This gives the *maximum magnitude*.

Here's a test to see if you understand how to use the RHR
In each case, decide whether **A**x**B** points up, down, left, right, into the page, or out of the page. The symbol ⊙ means a vector pointing out of the page, and ⊗ means a vector pointing into it.



Solutions to practice RHR problems on the front page. (1) out of page, (2) left, (3) down, (4) zero magnitude so the direction is undefined, (5) right, (6) zero magnitude.

Differences between the dot- and cross-products: The biggest difference, of course, is that $\vec{A}\cdot\vec{B}$ is a number and $\vec{A}\times\vec{B}$ results in a new vector. Also, when the magnitude of the dot product is a maximum, the magnitude of the cross-product is zero and *vice versa*.

Moreover, because $\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$, the dot product is proportional to:

The magnitude of $|\vec{A}|$ times the *component* of $|\vec{B}|$ that is *parallel* to $|\vec{A}|$.

On the other hand, the cross-product magnitude is given by $\vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\sin\theta$, so it is proportional to:

The magnitude of $|\vec{A}|$ times the *component* of $|\vec{B}|$ that is *perpendicular* to $|\vec{A}|$.

Example: Calculate the result of the expression $(\hat{j}\times\hat{k})\times(\hat{k}\times\hat{i})$.

Answer:

$$(\hat{j}\times\hat{k})\times(\hat{k}\times\hat{i}) = \hat{i}\times\hat{j} = \hat{k}$$

Example: Three vectors are given by: $\vec{a} = (3.0\hat{i} + 3.0\hat{j} - 2.0\hat{k})$, $\vec{b} = (-1.0\hat{i} - 4.0\hat{j} + 2.0\hat{k})$ and $\vec{c} = (2.0\hat{i} + 2.0\hat{j} + 1.0\hat{k})$. Find

(a) $\vec{a}\cdot(\vec{b}\times\vec{c})$, (b) $\vec{a}\cdot(\vec{b}+\vec{c})$, and (c) $\vec{a}\times(\vec{b}+\vec{c})$

Answer: Applying definitions of addition, dot and cross products of vectors:

$$\vec{a}+\vec{b} = (a_x+b_x)\hat{i} + (a_y+b_y)\hat{j} + (a_z+b_z)\hat{k}$$

$$\vec{a}\cdot\vec{b} = a_xb_x + a_yb_y + a_zb_z$$

$$\vec{a}\times\vec{b} = (a_yb_z + a_zb_y)\hat{i} + (a_zb_x - a_xb_z)\hat{j} + (a_xb_y + a_yb_x)\hat{k}$$

a- We note that $\vec{b}\times\vec{c} = -8.0\hat{i} + 5.0\hat{j} + 6.0\hat{k}$. Thus,

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = (3.0)(-8.0) + (3.0)(5.0) + (-2.0)(6.0) = -21.$$

b- We note that $\vec{b}+\vec{c} = 1.0\hat{i} - 2.0\hat{j} + 3.0\hat{k}$. Thus,

$$\vec{a}\cdot(\vec{b}+\vec{c}) = (3.0)(1.0) + (3.0)(-2.0) + (-2.0)(3.0) = -9.0.$$

c- Finally,

$$\vec{a}\times(\vec{b}+\vec{c}) = [(3.0)(3.0)-(-2.0)(-2.0)]\,\hat{i} + [(-2.0)(1.0)-(3.0)(3.0)]\,\hat{j}$$
$$+[(3.0)(-2.0)-(3.0)(1.0)]\,\hat{k}$$
$$= 5\hat{i} - 11\hat{j} - 9\hat{k}$$

Summary of the some rules

- $\vec{B}+\vec{A}=\vec{A}+\vec{B}$
- $\left(\vec{B}+\vec{A}\right)+\vec{C}=\vec{A}+\left(\vec{B}+\vec{C}\right)$
- $\vec{B}-\vec{A}=\vec{B}+(-\vec{A})$
- $m\left(\vec{B}+\vec{A}\right)=m\vec{A}+m\vec{B}$
- $\vec{B}\cdot\vec{A}=\vec{A}\cdot\vec{B} = \left|\vec{B}\right|\left|\vec{A}\right|\cos\theta = A_x B_x + A_y B_y + A_z B_z$
- The component of $\vec{A}$ in the direction of $\vec{B}$ = $\left|\vec{A}\right|\cos\theta$ = $\dfrac{\vec{A}\cdot\vec{B}}{\left|\vec{B}\right|}$
- $\vec{C}\cdot\left(\vec{B}+\vec{A}\right)=\vec{C}\cdot\vec{A}+\vec{C}\cdot\vec{B}$
- $\vec{B}\times\vec{A}=-\vec{A}\times\vec{B} = \left|\vec{B}\right|\left|\vec{A}\right|\sin\theta\,\hat{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$
- $\left|\vec{A}\times\vec{B}\right|$ = area of the parallelogram generated by $\vec{A}$ and $\vec{B}$

Exam's problems: **PHYS101** – Chapter 3 (Instructor: Dr. Al–Shukri)

1. Vector **A** = (5.0***i*** + 3.0**j**) m, and vector **B** is 6 m in length and making 120° with +ve x–axis. Find A–B.

a. (8.0***i*** – 2.2***j***) m b. (8.0***i*** + 8.2***j***) m
c. (–2.0***i*** + 8.2***j***) m d. (2.0***i*** – 5.6***j***) m

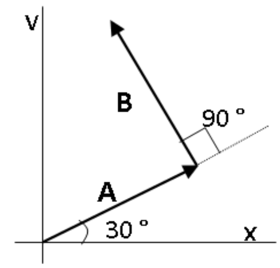
2. If **a** = (3.0***i*** + 4.0***j***) m and **b** = (5.0***i*** –2.0***j***) m, find the angle between the two vectors.

a. 75 ° b. 31 ° c. 82 ° d. 55 ° e. 93 °

3. For the following three vectors; **A** = 2***i***+3***j***+4***k***, **B** = 4***i***+4***j*** and **C** = 2***i***+2***k***, find **A.(B×A)**.

a. 0 b. –16***i*** +16***j*** –8***k*** c. 16***i*** –16***j*** +8***k***
d. 8***i*** – 8***j*** –8***k*** e. –8***i*** + 8***j*** +8***k***

4. The two vectors **A** and **B** shown in the Figure have equal magnitudes of 10.0 m. Find the magnitude of the resultant of these vectors and the angle it makes with the positive x-axis.



a. 14.1 m, 75 ° b. 10.0 m, 90 °
c. 12.0 m, 60 ° d. 16.0 m, 30 °
e. 20.0 m, 45 °

5. A vector in the xy-plane has a magnitude of 25.0 and an x-component of 12.0. The angle that it makes with the positive x-axis is:

a. 61.3 ° b. 25.6 ° c. 28.7 ° d. 64.3 °

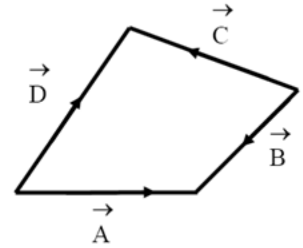
6. The unit vectors in the positive directions of the x, y, and z axes are labeled ***i***, ***j***, and ***k***. The value of ***i*.(*j* × *k*)** is:

a. +1 b. –1 c. 0 d. –i e. +j

7. Unit vectors ***i***, ***j***, ***k*** have magnitudes of unity and are directed in the +ve directions of the x, y, z axes. The value of ***k***.(***k*** ×***i***) is:

a. 0 b. –1 c. +1 d. i e. j

8. The Figure shows four vectors **A**, **B**, **C**, **D**. Which one of the following statements is correct?



a. C = **D** + **B** – **A**
b. **C** = **A** + **B** + **D**
c. **C** = –**D** – **B** + **A**
d. **C** = **A** – **B** + **D**
e. **C** = –**A** – **B** – **D**

9. If we have two vectors **A** = (a $\boldsymbol{i}$ – 2 $\boldsymbol{j}$) and **B** = (2 $\boldsymbol{i}$ + 3 $\boldsymbol{j}$) such that **A.B** = 4, find the value of a.

a. 5 b. 4 c. 0 d. –5 e. –4

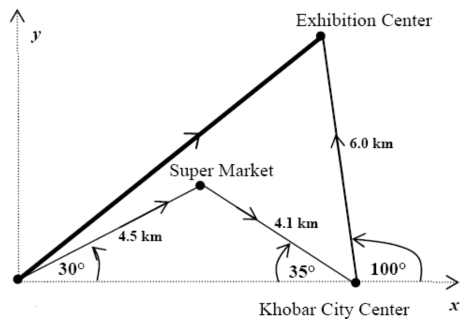
10. Which of the following is NOT a unit vector?

a. ½ ($\boldsymbol{i}$ + $\boldsymbol{j}$) b. vector **a** / |a| c. $\boldsymbol{j} \times \boldsymbol{i}$
d. $(\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}) / \sqrt{3}$ e. 0.6 $\boldsymbol{j}$ + 0.8 $\boldsymbol{k}$

11. What is the angle between the two vectors **A** = ($\boldsymbol{i}$ –2$\boldsymbol{j}$ +2$\boldsymbol{k}$) and **B** = (–2$\boldsymbol{i}$ +$\boldsymbol{j}$ +2$\boldsymbol{k}$) ?

a. 90 ° b. 30 ° c. 45 ° d. 60 ° e. 0 °

12. A student makes the journey from KFUPM to a Super Market and then to Khobar City Center and finally to Exhibition Center. The magnitude and the direction of each of these displacements are indicated in the Figure. Give the resultant displacement from KFUPM to the Exhibition Center in unit vector notation.

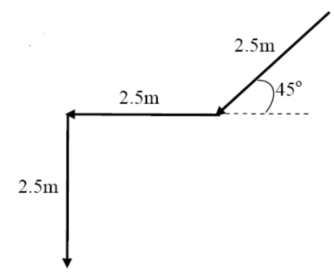


a. (6.2 $\boldsymbol{i}$ + 5.8 $\boldsymbol{j}$) km b. (–0.5$\boldsymbol{i}$ +12.1 $\boldsymbol{j}$) km
c. (5.2$\boldsymbol{i}$ + 5.8 $\boldsymbol{j}$) km d. (13.2$\boldsymbol{i}$ +12.1 $\boldsymbol{j}$) km
e. (9.1$\boldsymbol{i}$ + 8.7 $\boldsymbol{j}$) km

13. The angle between the two vectors **A** = 2 $\boldsymbol{i}$ + 4 $\boldsymbol{j}$ and **B** = 4 $\boldsymbol{i}$ – 2 $\boldsymbol{j}$ is:

a. 90 ° b. 27 ° c. 39 ° d. 180 °

14. As shown in the Figure, a block moves down on a 45° inclined plane of 2.5 m length, then horizontally for another 2.5 m, and then falls down vertically a height of 2.5 m. Find the magnitude and direction of the resultant displacement vector of the block.



a. 6.0 m and 45 ° below horizontal axis
b. 3.5 m and 30 ° degrees below horizontal axis
c. 6.0 m and 30 ° below horizontal axis
d. 3.5 m and 45 ° below horizontal axis
e. 5.5 m and 60 ° below horizontal axis

15. Given the vectors **A** = 3 $\boldsymbol{j}$ + 6 $\boldsymbol{k}$, **B** = 15 $\boldsymbol{i}$ + 21 $\boldsymbol{k}$. Find the magnitude of vector **C** that satisfies equation 2**A** + 3**C** - **B** = 0.

a. 6.16 b. 5.48 c. 18.5 d. 6.71

16. Two vectors are given as: **A** = –3.0 $\boldsymbol{i}$ + 5.0 $\boldsymbol{j}$ + 4.0 $\boldsymbol{k}$ and **B** = 4.0 $\boldsymbol{i}$ + 5.0 $\boldsymbol{j}$ + 3.0 $\boldsymbol{k}$, where $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are the unit vectors in the positive x, y and z directions. Find the angle between the vectors **A** and **B**.

a. 60 ° b. 45 ° c. 150 ° d. 30 ° e. 90 °

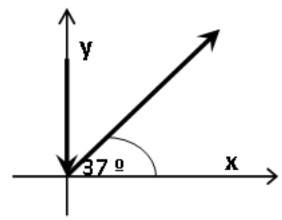
17. In the cross product **F** = **v** × **B**, take **v** = 2.0 ***i***, F = 6.0 ***j*** and the x-component of vector **B** equals zero. What then is **B** in unit-vector notation?

a. –3.0 ***k*** b. 3.0 ***k*** c. 2.0 ***j*** + 6.0 ***k***
d. 2.0 ***j*** – 6.0 ***k*** e. –2.0 ***j*** + 6.0 ***k***

18. Two displacement vectors **A** and **B** have equal magnitudes of 10 m. Vector **A** is along the +y axis and vector **B** makes 45 ° counterclockwise with +x axis. Find the vector **C** such that **B** + **C** = 2**A** .

a. **C** = –7 ***i*** + 13 ***j*** b. **C** = –7 ***i*** + 3 ***j***
c. **C** = 7 ***i*** + 13 ***j*** d. **C** = 7 ***i*** + 3 ***j*** e. **C** = 7 ***i*** + 27 ***j***

19. An object is displaced initially by -30 ***j*** m then by 50 m in a direction making an angle of 37 ° with +x axis (see the Figure). What is the resultant displacement?



a. (40***i***) m b. (40***i*** +30***j***) m
c. (–40***i*** –60***j***) m d. (– 40***i***) m e. (–40***i*** –30***j***) m

Ans:
$$\boldsymbol{R} = \boldsymbol{A} + \boldsymbol{B} = -30\,\boldsymbol{j} + 50\cos(37^o)\boldsymbol{i} + 50\sin(37^o)\,\boldsymbol{j}$$
$$= -30\,\boldsymbol{j} + 40\boldsymbol{i} + 30\,\boldsymbol{j} = 40\boldsymbol{i}$$

20. Vector **A** has a magnitude of 40.0 cm and is directed 60.0 degrees above the negative x-axis. Vector **B** has magnitude of 20.0 cm and is directed along the positive x-axis. Find the resultant vector (***i*** and ***j*** are unit vectors along positive x and y axes, respectively).

a. 34.6 ***j*** cm b. 34.6 ***i*** cm c. 20.0 ***i*** cm
d. 20.0 ***j*** cm e. (20.0 ***i*** + 34.6 ***j***) cm

21. Consider two vectors **A** = (3 ***i*** + 4 ***j***) cm and **B** = (- 4 ***i*** + 3 **j**) cm. Find the angle between these two vectors.

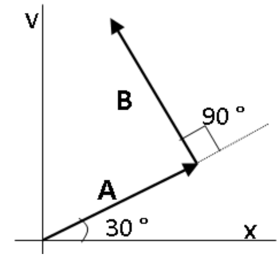
a. 90 ° b. 45 ° c. 120 ° d. 0 ° e. 25 °

22. If vector **A** is added to vector **B**, the result is (6 ***i*** + 1***j***) m. If **A** is subtracted from **B**, the result is (- 4 ***i*** + 7 ***j***) m. Find the magnitude of **B**.

a. 4 m. b. 8 m. c. 2 m. d. 1 m. e. 9 m.

1- Vector **A** = (5.0***i*** + 3.0**j**) m, and vector **B** is 6 m in length and making 120° with +ve x–axis. Find A–B.

2- If **a** = (3.0***i*** + 4.0***j***) m and **b** = (5.0***i*** –2.0***j***) m, find the angle between the two vectors.



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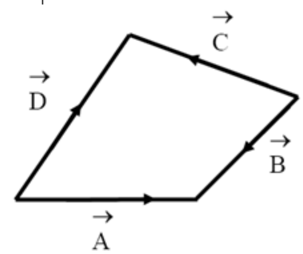

a. **C** = **D** + **B** – **A**
b. **C** = **A** + **B** + **D**
c. **C** = –**D** – **B** + **A**
d. **C** = **A** – **B** + **D**
e. **C** = –**A** – **B** – **D**

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7- Which of the following is NOT a unit vector?

a. ½ (***i*** + ***j***) b. vector **a** / |a| c. ***j*** × ***i***
d. (***i*** + ***j*** + ***k***) / √3 e. 0.6 ***j*** + 0.8 ***k***

8- What is the angle between the two vectors **A** = (***i*** –2***j*** +2***k***) and **B** = (–2***i*** +***j*** +2***k***) ?

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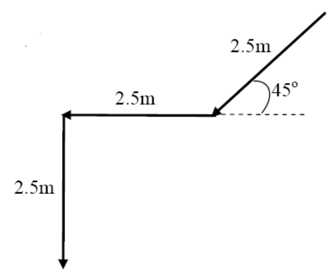

10- Given the vectors **A** = 3 ***j*** + 6 ***k*** , **B** = 15 ***i*** + 21 ***k***. Find the magnitude of vector **C** that satisfies equation 2**A** + 3**C** - **B** = 0.

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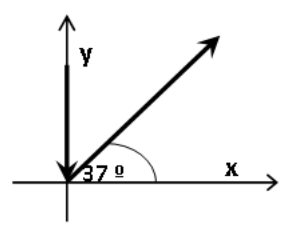

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15- Consider two vectors **A** = (3 ***i*** + 4 ***j***) cm and **B** = (- 4 ***i*** + 3 **j**) cm. Find the angle between these two vectors.

16- If vector **A** is added to vector **B**, the result is (6 ***i*** + 1***j***) m. If **A** is subtracted from **B**, the result is (- 4 ***i*** + 7 ***j***) m. Find the magnitude of **B**.